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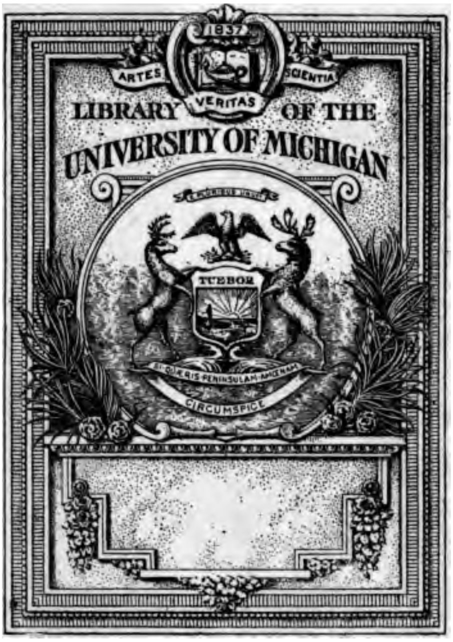
INDUSTRIAL MATHEMATICS

HORACE WILMER MARSH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CONSTRUCTIVE TEXT-BOOK
OF
PRACTICAL MATHEMATICS

BY

HORACE WILMER MARSH

Head of Department of Mathematics, School of Science and Technology, Pratt Institute

WITH THE COLLABORATION OF

ANNIE GRISWOLD FORDYCE MARSH

VOLUME I
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PREFACE

THIS book has been written for the use of those who are employed or anticipate employment in technical industries, who desire mathematical preparation for technical courses, or who are taking industrial courses of study. It has been "tried out" in mimeographed form for seven years in both day and evening classes, which now enroll upward of two hundred students. During this time the book has grown from a small volume containing mostly industrial problems with little descriptive and explanatory matter to a volume which provides a wealth of mathematical and technical material with thousands of examples and applied problems.

The aim of the author has been to make the student somewhat proficient in the various mathematical operations and their employment in computation from numerous technical formulas with actual *commercial data.*

The book therefore gives the solution of equations and triangles, and their use in arithmetic, logarithmic, and slide-rule computation. The chapters covering these subjects are not abridged from more comprehensive works, but have been especially written in simple, direct form for this book. Throughout, the attempt has been made to present the subject matter in such form that the student with little knowledge of mathematics shall find the book self instructive.

For the data on which the problems are based, the author is indebted to engineering handbooks, periodicals,

326045

and books on the various subjects represented. Among the handbooks of special value are Foster and Kent, and among the periodicals are Machinery, American Machinist, and Engineering Magazine.

The drawings for the cuts were made by the students in the author's classes.

The four-place tables have been prepared from the new tables of Bauschinger and Peters, each reading being carefully checked with Loomis and Bruhn. The arrangement of the four-place trigonometric function table was suggested by Vega's seven-place tables.

The table of the weights of the materials of construction has been compiled from many sources, the range of specific gravities being from the Cambria Handbook.

The text is designed to furnish the mathematics which is indispensible to the shop foreman or master mechanic, to the pursuit of industrial studies, the use of an engineering handbook, and the intelligent reading of technical periodicals and other publications regarding the industries in which one is seeking advancement or information.

HORACE WILMER MARSH.

BROOKLYN, N. Y., August 19, 1912.

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INDUSTRIAL MATHEMATICS

THE WORK-BOOK

NOTE.—The pages immediately following contain the detailed instructions which govern the use of the mathematics' work-book in Pratt Institute. In schools where written work does not seem feasible, these pages may not be of service. They are inserted here in the hope that they may suggest ways of making the teaching of mathematics of the same significance as shop and laboratory work, and at the same time pleasurable and useful both to student and teacher.

INDUSTRIAL MATHEMATICS

THE WORK-BOOK

1. Description. The constructive form of this text, requires that each student shall prepare the work daily in accordance with the suggestions, questions, and directions in numerous developing exercises. These exercises force a student not only to do his own thinking but to express his thought in written form.

For this purpose the mathematics work-book is used, which both in size and form is the result of fifteen years of experiment with over 2000 students. It consists of the daily record sheet, described in a subsequent paragraph, and 200 removable sheets of 16-pound, unruled, linen paper measuring $5\frac{1}{2} \times 8\frac{1}{2}$ inches, with fasteners for attaching the sheets to the back cover and the student's written work to the front cover.

2. Instruments. In order to prepare the work in a satisfactory manner and to secure the greatest educational benefit, each student will require the following equipment:

blotter,
12-inch triangular scale with U.S. and metric graduations,
medium lead pencil,
ink and pencil erasers,
fountain pen,
ruling pen,

protractor,
compasses,
red, black, and India ink,
slide-rule.

3. Value of Careful Work. It is obvious that skill is never acquired by careless, indifferent effort. Therefore, as in manual-training, the desideratum in every exercise, example, and problem in this text, is perfection in the finished work.

The instructions in the two subsequent paragraphs and throughout the text, have accordingly been written to help each student to a clear vision of an enlarging ideal of beauty and efficiency, and to an increasing ability and enthusiasm for its realization.

4. Instructions for Work-book Entries. (1) What to do First. On the inside of the front cover of the work-book, write your full name and home and rooming address, and name of school and course.

Attach all record sheets except one, to the back cover, under the blank sheets.

(2) Use of Ink. With the exception of the drawing, all work is to be done directly with pen and ink, whether in the classroom or outside.

(3) Date, and Page Number. Enter date on which work is prepared, in the upper right corner of the page about one and one-half inches from the top.

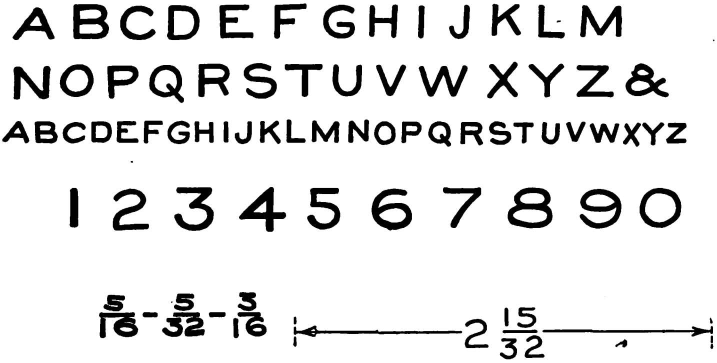
Number each page in the lower right corner when finished.

(4) Headings. Enter all work under the same heading as in the text or as otherwise specified.

Begin paragraph numbers and headings about one-half inch from the left margin.

(5) Lettering. The greater legibility of lettered headings and the practical value of the ability to letter neatly and rapidly, justify the requirement that title pages, chapter, section, and paragraph headings, and problem titles, shall

be lettered. The best style of letter for this purpose, because the simplest and the most easily made, is the "Engineering News" alphabet, shown below and used quite generally in drafting rooms throughout the country.



The only principle involved in learning to use this alphabet, is that each letter consists of straight lines, or arcs of circles, or both, and that the width and height are the same.

Be sure to observe that W is not an inverted M, nor M an inverted W. Observe also particularly how R and G are made.

(6) Title-pages. Letter title-pages in India ink without punctuation, for the subject and for each chapter, corresponding to the title-pages in the text. Insert these in the work-book preceded by a blank sheet.

(7) Spacing. Indicate a new topic both by heading and by extra space. Keep all work in straight lines with no irregular spacing between words or lines.

If straight lines are difficult without a guide, rule a page of the work-book in India ink with lines from three-sixteenths to one-fourth of an inch apart, and place it under the page when writing.

Examples and problems are best separated by extra space only, but if preferred the separation may be emphasized by a hair-line not over two inches long, lightly drawn with straight-edge and ruling pen.

(8) Drawing. Drawings which are to be measured, are to be drawn lightly and carefully with pencil and straightedge, and after measurement, are to be inked in with India ink and a ruling pen. Other drawings may be done directly in India ink.

As soon as possible learn to draw a light, smooth, draftsman's line.

(9) Symbols. Make parentheses, equality signs, and other symbols carefully: parentheses with regular curves of the same height as the quantities enclosed, the lines of the equality sign exactly the same length and about one-eighth of an inch long.

Learn to make comparatively small, neat figures and to draw free-hand, smooth, light, straight lines when performing the four fundamental operations.

Work slowly, seriously, and steadily, and thereby become expert by avoiding careless mistakes.

5. Instructions for the Record Sheet. One dozen daily record sheets, a year's supply, are furnished with the workbook. These have columns for the instructor's stamp and for the daily entry by the student, of date, paragraph and problem numbers, and number of hours spent in outside preparation of studies.

Submit work for inspection as follows: On the first day prepare a record sheet by filling in the blanks as indicated. At the top of the time columns letter the names of the studies in which outside preparation is required, as Math, Phys, C L (chemistry laboratory), etc.

Observe that while the record sheet is for mathematics only, it is a time sheet for all studies.

Whenever work is to be submitted, attach it to the record sheet on which fill in the entries denoted by the column headings, making no entries in the remarks column.

When an additional record sheet is needed, place it on top of those already filled.

6. Excuse for Non-performance and Absence. If unable to do assigned work, give to the instructor at the beginning of the period, a written excuse with date, exact reason for failure, and signature. Make the usual record sheet entries and write the word " Excuse " in the remarks column.

In case of *absence* make the usual entries for each day's absence and write " Absent " in the remarks column.

7. Collection and Distribution of Work-books. (*a*) Collection. On the stroke of the bell at the beginning of the period, each student will pass his work-book along the row in reverse order from which the chairs or desks are numbered, placing it on *top* of the books passed to him.

The student receiving the books in the last row, will collect each row's books and will place them in the file.

Work-books may be taken from the classroom only when permission is noted on the record sheet by the instructor.

(*b*) Distribution. At the beginning of the mathematics period, the collector will place the books at the end of the rows so that each student may receive his book from the pile as it is passed.

Books of absentees will be reported directly to the instructor's desk by the collector. In the collector's absence, the next student in the row will attend to the books.

8. Inspection. The remarks column on the record sheet is for the instructor's stamp. When the dater is used instead of the " accepted " stamp, it signifies that the work is incomplete, or unsatisfactory, or incorrect. Changes in such work, unless obvious or indicated, must be arranged with the instructor before the close of the period.

9. Corrected Work. Incorrect or rejected work is due in correct form at the beginning of the next period.

Make corrections in red ink, on the same page with the incorrect work. If numerous mistakes have been made, prepare a new page and insert it following the incorrect one on which write in red ink " Corrected on next page," with date of correction.

10. Solution of Equations. Number equations at the left in a vertical column with Arabic numerals in parentheses.

Separate equations from specified operations, by a horizontal, rippled line not shorter than half an inch. It must follow each equation except those taken from the text or formed from the conditions specified in the problem.

The following model solution shows how to number, how to use the rippled line, and how to abbreviate.

(a) $\frac{5x}{8}-3x=\frac{1}{4}$

LCD 8

(1) $5x-24x=2 \sim\sim\sim\sim\sim\sim$ Mul (a) by 8

(2) $-19x=2 \sim\sim\sim\sim\sim\sim$ Col in (1)

(3) $x=-2/19 \sim\sim\sim\sim\sim\sim$ Div (2) by -19

As illustrated, equations from the text are numbered with the first letters of the alphabet.

Equations resulting from operations, are numbered in succession in Arabic numerals.

11. Indication of Results. Indicate final results in all problems by double underlining with parallel hair-lines not over one-sixteenth of an inch apart. Do *not* write the word " Answer."

On the same line with the result write a statement in initial capitals specifying exactly what the result represents, whether horse-power, or speed, etc.

12. Index. At the end of the school year arrange all the work-books of the year in order, make an alphabetic index for your combined book, cut the work-book cover in two through the back, and bind all together with one pair of fasteners.

On the front cover, attach a label about 4×5, with a line border and lettered title, enumerating the subjects covered.

CHAPTER I

THE FUNDAMENTAL OPERATIONS

Section 1, Addition. Section 2, Subtraction. Section 3, Multiplication. Section 4, Division. Section 5, Applied Problems

1. The Symbols of Arithmetic. All computation is carried on through the use of the following symbols:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

These symbols are called numbers, digits, or figures, figures being the usual name.

The last is called cipher, nought, or zero, as may be convenient. The name ought, by which it is sometimes known, is incorrect and should not be used.

The value of these symbols depends entirely on how they are written with respect to each other. When used separately, or together with commas between them as above, they denote one, two, three, four, five, six, seven, eight, nine, nought. When written one after the other with no marks between, their value is determined by their position.

For example, 75483 is read seventy-five thousand, four hundred eighty-three. 3, being in the first place, counting from the right, means three units, the same as when written alone. 8, being in the second place from the right, means ten times eight, or eighty. 4 in the third place means four hundred; 5 in the fourth place means five thousand; 7 in the fifth place means seventy thousand.

Below is shown the values of figures to right and left from the decimal point.

Millions	100-Thousands	10-Thousands	Thousands	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths
3	9	7	2	1	6	9	.	1	8	4

This number is read, three million, nine hundred seventy-two thousand, one hundred sixty-nine, and one hundred eighty-four *thousandths*. It should be noted that “and” is used in place of the decimal point in the reading, and that the figures following the decimal point are read as thousandths, because the last figure is in thousandths place.

.1364 is read thirteen and sixty-four ten-thousandths because the last figure is in ten-thousandths place.

Another way of reading is illustrated below, and is recommended because it makes it unnecessary to think the name of the place in which the last figure of the decimal stands.

.224, 5.17, 381.03,

may be read

point two twenty-four,
five point seventeen,
three eighty-one point nought three;

or

two hundred twenty-four thousandths,
five and seventeen hundredths,
three hundred eighty-one and three hundredths.

2. Examples in Reading Numbers. Between the holes at the top of the first page of your work-book, write your last name.

In the upper right corner, not less than one inch from the top of the page, enter the date on which you are doing the work required in this paragraph.

Beginning not less than one and one-half inches from the top of the page, near the left margin, enter the paragraph number and title.

Under the paragraph title, copy any five of the following numbers and write two different readings for each.

1. .34
2. .5
3. .342
4. 174.4
5. 289.02
6. 39.005
7. 4.0008
8. 3000.6
9. 7.8009
10. 1005.0654
11. 10.098
12. 5.3107
13. 295.4008
14. 72.01509
15. 131.2906
16. 3.60105
17. .8906
18. 13.00006
19. $.84
20. $164.25
21. $596.83

3. The Four Fundamental Operations. The operations of arithmetic by means of which all computations are made, are

Addition, or finding the sum of numbers,
Subtraction, or finding the difference of numbers,
Multiplication, or finding the product of numbers,
Division, or finding the quotient of numbers.

§ 1. ADDITION

4. How to Add. In setting down numbers having no decimal points, so that they may be added, the margin is kept straight on the right so that units shall be under units, ten under tens, hundreds under hundreds, etc. In setting down numbers having decimal points, no attention is paid to right or left margins, the only requirement being that the *decimal points* shall be *in a vertical column*, so that units shall be under units, tens under tens, etc.

Thus to add 134, 17, 1763, 21783, 9 and 10005;

Also 17.962, .00073, 184.005, 9.64, 1732.4, and 18, set them down as follows:

```
  134         17.962
   17           .00073
 1763        184.005
21783          9.64
    9       1732.4
10005         18.
-----       ----------
33711
```

Each column is then added, beginning at the right. The sum of the units column of the first group of numbers, is 31 which is one unit and three tens. 1 is therefore entered under the units column and the 3 is added (or carried as it is called) to the tens column.

5. Examples in Addition. While one may learn to add from one example, to gain accuracy and rapidity much practice is necessary.

Under the paragraph number and title, copy and add the following numbers:

```
1.   349    2. 1896    3. 3826    4.  1374
    1783       3452       4392        7860
      68       7891       6109        2918
    1092       2907       8319        1593
    1427       6341       7825        5671
     659        188       5490        8192
    ----       ----       ----        3948
                                      1967
                                      ----

5.  2347    6. 3002    7. 3158    8. 14925
    3672       8516       7342       17894
    8491       1898       6804       29602
    6308       6324       1555       16418
    5496       1673       1109       22483
    1297       1929       2160       96510
    5183       3441       4828       32065
    3609       2558       1347       18999
    ----       ----       ----       -----
```

9. 2308
2197
1342
9671
8344
8190
1783
5448

10. 17284
32560
18298
42672
29181
54672
34086
78096

11. 21985
395
1472
63048
916
91007
8614
31985

12. 54820
439
1723
21734
68090
54372
1898
56

13. 3478
1929
3693
4905
2364
1990
1528
8196
6344
1729

14. 2918
7129
3504
4263
1721
1568
2784
9007
2865
3419

15. 5163
7829
6431
2286
7540
4481
2932
4374
1892
2460

16. 4820
5105
1872
4008
3444
2968
3400
5942
9186
7855

17. 4271
1684
5525
1995
5481
6172
4231
1873
4008
1995

18. 5128
4832
8344
2817
1422
2205
5371
7219
4523
6425

19. 34.72
135.08
29.63
240.75
381.76
8.90
1500.00
.86
72.21

20. 54.27
16.83
198.27
4.59
.85
340.16
84.22
2840.15
3768.29

21. 119.46
409.16
528.34
1783.9
4862.34
196.00
826.15
63.08
129.14

22. $348.67
709.62
155.84
236.49
807.50
112.87
342.68
780.08
328.12
594.08
234.29

23. $199.29
384.96
609.05
343.68
782.96
39.40
118.57
860.51
427.48
763.94
1290.58

24. $320.51
192.81
402.96
308.78
266.45
144.84
569.28
674.67
298.47
1842.34
568.53

25. $1719.29
593.17
1200.09
2418.50
5733.42
8651.94
7208.65
1517.87
4119.25

26. $5307.54
193.76
875.32
2108.50
49.76
2492.65
8076.18
2142.84
7306.25

27. $8496.21
1598.33
4205.56
1892.80
5691.93
2464.78
3429.44
1255.18
9304.47

28. $5460.81
1735.52
1684.90
7207.62
2481.73
6375.59
3418.78
4093.16

29. $8674.06
1865.24
2908.15
4354.27
7583.90
3816.75
985.56
2160.84

30. $1281.45
4256.25
3682.50
1497.34
5643.18
6724.25
2685.94
8064.58

§ 2. SUBTRACTION

6. How to Subtract. A minus between two numbers means that the number following the minus is to be taken away from the number before the minus.

Thus

$$1824-392$$

denotes that 392 is to be taken away or subtracted from 1824.

In order to perform the subtraction the numbers should be set down the same as for addition.

Thus

$$\begin{array}{r}1824\\ \underline{392}\\ 1432\end{array}$$

Since it is not possible to take 9 from 2, 100 or 10 tens, was borrowed from 800, making 12 tens, from which 9 was taken away, leaving 3 in the tens column.

When the numbers have a decimal point, like

$$34.375-2.29,$$

they are set down as in addition with the decimal points under each other.

Thus

$$\begin{array}{r} 34.375 \\ 2.29 \\ \hline 32.085 \end{array}$$

7. Examples in Subtraction. Under this paragraph number and title, set down the following numbers and perform the subtraction. Be sure to set down the numbers so that decimal points are under each other, and to place a decimal point in the result exactly in the same vertical line with the decimal points in the numbers.

1. $34963-282$
2. $786129-127614$
3. $5.84-2.91$
4. $137.62-36.72$
5. $13.748-5.925$
6. $6.429-2.518$
7. $21.0093-9.18$
8. $2.4286-.1854$

The words from and take are sometimes used instead of a minus between the numbers. When copying the examples into the work-book, use the minus sign.

9. From 3.72908 take 1.05416.
10. From .70861 take .55491.
11. From 1.844134 take .97635.
12. From 11.009 take 8.21768.
13. From 34684.01 take 1396.725.

§ 3. MULTIPLICATION

8. How to Multiply. A times symbol ×, between two numbers, means that they are to be multiplied together. The result of multiplication is called the product.

Thus

$$125 \times 116 \times 92$$

denotes that 125 is to be multiplied by 116 and the product by 92.

In order to perform the multiplication, the numbers are set down as follows:

$$\begin{array}{r}
125 \\
116 \\
\hline
750 \\
125 \\
125 \\
\hline
14500 \\
92 \\
\hline
29000 \\
130500 \\
\hline
1334000
\end{array}$$

When the numbers have a decimal point, like 12.961×32.5, they are set down so that the last figure of the second number stands under the last figure of the first number. Thus

$$\begin{array}{r}
12.961 \\
32.5 \\
\hline
64805 \\
25922 \\
38883 \\
\hline
421.2325
\end{array}$$

Four decimal places are pointed off in the result or product, because there is a total of four decimal places in the numbers which were multiplied.

9. Rule for Pointing off in Multiplication. The product must be pointed off so that the number of decimal places in it shall equal the number of decimal places in both the numbers that are multiplied.

10. Multiplication Table. The following table gives the products of numbers through the 20's.

It should be noted that the squares from 1 to 400 may be read on the diagonal from 1 to 20×20.

All products in this table which are not already known should be learned as quickly as possible, at least through the 12's.

One illustration will show how the table is used:

9×13, which is 117, may be read in column 13 in line 9, or in column 9 in line 13.

11. Examples in Multiplication. Under the paragraph number and title, set down the following numbers and perform the multiplication:

1. 182×9
2. 324×12
3. 793×11
4. 786×17
5. 960×13
6. 548×8
7. 341.3×15
8. 708×16
9. 47×1456
10. 910.8×34
11. 119×28
12. 345×1.8
13. 396.1×4.5
14. 380×12.2
15. 86×19.5
16. 348.8×19
17. 607.3×1.7
18. 980.1×15.3
19. 4009.5×4.8
20. 38.06×4.22
21. 7.19×38.9
22. 82.5×19×28.3
23. 44.92×3.4×827
24. 810.56×15.7
25. 308.72×13×4.91
26. 2.98×13.7×40.25
27. 82.6×91.7×1.4
28. 3.008×4.09×.128
29. .0092×.148×1.5
30. .723×.09×3.275
31. .387×17.02×12.93
32. 38.12×1.25×.0031
33. 7.296×1.41×384.3
34. 1.875×3.1416×.08
35. 24.107×1.312×.0009
36. .7854×.03×113.42
37. 5.678×1.28×1.0025
38. 3.009×754.3×22.09
39. 412.07×.731×9.189
40. 816.5×1910×.7268
41. 5.173×8.02×16.01
42. 7.92×37.1×.0256
43. 37.04×213.1×39.21

TABLE I

MULTIPLICATION TABLE

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2	**4**	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	**9**	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	**16**	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	**25**	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	**36**	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	**49**	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	**64**	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	**81**	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	**100**	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	**121**	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	**144**	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	**169**	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	**196**	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	**225**	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	**256**	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	**289**	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	**324**	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	**361**	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	**400**

12. Powers of 10. The powers of a number are obtained by multiplying it by itself. They are indicated by a small figure, called an exponent, slightly above the number and to the right of it.

Thus

$$10^2=10\times10=100$$
$$10^3=10\times10\times10=1000$$
$$10^4=10\times10\times10\times10=10000$$

The fact that the number of ciphers in the power is always the same as the number of units in the exponent, makes it easy to write any power of 10 when its exponent is given.

Thus 10^5 in which there are five units in the exponent, means 1 followed by five ciphers: or $10^5=100000$, which is the same as would be obtained from $10\times10\times10\times10\times10$.

13. Meaning of a Minus Exponent. When the exponent has a minus before it, like 10^{-3}; we write

$$\frac{1}{10^3} \text{ or } \frac{1}{1000}$$

Thus

$$10^{-2}=\frac{1}{10^2}=\frac{1}{10\times10}=\frac{1}{100}$$
$$10^{-3}=\frac{1}{10^3}=\frac{1}{10\times10\times10}=\frac{1}{1000}$$
$$10^{-4}=\frac{1}{10^4}=\frac{1}{10\times10\times10\times10}=\frac{1}{10000}$$

14. Multiplication by 10 or its Powers. If a number is multiplied by 10, like

$$\begin{array}{r} 384.5 \\ 10 \\ \hline 3845.0 \end{array}$$

we have the same result that we would get by moving each of its figures *one* place to the *left*, and keeping the decimal point fixed.

If a number is multiplied by 10^2 or 100, like

$$\begin{array}{r} 98741 \\ 100 \\ \hline 9874100 \end{array}$$

we have the same result that we would get by moving its figures *two* places to the left, and keeping the decimal point fixed.

Instead of moving the figures of the number to the left, however, we move the decimal point to the *right*, according to the following

RULE. To multiply a number by 10 or its powers, write the number with the decimal point moved to the *right* as many places as the number of ciphers in the power of 10.

If the exponent of 10 is *minus*, or negative as it is called, write the number with the *decimal point moved to the left*, instead of the right, as many places as the number of units in the minus exponent.

15. Examples in Multiplication by Powers of 10. Under paragraph number and heading, set down by inspection the results of the following indicated multiplications:

1. 2968×10^{1} **2.** 34.45×100 **3.** 3.89×1000
4. 78.64×10^{2} **5.** 4.2905×10^{3} **6.** $.3197\times10^{4}$
7. $.78\times10000$ **8.** $.034\times10^{6}$ **9.** 4.897×10^{2}
10. 3.290×1000 **11.** $.00928\times10^{3}$ **12.** 38×1000
13. 4.34×10^{2} **14.** 3.2568×10^{4} **15.** 45×10^{-2}
16. 6.345×10^{4} **17.** 256.8×10^{-3} **18.** 1876×10^{-8}
19. 12.876×10^{3} **20.** 31.7×1000 **21.** 48.961×10^{-2}
22. $.0076\times10000$ **23.** $.0875\times10^{-4}$ **24.** 3716.8×10^{-5}
25. 52.004×10^{3} **26.** 726.17×10^{-6} **27.** 46.87×10^{-7}

16. Multiplication by Numbers Ending with One or More Ciphers. To multiply 34.58 by 400, it is only necessary to move the decimal point *two* places to the *right,* and multiply by 4.

Thus

$$38.58\times 400=3858\times 4=15432.00$$

Rule. To multiply any number by a number ending with one or more ciphers, move the decimal point to the right in the number to be multiplied, as many places as the number of ciphers ending the multiplier, and multiply by all the figures which precede the ciphers in the multiplier.

Thus

$$.7821\times 700=78.21\times 7=547.47.$$

The multiplier preceding ends with two ciphers; therefore the decimal point is moved *two* places to the right, and 7 is used as a multiplier as shown.

Another Illustration. To compute the product of 28.543×16000 we set down the work as follows:

$$\begin{array}{r}
28543\\
16\\
\hline
171258\\
28543\\
\hline
456688
\end{array}$$

The decimal point was moved three places to the right because the multiplier ends with three ciphers.

17. Examples, Multiplier Ending with Ciphers. Under paragraph number and heading, work the following examples by the rule of the preceding paragraph:

1. 298×60 **2.** 23.01×800 **3.** 1.475×1100
4. 3.409×2900 **5.** 5.38×3400 **6.** $.4267 \times 14000$
7. $.125 \times 5200$ **8.** $.875 \times 1900$ **9.** $.3481 \times 42000$
10. $.00829 \times 13000$ **11.** $.0734 \times 90000$ **12.** 1.8642×7300
13. $.00052 \times 6500$ **14.** $.01904 \times 1800$ **15.** $.324 \times 98000$
16. $.00846 \times 9000$ **17.** $.00128 \times 11000$ **18.** 3.74×8000
19. 4.896×7000 **20.** 5.8724×1300 **21.** $.004853 \times 900$
22. $.005967 \times 14000$ **23.** 0.96×125000 **24.** $.07235 \times 1910$
25. $.08342 \times 38000$ **26.** 1.341×56100 **27.** 4.729×54000
28. 3.784×62100 **29.** $.1893 \times 4250$ **30.** $.00012 \times 120000$

§ 4. DIVISION

18. Ways of Indicating Division. There are five ways of indicating the division of one number by another:

Thus,

$12\overline{)172}$ read divide 172 by 12, or
12 is contained in 172.
$172 : 12$ read 172 is to 12, or
the ratio of 172 and 12,
$172 \div 12$ read 172 divided by 12, or
the quotient of 172 and 12.
$172/12$, $\frac{172}{12}$ read 172 12ths, or
172 over 12.

Each of the five forms denotes exactly the same thing; viz., that 172 is to be divided by 12. The last is the best way to indicate division and is therefore to be preferred.

19. Kinds of Division. There are two ways of dividing; one known as short division, the other as long division.

Short division is generally used when the divisor is not greater than 12. Long division is used for divisors greater than 12.

20. Short Division. The following illustrates the use of short division:

1.

$$\begin{array}{r} 7\underline{)896} \\ 128 \end{array}$$

7 is the divisor,
896 is the dividend,
128 is the quotient.

The operations are as follows:

7 is contained in 8, once, and 1 to carry,
7 is contained in 19, twice, and 5 to carry,
7 is contained in 56, eight times.

2.

$$\begin{array}{r} 9\underline{)4518787} \\ 502087\frac{4}{9} \end{array}$$

When there is a remainder as in the 2d illustration, it is written over the divisor, or reduced to decimal form to as many places as may be desired.

Thus

$$\begin{array}{r} 9\underline{)4518787.00} \\ 502087.44+ \end{array}$$

21. Examples in Short Division. Under this paragraph number and heading, work not less than five of the following examples.

When there is a remainder, write it in the quotient, over the divisor.

1. $4\underline{)131948}$
2. $6\underline{)4704}$
3. $8\underline{)50592}$
4. $5\underline{)1520}$
5. $9\underline{)426769}$
6. $8\underline{)3045400}$
7. $7\underline{)552468}$
8. $3\underline{)29954295}$
9. $6\underline{)45064944}$
10. $11\underline{)142824}$
11. $5\underline{)40134595}$
12. $7\underline{)19890724}$
13. $8\underline{)39173752}$
14. $4\underline{)7586972}$
15. $9\underline{)30862656}$
16. $11\underline{)3186387}$
17. $12\underline{)2170944}$
18. $5\underline{)2425985}$
19. $9\underline{)48608703}$
20. $8\underline{)476145}$
21. $9\underline{)2807651}$

22.)7243145
23. 6)9407263
24. 11)4617382
25.)391001269
26. 5)1564355
27. 4)5420009
28.)96348722
29. 6)7392108
30. 12)8612076
31.)734126759
32. 8)2330492
33. 6)54487254
34.)81307344
35. 12)682091
36. 13)549678
37. $\frac{15}{8}$
38. $\frac{11}{4}$
39. $\frac{3}{2}$
40. $\frac{22}{7}$
41. $\frac{22}{5}$
42. $\frac{19}{6}$
43. $\frac{19}{3}$
44. $\frac{85}{3}$
45. $\frac{100}{3}$
46. $\frac{97}{6}$
47. $\frac{149}{12}$
48. $\frac{1872}{11}$
49. $\frac{34.735}{12}$
50. $\frac{.18392}{9}$
51. $\frac{.007963}{7}$

22. Division by 10 or its Powers. The powers of 10 are 100, 1000, 10000, etc., since they are obtained by raising 10 to some power, as follows:

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$10^4 = 10 \times 10 \times 10 \times 10 = 10000$$

Each of the small figures (exponents) to the right of 10, tells how many times 10 is to be used as a factor.

RULE. To divide a number by 10 or its powers, move the decimal point to the *left* as many places as the number of ciphers in the divisor or as the number of units in the exponent of 10.

Thus

(1) $\frac{34298}{10} = 3429.8$ (2) $\frac{594437}{100} = 5944.37$

(3) $\frac{8716.52}{1000} = 8.71652$ (4) $\frac{.048765}{10^2} = .00048765$

23. Examples in Division by Powers of 10. Under paragraph number and heading, set down by inspection, the results of the following indicated divisions:

1. $\frac{42968}{10}$ **2.** $\frac{34967}{10^3}$ **3.** $\frac{29728}{100}$

4. $\frac{9671.43}{10}$ **5.** $\frac{418.43}{10^2}$ **6.** $\frac{7.2639}{10}$

7. $\frac{51.849}{100}$ **8.** $\frac{.31462}{10}$ **9.** $\frac{.00785}{1000}$

10. 72.98×10^{-2} **11.** $.7856 \times 10^{-2}$ **12.** 8412×10^{-3}

13. 139.008×10^{-5} **14.** 12.245×10^{-7} **15.** $.428 \times 10^{-4}$

16. $.00728 \times 10^{-2}$ **17.** 13.927×10^{-3} **18.** 346.71×10^{-7}

19. 42.89×10^{-1} **20.** 1.5439×10^{-2} **21.** 1842.7×10^{-5}

22. 41.672×10^{-2} **23.** 5189.3×10^{-3} **24.** 18967×10^{-4}

25. 2180.02×10^{-5} **26.** 671095×10^{-7} **27.** 1420.96×10^{-3}

28. 9.0007×10^{-2} **29.** 154.63×10^{-8} **30.** 18.2601×10^{-1}

31. 12964×10^{-5} **32.** 1675.01×10^{-2} **33.** 4.00735×10^{-3}

34. $.0005 \times 10^{-2}$ **35.** $.0108 \times 10^{-4}$ **36.** 865217.9×10^{-4}

37. $\frac{2918.73}{10^2}$ **38.** $\frac{182.63}{10^4}$ **39.** $\frac{.000118}{10^2}$

40. $\frac{13.7298}{1000}$ **41.** $\frac{.071608}{10}$ **42.** $\frac{18956}{10^4}$

43. $\frac{78.492}{10^2}$ **44.** $\frac{3165.83}{10000}$ **45.** $\frac{56297.48}{10^3}$

24. Division by Numbers Ending with One or More Ciphers.

To divide 8496 by 600, it is only necessary to move the decimal point *two* places to the *left*, and divide by 6.

Thus

$$\begin{array}{r} 6\overline{)84.96} \\ \hline 14.16 \end{array}$$

Rule. To divide any number by a number ending with one or more ciphers, move the decimal point in the dividend as many places to the *left* as the number of ciphers ending the divisor, and divide by all the figures which precede the ciphers in the divisor.

Thus

$$\frac{849.684}{12000}=\frac{.849684}{12}=.070807$$

The divisor preceding, ends with *three ciphers.* Therefore the decimal point is moved three places to the *left,* and 12 is used as the divisor as shown.

Another Illustration:

$$\frac{3.9618}{800}=\frac{.03968}{8}=.00496$$

In dividing the preceding, we put down the decimal point in the quotient, and then think

8 goes into 0, 0 times;
into 3, 0 times;
into 39, 4 times;
into 76, 9 times;
into 48, 6 times.

25. Examples, Divisor Ending with Ciphers. Under the paragraph number and heading, work the following examples by the rule of the preceding paragraph:

1. $\frac{98726}{20}$ **2.** $\frac{29628}{400}$ **3.** $\frac{3265}{5000}$

4. $\frac{726.25}{700}$ **5.** $\frac{.514206}{60}$ **6.** $\frac{389.28}{12000}$

7. $\frac{17.622}{3\times10^3}$ **8.** $\frac{.9128}{8000}$ **9.** $\frac{129600}{3000}$

10. $\frac{37125}{500}$ **11.** $\frac{18768}{800}$ **12.** $\frac{448562}{6000}$

13. $\frac{17346.2}{80000}$ **14.** $\frac{4100829}{9000}$ **15.** $\frac{29684.44}{1100}$

16. $\frac{298723.9}{7000}$ **17.** $\frac{1843.2}{8000}$ **18.** $\frac{16874.55}{9000}$

19. $\frac{34000.76}{4000}$ **20.** $\frac{559672.8}{120000}$ **21.** $\frac{34676.49}{60000}$

22. $\frac{517842}{70000}$ **23.** $\frac{187154.6}{11000}$ **24.** $\frac{1348928.72}{900000}$

26. Long Division. The following illustrates the use of long division:

1.

$$
\begin{array}{l}
\underline{14})88536(\underline{6324} \\
\underline{84} \\
45 \\
\underline{42} \\
33 \\
\underline{28} \\
56 \\
\underline{56}
\end{array}
$$

14 is the divisor,
88536 is the dividend,
6324 is the quotient.

The operations are as follows:

14 is contained in 88, 6 times;
6 × 14 is 84, which subtracted from 88, leaves 4;
Bring down 5, the next figure of the dividend.

14 is contained in 45, 3 times;
3 × 14 is 42, which subtracted from 45 leaves 3;
Bring down 3, the next figure of the dividend.

14 is contained in 33, 2 times;
2 × 14 is 28, which subtracted from 33, leaves 5;
Bring down 6, the next figure of the dividend.

14 is contained in 56, 4 times;
4 × 14 is 56.

2.

```
38)185849035(4890764 3/38
   152
   ---
    338
    304
    ---
     344
     342
     ---
     * 290
       266
       ---
        243
        228
        ---
         155
         152
         ---
           3
```

Where the star is inserted, the remainder was 2, giving only 29 when the next figure was brought down. 33 is contained in 29, 0 times, which is the reason for the cipher in the quotient.

The figure 0 in the dividend was then brought down, giving 290, in which 38 is contained 7 times.

The remainder 3 is written over the divisor 38, in the quotient, the same as in short division.

27. Pointing Off the Quotient. When dividend or divisor, or both of them, have decimal figures, students frequently make mistakes in pointing off the quotient. This need never happen if the following rule is observed:

RULE. Point off as many decimal places in the quotient, as the number of decimal places in the dividend *minus* the number of places in the divisor.

When the number of decimal places in the dividend is less than the number of decimal places in the divisor *annex ciphers to the dividend until the number of decimal places is the same as in the divisor, before attempting to divide.*

Thus

$$\underline{38.96})129.7(\underline{\quad\quad}$$

There are *two* decimal places in the divisor, but only *one* in the dividend. One cipher is therefore annexed to the dividend, making

$$\begin{array}{r@{}l}
\underline{38.96})129.70000(&\underline{3.329+} \\
\underline{116\ 88} & \\
12820 & \\
\underline{11688} & \\
11320 & \\
\underline{7792} & \\
35280 & \\
\underline{35064} & \\
216 &
\end{array}$$

Three decimal places are pointed off in the quotient because three ciphers were brought down, making three more decimal places in the dividend than in the divisor, and therefore three decimal places in the quotient.

28. Examples in Long Division. Under paragraph number and heading, work the following examples:

1. $\frac{38219}{17}$ **2.** $\frac{296035}{15}$ **3.** $\frac{18726}{13}$

4. $\frac{418347}{25}$ **5.** $\frac{526350}{35}$ **6.** $\frac{780096}{14}$

7. $\frac{296045}{42}$ **8.** $\frac{307185}{62}$ **9.** $\frac{906529}{56}$

10. $\frac{608929}{18}$ **11.** $\frac{526095}{45}$ **12.** $\frac{3126788}{48}$

13. $\frac{1206973}{24}$ **14.** $\frac{4210738}{35}$ **15.** $\frac{7089238}{24}$

16. $\frac{1.7238}{58}$ **17.** $\frac{397.35}{1.225}$ **18.** $\frac{217634}{3.8}$

19. $\frac{437981}{1.24}$ **20.** $\frac{5298.007}{34.8}$ **21.** $\frac{43.2198}{41.8}$

22. $\frac{766.3498}{2.318}$ **23.** $\frac{30768.4}{54.24}$ **24.** $\frac{21009.67}{128.1}$

25. $\frac{1923.45}{3.461}$ **26.** $\frac{15.7204}{91.82}$ **27.** $\frac{5349.61}{.341}$

28. $\frac{830.075}{75.04}$ **29.** $\frac{517340}{873.4}$ **30.** $\frac{128.942}{.4427}$

31. $3972.1 \div 371.2$ **32.** $7162.4 \div 3.009$

33. $4208.7 \div 3.45$ **34.** $521.081 \div .672$

35. $4.1072 \div 59.6$ **36.** $346.782 \div 917.6$

37. $8975.1 \div 910.07$ **38.** $5.1783 \div 5.1$

39. $\frac{182.4 \times 3.45}{29.7}$ **40.** $\frac{230.4 \times 1.75}{31.42}$

41. $\frac{41.96 \times 3.4 \times 53}{72.08 \times 14.1}$ **42.** $\frac{78.007 \times .34 \times 181}{31 \times 72.5}$

43. $\frac{5.473 \times 3.1416}{14.8 \times 3.7}$ **44.** $\frac{70.81 \times 4.26 \times 5.7}{23.4 \times 17.28}$

45. $\frac{53.09 \times 1.74 \times 35.7}{.4105 \times 54.3}$ **46.** $\frac{18.003 \times 19.8}{4208 \times 10^{-3}}$

47. $\frac{2204.2 \times 17.5}{568.1 \times 10^{-4}}$ **48.** $\frac{.34005 \times 91.8}{97.1 \times 54}$

49. $\frac{530.7 \times 19.1}{45.81 \times 17}$ **50.** $\frac{34.46 \times 10^{-2} \times 91}{51.760 \times 42.1}$

51. $\frac{5.0872 \times 4307.6}{10^{-3} \times 187}$ **52.** $\frac{178.41 \times 41.8}{308.3 \times 10^{-2}}$

§ 5. APPLIED PROBLEMS

29. Problems. Work the following problems according to the directions given in each case. Where tabulation is required, the table should be neatly ruled in pencil in the work-book, the outside lines being made heavier than those that separate the entries.

Show all computations in full and make them directly on the work-book paper.

1. Weight of Materials of Construction. When the weight of material is expressed as so many pounds per cubic foot, the weight per cubic inch is computed from the following formula:

$$W_i = \frac{W_f}{1728}$$

in which

W_i = weight per cubic inch;

W_f = weight per cubic foot.

Compute the omitted entries and tabulate in the following table:

TABLE 2

WEIGHT OF MATERIALS

No.	Material.	Average Weight, Cu.ft.	Average Weight, Cu.in.	No.	Material.	Average Weight, Cu.ft.	Average Weight, Cu.in.
1	Brass, cast.	504		8	Bronze....	529	
2	Copper, rld.	555		9	Iron, cast..	450	
3	Lead......	709.6		10	Steel......	490	
4	Gun metal.		.306	11	Antimony..		.242
5	Tin.......	455.1		12	Ash.......		.027
6	Oak......		.03	13	Granite....		.094
7	Cement....		.054	14	Mortar....		.057

2. Sharp V Screw-threads. In the sketch P denotes the distance between the threads, or the pitch of the screw; d denotes the depth of the thread; n denotes the number of threads per inch of length of screw.

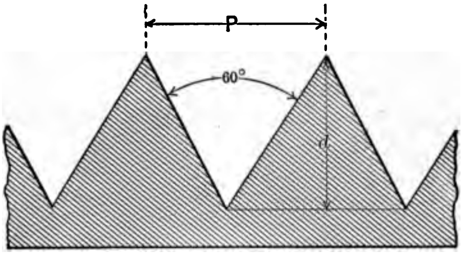


FIG. 1.

When the number of threads to the inch is known, the pitch and the depth may be computed from the following:

$$\text{Pitch equals } \frac{1}{\text{No. of threads to the inch}}$$

$$\text{Depth equals .86603 times the pitch}$$

or

$$P=\frac{1}{n} \tag{1}$$

$$d=.86603\times P \tag{2}$$

The first expression is the *law of pitch,* and tells us that the pitch of a screw may be determined by dividing 1 by the number of threads to the inch.

The second expression is the *law of depth,* and tells us that the depth of a sharp V-thread screw may be determined by multiplying .86603 by the pitch.

Expressions (1) and (2) are the formulation of the two laws, and are called *formulas.* Why they are so called will be evident from the following:

DEFINITION. A formula is an expression in symbols, of the exact mathematical relation of two or more quantities.

Thus P is the symbol for pitch, n the symbol for the number of threads per inch, and d the symbol for depth. You will observe that each symbol in this instance, is the initial letter of the name of the quantity.

One of the greatest advantages of the study of mathematics, is that it teaches how to make computations by means of formulas.

For instance, if we wish to know the pitch of a screw which has 7 sharp threads to the inch, the computation may be made by the use of formula (1) as follows:

$$P=\frac{1}{n}$$

$$P=\frac{1}{7}=.1429''$$

To compute the depth of the thread, we use

$$d=.86603\times P$$

$$d=.86603\times\frac{1}{n}$$

$$d=.86603\times\frac{1}{7}$$

$$d=.1237''$$

Whenever a result is carried to four decimal places, it is necessary to determine the fifth figure. If this is 5 or greater, the preceding figure is increased by 1; if it is less than 5, it is dropped and no increase is made to the preceding figure.

By the use of formulas (1) and (2) compute and fill in all omitted entries to four decimal places in the following table except the last column, in which

D = diameter of screw;
n = number of threads per inch;
P = pitch;
d = depth of the thread;
D_r = diameter of screw at the root of the thread.

TABLE 3

SHARP V-SCREW THREADS

No.	Diam. Screw.	n	P	d	D_r	No.	Diam. Screw.	n	P	d	D_r
1	$\frac{5}{16}$	18				6	$2\frac{1}{8}$	$4\frac{1}{2}$			
2	$\frac{7}{16}$	14				7	$3\frac{1}{8}$	$3\frac{1}{2}$			
3	$\frac{7}{8}$	9				8	$3\frac{5}{8}$	$3\frac{1}{4}$			
4	$1\frac{1}{4}$	7				9	$\frac{11}{16}$	11			
5	$1\frac{1}{2}$	6				10	$3\frac{7}{8}$	3			

3. The symbol at the head of the last column in the table is read D sub r.

Write a law and a formula for D_r and compute and fill in the column in the table, carrying all results to four places.

4. U.S. Standard Screw-Threads. Computations on United States standard screw-threads may be made from the formulas below, in which F = the flat or width at the point and the root of the thread; the other symbols are the same as in Problems 2 and 3.

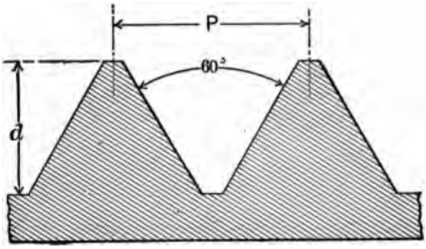


FIG. 2.

$$F=\frac{P}{8}$$

$$P=\frac{1}{n}$$

$$d=.64952\times P$$

$$D_r=?$$

Copy the formulas into the work-book, together with the formula for D_r.

5. Compute and fill in, to four decimal places, the omitted entries in the following table:

TABLE 4

U. S. STANDARD SCREW THREADS

No.	D	n	P	d	D_r	F	No.	D	n	P	d	D_r	F
1	$\frac{5}{16}$	18					6	$2\frac{1}{4}$	$4\frac{1}{2}$				
2	$\frac{7}{16}$	14					7	$3\frac{1}{2}$	$3\frac{1}{4}$				
3	$\frac{5}{8}$	11					8	$4\frac{3}{4}$	$2\frac{5}{8}$				
4	$1\frac{1}{8}$	7					9	$5\frac{1}{2}$	$2\frac{3}{8}$				
5	$1\frac{5}{8}$	$5\frac{1}{2}$					10	6	$2\frac{1}{4}$				

6. Metric Standard Screw-Threads. The dimensions of a metric or French screw-thread are expressed in millimeters, the symbol for which is mm.

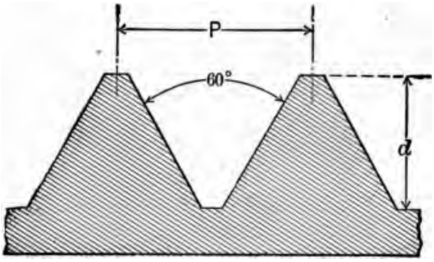


FIG. 3.

The formulas are as follows:

$$d = .6495 \times P$$

$$F = \frac{P}{8}$$

$$D_r = ?$$

The symbols have the same meaning as in the preceding problems.

Copy the formulas and write a formula for D_r.

7. Compute and fill in, to two decimal places, the omitted entries in the following table:

TABLE 5

METRIC STANDARD SCREW-THREADS

No.	D mm.	P mm.	d	D_r	F	No.	D mm.	P mm.	d	D_r	F
1	4	.75				6	30	3.5			
2	8	1.25				7	39	4.0			
3	12	1.5				8	44	4.5			
4	12	1.75				9	56	5.5			
5	18	2.5				10	72	6.5			

8. Wire Measurement of 60° V-Thread. This thread is measured by a wire laid in the thread groove as shown.

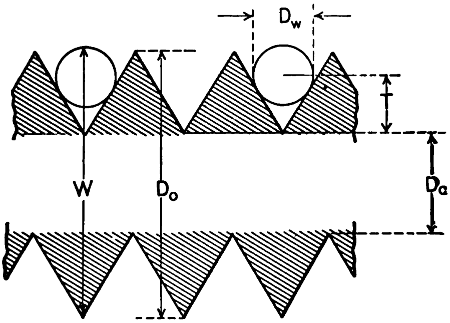


FIG. 4.

W may be computed from the following formula:

$$W = T + \frac{D_w}{2} + \frac{D_o}{2} + \frac{D_c}{2}$$

W = distance from top of thread on one side, to top of wire on opposite side,
T = distance from root of thread to center of wire,
D_w = diameter of wire,
D_c = diameter at root of thread,
D_o = outside diameter over thread.

Compute and fill in, to four decimal places, the omitted entries in the following table:

TABLE 6

MEASUREMENT 60° V-THREADS

No.	D_c	D_o	D_w	T	W	No.	D_c	D_o	D_w	T	W
1	.2163	$\frac{5}{16}$	.04	.04		6	1.615	2	.15	.15	
2	.3557	$\frac{1}{2}$	.06	.06		7	1.740	$2\frac{1}{8}$	.15	.15	
3	1.0026	$1\frac{1}{4}$	.10	.10		8	2.317	$2\frac{3}{4}$	.20	.20	
4	1.0863	$1\frac{3}{8}$	.15	.15		9	2.967	$3\frac{1}{2}$	.20	.20	
5	1.490	$1\frac{7}{8}$	.15	.15		10	3.1727	$3\frac{3}{4}$	.20	.20	

9. Dimensions of Flat-head Machine Screws. The dimensions of A.S.M.E. (American Society of Mechanical Engineers) standard, flat-head machine screws, may be computed from the formulas below:

$$D_h = 2D_b - .008$$

$$H = \frac{D_b - .008}{1.739}$$

$$w = .173D_b + .015$$

$$d = \frac{H}{3}$$

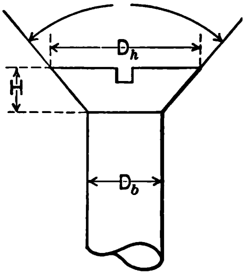


FIG. 5.

in which

D_b = diameter of the body or shank,
D_h = diameter of the head,
H = depth of the head,
w = width of the slot,
d = depth of the slot.

Compute and fill in, to three decimal places, the omitted entries in the following table when drawn in the work-book:

TABLE 7

FLAT-HEAD MACHINE SCREWS

No.	D_b	D_h	H	d	w	No.	D_b	D_h	H	d	w
1	.086					6	.320				
2	.112					7	.346				
3	.138					8	.372				
4	.177					9	.398				
5	.268					10	.424				

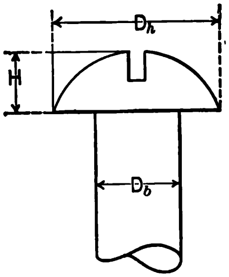


FIG. 6.

10. Dimensions of Round-head Machine Screws. The formulas below may be used for computing the dimensions of round-head machine screws, A.S.M.E. standard, the significance of the symbols being the same as in Problem 9.

$$D_h = 1.85D_b - .005$$

$$H = .7D_b$$

$$d = \frac{H}{2} + .01$$

$$w = .173D_b + .015$$

Draw the following table in the work-book, and compute and fill in, to three decimal places, the omitted entries:

TABLE 8

ROUND-HEAD MACHINE SCREWS

No.	D_b	D_h	H	d	w	No.	D_b	D_h	H	d	w
1	.060					6	.216				
2	.099					7	.242				
3	.125					8	.294				
4	.151					9	.424				
5	.164					10	.450				

11. Weight of Sheet Steel. The weight of sheet steel is about 487.7 lbs. per cubic foot.

Compute the weight per square foot, 1 inch thick.

12. Weight of Sheet Iron. Iron plates are estimated to weigh 480 lbs. to the cubic foot.

Compute the weight per square foot, 1 inch thick.

13. Formula for Weight per Square Foot. The weight of iron and steel plates per square foot, 1 inch thick, equals the weight per cubic foot divided by what?

Formulate this law, using the following notation:

W_s = weight per sq.ft.

W_c = weight per cu.ft.

If the plates were 2 inches thick, the weight per square foot would be how many times as much as when 1 inch thick?

If 5 inches thick, the weight per square foot would be how many times the weight per square foot, 1 inch thick?

If t = thickness of plate in inches, write a formula for the weight per square foot for any thickness.

14. By the formula compute and fill in, to three decimal places, the omitted entries in the following table:

Table 9

WEIGHT SHEET IRON AND STEEL

No.	U.S. Gage.	t	W_s		No.	U.S. Gage.	t	W_s	
			Iron.	Steel.				Iron.	Steel.
1	000	.375			6	9	.15625		
2	00	.34375			7	11	.125		
3	1	.28125			8	13	.09375		
4	6	.20312			9	33	.00937		
5	8	.17187			10	36	.00664		

15. Safe Working Boiler Pressure. The safe working pressure for a boiler is commonly taken as $\frac{1}{5}$ the bursting pressure. When such is the case, 5 is called the factor of safety.

P_b = bursting pressure in pounds per square inch;
T = tensile strength of the boiler plate in pounds per square inch;
t = thickness of the boiler plate or shell in inches;
R = radius of boiler in inches.
C = a constant which has the following values:
.56 for single-riveted boilers;
.70 for double-riveted boilers;
.88 for triple-riveted boilers.

$$P_b = \frac{T \times t \times C}{R}$$

Compute the safe working pressure for single-, double-, and triple-riveted boilers 58 inches in diameter, when made of $\frac{5}{16}$-inch steel plate, having a tensile strength of 55000 lbs. per square inch.

16. What is the bursting pressure P_b, for each of the boilers in Problem 15?

17. Thickness of a Wrought-iron Tube. The actual inside diameter of a Briggs standard, wrought-iron, welded tube, is 0.623 inch; the actual outside diameter is 0.840 inch.

Compute the thickness of the metal to three decimal places.

18. Efficiency of a Motor. The efficiency of a motor, Eff, equals its output in horse-power HP_o, divided by its input in horse-power HP_i.

Formulate this law and compute Eff to two decimal places, when a motor, driven by 5 kilowatts, delivers 6 horse-power. (To reduce kilowatts to horse-power, multiply by 1.34.)

19. Time Required to Plane Metals. The time in minutes, in which a piece of work can be planed, equals the width of the work in inches, divided by the product of the number of strokes per minute and the feed per stroke (width of the cut) in inches.

Formulate this law and compute the time required to plane a piece of work 30 inches wide, when the feed per stroke is $\frac{5}{16}$ inch, and the planer makes 8 strokes per minute.

CHAPTER II

POWERS AND ROOTS

SECTION 1. POWERS. SECTION 2. ROOTS. SECTION 3. SQUARE ROOT. SECTION 4. CUBE ROOT. SECTION 5. APPLIED PROBLEMS.

§ 1. POWERS

30. Definition and Illustration. A power is the product obtained by multiplying a number one or more times by itself.

Thus

$$25=5\times 5$$
$$125=5\times 5\times 5$$

In the preceding, 25 is a power because it is the product obtained when 5 is multiplied once by itself; 125 is a power because it is the product obtained by multiplying 5 twice by itself.

When a number is multiplied, it is said to be used as a factor.

Thus in 5×4, both the 5 and the 4 are factors.

31. An Exponent. When a number is used more than once as a factor, it is denoted by a small figure at the right of and a little above the number, called an exponent.

Thus

3^2 denotes the square or second power of 3,
5^3 denotes the cube or third power of 5.

Therefore

$$3^2=3\times3=9$$
$$5^3=5\times5\times5=125$$

32. Examples in Powers. Under the paragraph number and title, indicate and compute the following powers:

1. The eighth power of 2.
2. The sixth power of 4.
3. The fourth power of 8.
4. The fifth power of 7.
5. The third power of 12.
6. The second power of 225.
7. The third power of 134.

33. The Reciprocal of a Number. The reciprocal of a number is 1 divided by the number.

Thus

$\frac{1}{8}$ is the reciprocal of 8
$\frac{1}{12}$ is the reciprocal of 12
$\frac{1}{7}$ is the reciprocal of 7

34. Examples in Reciprocals. Express and compute to five decimal places, the reciprocals of the following numbers:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 32, 36, 40, 42, 48, 50, 56, 125, 250, 275, 500.

35. A Negative Exponent. When an exponent has a minus sign, it is called a negative exponent; it denotes the reciprocal of the number having the same exponent with the sign omitted.

Thus

$$10^{-2}=\frac{1}{10^2}$$
$$8^{-3}=\frac{1}{8^3}$$

36. Examples in Negative Exponents. Interpret the following negative powers:

1. 3^{-2} **2.** 2^{-4} **3.** 5^{-8}
4. 15^{-3} **5.** 18^{-5} **6.** 16^{-1}
7. 24^{-2} **8.** 10^{-7} **9.** 30^{-3}
10. 45^{-4} **11.** 92^{-3} **12.** 40^{-4}

37. Examples in Powers. Under the paragraph number and title, compute the following:

1. 12^3 **2.** 17^2 **3.** 18^3
4. 19^3 **5.** 25^3 **6.** 16.5^2
7. 13.4^3 **8.** 63.7^2 **9.** 82.19^2
10. 461.8^2 **11.** 18.76^2 **12.** 1.25^3
13. 18^{-2} **14.** 2^{-5} **15.** 14^{-3}
16. 32^{-1} **17.** 25^4 **18.** 3.1416^2
19. 3.1416^{-1} **20.** 3.1416^3 **21.** 3.1416^{-2}
22. 9^{-2} **23.** 7.296^2 **24.** 42.043^2
25. 71.925^2 **26.** 8.341^3 **27.** 28^4
28. $\frac{3.1416^2}{4}$ **29.** 300^4 **30.** $\left(\frac{3}{.4}\right)^2$

§ 2. ROOTS

38. Definition and Illustration. The root of any number is one of the equal factors of the number.

A square root is one of the two equal factors of a number.

A cube root is one of the three equal factors of a number.

A fourth root is one of the four equal factors of a number. Thus

5 is the square root of 25 because it is one of the two equal factors of 25;

6 is the cube root of 216 because it is one of the three equal factors of 216;

2 is the fourth root of 16 because it is one of the four equal factors of 16.

39. How a Root is Expressed. A root is indicated in two ways:

By the radical sign,
By a fractional exponent.

Thus the square root of 36 is denoted as follows:

$$\sqrt{36} \quad \text{or} \quad 36^{\frac{1}{2}}$$

Any other root than the square root is expressed by a small figure called the index of the root, at the top and to the left of the radical.

Thus the cube root is denoted by

$$\sqrt[3]{1728} \quad \text{or} \quad 1728^{\frac{1}{3}}$$

and the fourth root is denoted by

$$\sqrt[4]{243} \quad \text{or} \quad 243^{\frac{1}{4}}$$

40. A Fractional Exponent. In the preceding paragraph, it has been shown how a root may be indicated by a fractional exponent in which the denominator (the number below the fraction line) denotes the root.

Not only does the denominator denote the root, but the numerator (the number above the fraction line) denotes the power.

Thus $10^{\frac{2}{3}}$ denotes the cube root of the square of 10 or

$$10^{\frac{2}{3}} = \sqrt[3]{10^2}$$

Also

$$8^{\frac{3}{4}} = \sqrt[4]{8^3}$$

$$3^{\frac{1}{3}} = \sqrt[3]{3}$$

$$7^{-\frac{1}{2}} = \frac{1}{7^{\frac{1}{2}}} = \frac{1}{\sqrt{7}}$$

$$225^{-\frac{3}{2}} = \frac{1}{225^{\frac{3}{2}}} = \frac{1}{\sqrt{225^3}}$$

41. Examples in Fractional Exponents. Under this paragraph number and heading, express each of the following in another form:

1. $5^{\frac{1}{2}}$

2. $36^{\frac{1}{3}}$

3. $72^{\frac{2}{3}}$

4. $29^{-\frac{1}{2}}$

5. $48^{-\frac{2}{3}}$

6. $118^{-\frac{1}{4}}$

7. $\sqrt{96}$

8. $\sqrt[3]{173}$

9. $\sqrt[4]{786^3}$

10. $\sqrt[5]{32^2}$

11. $\sqrt[3]{189.2^2}$

12. $753.8^{-\frac{3}{4}}$

13. $\sqrt[4]{560^3}$

14. $\sqrt{1728}$

15. $\sqrt[6]{192.78^2}$

16. $\dfrac{1}{8^2}$

17. $\dfrac{1}{16^3}$

18. $\dfrac{1}{\sqrt[3]{12^2}}$

19. $\dfrac{1}{\sqrt[4]{428^3}}$

20. $\dfrac{1}{\sqrt[5]{86.41^3}}$

21. $\dfrac{1}{\sqrt[3]{789.20}}$

22. $18.765^{-\frac{2}{7}}$

23. $5089^{-\frac{1}{3}}$

24. $10^{-\frac{3}{4}}$

25. $30^{\frac{3}{2}}$

26. $\sqrt[4]{20.942^3}$

27. $428^{-\frac{1}{4}}$

28. $\sqrt[5]{53.846^4}$

29. $609.72^{\frac{3}{5}}$

30. $\sqrt[4]{91.3043}$

§ 3. SQUARE ROOT

42. How to Extract the Square Root. The square root of a number may be determined by the following process:

1. Separate the number into periods of two figures each, in both directions from the decimal point.

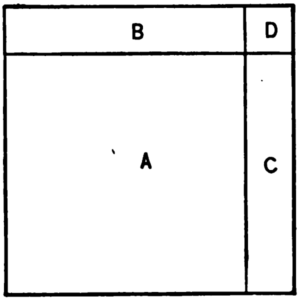


FIG. 7.

Thus

1875, 18′75
3.496, 3.49′60
118.0910, 1′18.09′10

2. Determine the number whose square is the largest which can be subtracted from the first period.

(*a*) Write this number as the first figure in the root.
(*b*) Subtract its square from the first period.
(*c*) Bring down the next period.

Thus

$$\begin{array}{l} 34'91.72(\underline{5} \\ \underline{25} \\ 991 \end{array}$$

3. At the left of the dividend, write the trial divisor as determined by the following law:

Trial divisor $= 20 \times$ root.

Thus

$$\begin{array}{rl} & 34'91.72(\underline{5} \\ & \underline{25} \\ 100) & \overline{991} \end{array}$$

In the above

991 is the dividend,
5 is the root,
100 or 20×5, is the trial divisor.

4. Determine the number of times the trial divisor is contained in the dividend.

(*a*) Write the number in the root,
(*b*) Add the number to the trial divisor.

Thus

$$\begin{array}{rl} & 34'91.72(\underline{59} \\ & 25 \\ \underline{109)} & \overline{\ 9\;91} \end{array}$$

5. Multiply and subtract as in long division and bring down the next period,

Thus

```
         34'91.72(59
       • 25
     109) 9 91
          9 81
           1072
```

6. Repeat the process as directed in 3, 4, and 5.

Thus

```
         34'91.72(59.09+
         25
    109) 9 91
         9 81
   1180) 1072
  11809)107200
        106281
           919
```

The second trial divisor 1180 was not contained in 1072. Therefore a cipher was entered in the root, and 107200 was brought down, the two ciphers being the next period.

43. Examples in Square Root. Under this paragraph number and heading in the work-book, extract the square root of the following:

1. 3481
2. 144
3. 256
4. 289
5. 225
6. 361
7. 196
8. 441
9. 169
10. 841
11. 784
12. 484
13. 5625
14. 4624
15. 31.49

16. 321.96
17. 5.4741
18. 179.56
19. 63.807
20. 3163.18
21. 796.82
22. 12.456
23. 8.19207
24. 297.34
25. 4896.34
26. 180.078
27. 53647.1
28. 3972.64
29. .05396
30. .0083196

§ 4. CUBE ROOT

44. How to Extract the Cube Root. The cube root of a number may be determined by the following process:

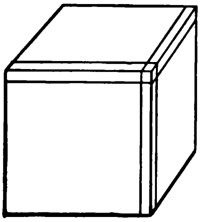


FIG. 8.

1. Separate the given number into periods of three figures each, in both directions from the decimal point.

Thus

34896,	34'896
129367,	129'367
1448.960,	1'448.960
.008371,	.008'371

2. Determine the number whose cube is the largest which can be subtracted from the first period.

(*a*) Write this number as the first figure in the root.

(*b*) Subtract its cube from the first period.

(*c*) Bring down the next period.

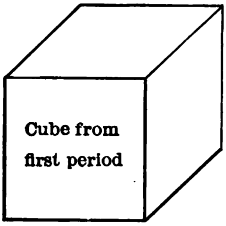


FIG. 9 (*a*).

Thus

$$\begin{array}{l} 13'284)\underline{2} \\ \underline{8\quad\ } \\ 5284 \end{array}$$

3. At the left of the dividend, write the trial divisor as determined by the following law:

Trial divisor = 300 × root^2.

Thus

$$\begin{array}{r@{}l} & 13'284)\underline{2} \\ & \underline{8} \\ \underline{1200)} & \;5284 \end{array}$$

In the above,

5284 is the dividend.
2 is the root,
1200, or 300 × 2^2, is the trial divisor.

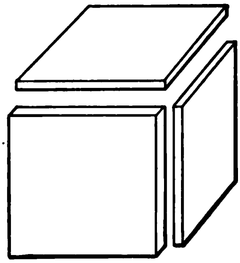


FIG. 9 (*b*).

4. Determine the number of times the trial divisor is contained in the dividend.

(*a*) Write the number in the root.

(*b*) Multiply it by 30 and by all the other figures of the root, and enter the result under the trial divisor.

(*c*) Square the number in (*a*) and write the result under the trial divisor.

(*d*) Make up the complete divisor by adding to the trial divisor the results obtained from (*b*) and (*c*).

Thus

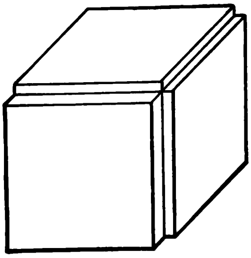


FIG. 9 (*c*).

$$\begin{array}{lrl} & & 13'284(\underline{23} \\ & 1200) & \underline{8} \\ (b) & 180 & 5284 \\ (c) & \underline{9} & \\ (d) & 1389 & \end{array}$$

(*b*) $3 \times 30 \times 2 = 180$

(*c*) $3^2 = 9$

(*d*) $1200 + 180 + 9 = 1389$

5. Multiply and subtract as in long division and bring down the next period.

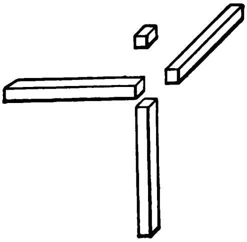


Fig. 9 (*d*).

Thus

$$\begin{array}{rl} & 13'284(\underline{23} \\ 1200 & \underline{8} \\ 180 & 5284 \\ \underline{9} & \underline{4167} \\ 1389 & 1117 \end{array}$$

6. Repeat the process as in 3, 4, and 5.

Thus

$$\begin{array}{lrl} & 1200 & 13'284(\underline{23.6+} \\ & 180 & \underline{8} \\ & \underline{9} & 5284 \\ & \underline{1389} & \underline{4167} \\ (a) & 158700 & 1117000 \\ (b) & 4140 & \underline{977256} \\ (c) & \underline{36} & 139744 \\ (d) & 162876 & \end{array}$$

(*a*). Trial divisor $=300\times 23^2 = 158700$

(*b*) Addition$_1$ $=6\times 30\times 23 = 4140$

(*c*) Addition$_2$ $=6^2 = \underline{36}$

(*d*) Complete divisor $= 162876$

45. Laws for Square Root.

1. Periods of two figures, in each direction from decimal point.
2. Trial divisor = 20 × root.
3. Complete divisor = trial divisor + last figure root.

46. Laws for Cube Root.

1. Periods of three figures, in each direction from decimal point.
2. Trial divisor $= 300 \times \text{root}^2$.
3. $\text{Addition}_1 = 30 \times \text{last figure root} \times \text{all except last}$.
4. $\text{Addition}_2 = (\text{last figure root})^2$.
5. Complete divisor $= \text{trial divisor} + \text{addition}_1 + \text{addition}_2$.

47. Examples in Cube Root. Under the paragraph number and title, extract the cube root of the following numbers.

The preceding page and the other explanatory pages of this section will tell you exactly what to do.

Do not waste time in trying to memorize the rules; you will soon know them by applying them to the examples.

1. 1728
2. 1331
3. 3375
4. 9261
5. 6859
6. 4096.
7. 12167
8. 7053.4
9. 25.3482
10. 21952
11. 22.934
12. 1694.52
13. 361.142
14. 438976
15. 804357
16. 18.9465
17. 681472
18. 56623104
19. 356.396
20. 2248091
21. 41063625
22. 12.9347
23. 32461759
24. 66430125
25. 44.1917
26. 6434.856
27. 36264691
28. 66430.125
29. 5802.958
30. 66923416
31. .0817563
32. .9572180
33. 1.181456
34. 3.14159
35. 4.2678309
36. .29600781
37. .7326841
38. 34.60084
39. 1.000695
40. .00096812
41. .000000934
42. 71.93408215
43. 17213.4954
44. 24036.871
45. 396082.963
46. 25.634008
47. 8630452.66
48. 7269.208645

§ 5. APPLIED PROBLEMS

48. Problems. Solve the following as indicated in each instance. Every result required should be so marked as to denote exactly what it represents, and should be double-underlined to be readily seen.

1. Area of Safety-valve for a Boiler. The area of the safety-valve for a boiler may be determined by allowing 1 square *inch* of valve to every 2 square *feet* of grate surface. The length of a grate multiplied by its width equals its surface or area.

Formulate and compute the surface of a grate measuring 12 feet by 10 feet, and the area of the safety-valve required, in square inches.

2. Size of Safety-valve for a Boiler. The size of a safety-valve is its diameter in inches, as a 3-inch valve, a 4-inch valve, etc.

When

D = diameter of valve in inches;
A = area of valve in square inches,

then

$$D=\sqrt{\frac{A}{.7854}}$$

Write the law.

3. Compute the diameter of the valve for the boiler in Problem 1.

4. A boiler is to be constructed with a grate measuring 6 feet by 8 feet. Compute the size of safety-valve required.

5. Size of Pop Valve for a Boiler. The rule for a pop safety-valve is that 1 square *inch* of valve shall be allowed for each 3 square *feet* of grate surface. Compute the area and the size of pop valve required for the boiler in Problem 1.

6. Formulate and compute the area and the size of the pop valve required for the boiler in Problem 4.

7. Velocity of Flow from a Pipe. The rate of discharge of water from a pipe in feet per second equals 8 times the square root of the head in feet.

Write a formula for this law.

8. Compute the velocity of flow from a pipe when the head is 90 feet.

9. Flow from a Tank. A tank 28 feet 9 inches high, has a discharge pipe on the side at a distance of 3 inches from the bottom.

Compute the rate of flow from the discharge pipe when the surface of the water is $3\frac{1}{2}$ feet below the top of the tank. The head in this instance is the distance of the discharge pipe from the surface of the water.

10. Flow Computed from Pressure Gage. When the pressure of the water at the place of discharge is denoted by a gage, the rate of discharge may be determined by the formula

$$V = 12.16\sqrt{P}$$

in which

V = velocity of flow in feet per second;

P = pressure in pounds per square inch, as read from the gage.

Write the law.

11. By the formula of Problem 10, compute the flow in feet per second, under a pressure of 45 lbs. per square inch.

12. If the pressure is increased to 225 lbs. per square inch, by high-pressure pumps, how much is the flow increased over that in Problem 11?

13. Diameter of a Chimney. The diameter of a chimney in inches equals 4, plus 13.54 times the square root of the effective area in square feet.

Write the formula, denoting diameter by D and effective area by E.

14. Compute the diameter of chimney required for an effective area of 6.57 square feet.

15. Effective Area of a Chimney. The effective area of a chimney may be determined by the formula,

$$E = A - 0.6\sqrt{A}$$

in which

E = effective area;

A = actual area.

Copy the formula and write the law.

16. By the formulas in the two preceding problems, compute and fill in, the omitted entries in the following table:

TABLE 10

CHIMNEYS

No.	*A**	*E*	*D*	No.	*A**	*E*	*D*
1	8.30			6	63.62		
2	15.90			7	70.88		
3	38.48			8	78.54		
4	44.18			9	95.03		
5	50.27			10	113.10		

* Entries in this column are from Swingle's 20th Century Handbook.

17. Volume of an Iron Ball. The volume or solid contents of an iron ball in cubic inches equals .5236 times the cube of the diameter of the ball in inches.

Formulate the law.

18. By the formula of Problem 17, compute the volume in cubic inches, of an iron ball 11 inches in diameter.

19. Weight of a Cast-iron Ball. If the iron ball of Problem 18 weighs .26 lb. per cubic inch, formulate and compute its weight to the nearest pound.

20. Time of Falling. The time in seconds, in which a body will fall under gravity from any height, equals the square root of the quotient of the height in feet divided by 16.08.

Formulate the law and compute the number of seconds required by a body to fall to the ground from a height of 1000 feet.

21. Time of Beat of a Pendulum. The time in seconds, in which a pendulum will make one beat, equals 3.1416 times the square root of the quotient of the length of the pendulum in feet divided by 32.16.

Write the formula.

22. Compute the time required for a pendulum 39.102 inches long, to make one beat.

23. Board Feet in a Log. The number of feet of boards which can be cut from a 12-foot log equals $\frac{4}{5}$ the number of square inches in half the square of its diameter.

How many feet of boards can be cut from a 12-foot log, 18 inches in diameter?

24. Altitude of an Equilateral Triangle. An equilateral triangle is one whose three sides are the same length. The altitude is the perpendicular distance from any vertex to the side opposite.

The altitude of an equilateral triangle equals $\frac{1}{2}$ the length of a side of the triangle, times the square root of 3.

Draw a figure in which the side is denoted by S and the altitude by h.

Write the formula.

25. Compute the altitude of an equilateral triangle whose side measures 125 inches.

26. Area of a Triangle. If the sides of a triangle are denoted by a, b, and c, and *half* the sum of the sides by s, the area is expressed by the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Draw the triangle to scale and compute the area when the sides are 18, 21, 26.

CHAPTER III

FRACTIONS

SECTION 1. REDUCTION. SECTION 2. ADDITION AND SUBTRACTION. SECTION 3. MULTIPLICATION. SECTION 4. DIVISION. SECTION 5. APPLIED PROBLEMS.

49. Illustration and Definition. A foot rule is marked off into twelve equal parts called inches.

Therefore

1 inch $= \frac{1}{12}$ of a foot,
2 inches $= \frac{2}{12}$ of a foot,
3 inches $= \frac{3}{12}$ of a foot,
4 inches $= \frac{4}{12}$ of a foot,

and so on.

Not only is the foot marked off into equal parts, but each inch is divided into 2 equal parts, 4 equal parts, 8 equal parts, 16 equal parts, etc.

Each of the 2 equal parts is called a half inch; each of the 4 equal parts is called a quarter inch, and so on.

The preceding numbers, $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ are called fractions, because they denote a part or fraction of some thing which has been divided.

We therefore have the following **definition.**

A fraction is a number which denotes one or more of the equal parts into which some thing has been divided.

You will observe that a fraction consists of three things:

(1) The fraction line, which may be horizontal or oblique, as $\frac{3}{4}$ or $^3/_4$.

(2) The numerator or number above the line, which denotes how many of the equal parts are taken.

(3) The denominator or number below the line, which denotes into how many equal parts the unit has been divided.

Thus in $\frac{5}{12}$, the denominator 12 denotes a division into 12 equal parts, and the numerator 5 denotes that 5 of these parts are taken.

A fraction therefore is an indicated division of the numerator by the denominator.

While for convenience, the oblique fraction line is sometimes used, your work will be more easily read if you use the horizontal instead of the oblique line.

§ 1. REDUCTION

50. Reduction to Lowest Terms. A fraction is said to be in its lowest terms when no number will evenly divide both numerator and denominator.

Thus $\frac{8}{9}$ is in its lowest terms because no number will evenly divide both its numerator and denominator.

On the contrary, $\frac{6}{9}$ is not in its lowest terms because 3 will evenly divide both numerator and denominator.

Thus $\frac{6}{9}=\frac{2}{3}$, both terms being divided by 3.

We therefore have the following

Rule for Reducing to Lowest Terms. To reduce a fraction to its lowest terms, divide both its numerator and denominator by any number that is evenly contained. Continue this process until further division is impossible.

Thus

$$\frac{\cancel{1}\cancel{9}\cancel{6}}{\cancel{2}\cancel{2}\cancel{4}}=\frac{\cancel{4}\cancel{9}}{\cancel{5}\cancel{6}}=\frac{7}{8}$$

The first divisor was 4, the second 7. Further division is impossible.

Work of this kind is sometimes exhibited as follows, the process being called cancellation:

$$\begin{array}{c} 7 \\ \cancel{4}\cancel{9} \\ \dfrac{\cancel{1}\cancel{9}\cancel{6}}{\cancel{2}\cancel{2}\cancel{4}} = \dfrac{7}{8} \\ \cancel{5}\cancel{6} \\ 8 \end{array}$$

When both numerator and denominator end in ciphers, the same number of ciphers should be cancelled in each, before cancelling other factors.

Thus

$$\frac{75\cancel{0}\cancel{0}}{350\cancel{0}\cancel{0}}=\frac{75}{350}$$

51. Examples in Reduction. Under the paragraph number and title, reduce the following fractions to the lowest terms by cancellation:

1. $\frac{6}{8}$ 2. $\frac{14}{16}$ 3. $\frac{15}{25}$

4. $\frac{6}{32}$ 5. $\frac{4}{64}$ 6. $\frac{14}{56}$

7. $\frac{16}{28}$ 8. $\frac{24}{32}$ 9. $\frac{48}{72}$

10. $\frac{50}{380}$ 11. $\frac{510}{1530}$ 12. $\frac{400}{1400}$

13. $\frac{13}{78}$ 14. $\frac{17}{51}$ 15. $\frac{125}{400}$

16. $\frac{63}{700}$ 17. $\frac{185}{725}$ 18. $\frac{118}{472}$

19. $\frac{288}{1728}$ 20. $\frac{3.1416}{4}$ 21. $\frac{1492}{8326}$

22. $\frac{428}{1476}$ 23. $\frac{968}{1244}$ 24. $\frac{13000}{78000}$

52. Reduction of a Mixed Number. A mixed number is a combination of an integer and a fraction.

Thus

$$18\frac{3}{4},\quad 3\frac{5}{9},\quad 124\frac{1}{2}.$$

A mixed number is reduced to full fractional form by the following

Rule for Reducing a Mixed Number. Multiply the integral part by the denominator, add the numerator to the product and write it over the denominator.

Thus

$$18\frac{3}{4}=\frac{75}{4}$$

The integral part 18, was multiplied by 4, giving 72, plus the numerator 3, making $\frac{75}{4}$.

53. Examples in Reduction of a Mixed Number. Under the paragraph number and title, reduce the following expressions to full fractional form.

1. $3\frac{1}{2}$ **2.** $5\frac{3}{4}$ **3.** $11\frac{1}{8}$
4. $6\frac{7}{16}$ **5.** $8\frac{2}{13}$ **6.** $9\frac{9}{15}$
7. $121\frac{10}{15}$ **8.** $10\frac{34}{41}$ **9.** $11\frac{3}{32}$
10. $7\frac{2}{11}$ **11.** $12\frac{3}{12}$ **12.** $13\frac{4}{13}$
13. $34\frac{3}{4}$ **14.** $2\frac{732}{900}$ **15.** $394\frac{3}{4}$
16. $17\frac{18}{25}$ **17.** $200\frac{4}{13}$ **18.** $85\frac{11}{30}$
19. $116\frac{7}{32}$ **20.** $421\frac{3}{4}$ **21.** $84\frac{15}{32}$
22. $890\frac{12}{31}$ **23.** $1292\frac{7}{8}$ **24.** $5000\frac{28}{40}$

§ 2. ADDITION AND SUBTRACTION OF FRACTIONS

54. How to Add or Subtract Fractions. Fractions can be added or subtracted only when their *denominators* are exactly alike. They are added by adding their numerators, and subtracted by subtracting their numerators.

Thus

$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4} \quad \text{and} \quad \frac{1}{16}+\frac{3}{16}+\frac{2}{16}=\frac{6}{16}=\frac{3}{8}$$

Subtracting

$$\frac{5}{8}-\frac{3}{8}=\frac{2}{8}=\frac{1}{4}$$

When the denominators are not alike, they must be made so before adding.

Thus $\frac{1}{4}+\frac{1}{2}+\frac{1}{8}$ can be added only when reduced to the same denominator. This for convenience should be the smallest number in which each of the denominators is evenly contained, and which for this reason is called

THE LEAST COMMON DENOMINATOR. Now 8 is the smallest number in which the denominators 4, 2, and 8 are contained.

Since 4 is contained in 8, 2 times, and 2 is contained in 8, 4 times, we multiply both numerator and denominator of $\frac{1}{4}$ by 2, and both numerator and denominator of $\frac{1}{2}$ by 4.

Thus

$$\frac{1\times 2}{4\times 2}=\frac{2}{8} \quad \text{and} \quad \frac{1\times 4}{2\times 4}=\frac{4}{8}$$

Therefore

$$\frac{1}{4}+\frac{1}{2}+\frac{1}{8}=\frac{2}{8}+\frac{4}{8}+\frac{1}{8}=\frac{7}{8}$$

55. Illustrations in Adding and Subtracting Fractions. The following illustrative examples will help you to understand how fractions are added or subtracted.

1. $$\frac{1}{3}+\frac{2}{3}+\frac{5}{3}=\frac{8}{3}=2\tfrac{2}{3}$$

In this example, the denominators are alike. The fractions were therefore added by adding their numerators as shown, and reducing the result to a mixed number.

2. $$\frac{9}{16}-\frac{4}{16}=\frac{5}{16}$$

The denominators are alike. Therefore the second numerator was subtracted from the first as shown.

3. $$\frac{1}{12}+\frac{1}{6}+\frac{2}{3}+\frac{5}{24}=\frac{2}{24}+\frac{4}{24}+\frac{16}{24}+\frac{5}{24}=\frac{27}{24}=1\tfrac{3}{24}=1\tfrac{1}{8}$$

The least common denominator is 24, as shown.

56. How to Determine a Least Common Denominator. When the least common denominator or L.C.D. is difficult to determine by inspection, the following process should be used:

$$\begin{array}{r|cccccc}
 & \frac{1}{5}+ & \frac{3}{16}+ & \frac{5}{48}+ & \frac{4}{9}+ & \frac{7}{30}+ & \frac{2}{45} \\
5) & 5 & 16 & 48 & 9 & 30 & 45 \\ \hline
9) & 1 & 16 & 48 & 9 & 6 & 9 \\ \hline
16) & 1 & 16 & 48 & 1 & 6 & 1 \\ \hline
3) & 1 & 1 & 3 & 1 & 6 & 1 \\ \hline
 & 1 & 1 & 1 & 1 & 2 & 1
\end{array}$$

$$2\times3\times16\times9\times5=\text{L.C.D.}=4320$$

The L.C.D. is the product of the divisors 5, 9, 16, and 3, and the numbers under the last line. Any number may be used as a divisor, if evenly containable in two or more denominators.

57. Examples in Addition and Subtraction of Fractions. Under the paragraph number and title, solve the following examples. Be sure to reduce the results to their lowest terms.

1. $\frac{1}{2}+\frac{3}{8}+\frac{1}{4}+\frac{5}{16}$ **2.** $\frac{3}{64}+\frac{5}{8}+\frac{7}{8}+\frac{9}{32}+\frac{3}{4}$

3. $\frac{2}{3}+\frac{8}{15}+\frac{17}{35}+\frac{7}{9}$ **4.** $\frac{5}{7}+\frac{5}{8}+\frac{9}{14}+\frac{13}{56}$

5. $\frac{3}{8}+\frac{7}{9}+\frac{15}{36}+\frac{11}{72}$ **6.** $\frac{3}{4}+\frac{5}{32}+\frac{11}{64}+\frac{13}{16}$

7. $\frac{8}{15}+\frac{7}{45}+\frac{23}{90}$ **8.** $\frac{7}{12}+\frac{11}{60}+\frac{4}{5}-\frac{27}{120}$

9. $\frac{1}{9}+\frac{4}{27}+\frac{5}{81}+\frac{7}{18}$ **10.** $\frac{5}{18}+\frac{7}{90}+\frac{11}{54}+\frac{2}{3}+\frac{5}{6}$

11. $\frac{198}{210}-\frac{3}{14}$ **12.** $\frac{12}{39}-\frac{5}{78}$

13. $\frac{29}{64}-\frac{15}{72}$ **14.** $\frac{22}{35}-\frac{21}{49}$

15. $\frac{3}{8}+\frac{5}{64}+\frac{11}{32}-\frac{1}{2}$ **16.** $\frac{18}{25}-\frac{3}{15}+\frac{13}{75}-\frac{7}{25}$

17. $\frac{15}{16}-\frac{11}{32}-\frac{1}{4}+\frac{5}{64}$ **18.** $\frac{1}{9}+\frac{8}{81}-\frac{11}{162}+\frac{3}{9}$

19. $18\frac{5}{8}+3\frac{1}{2}+6\frac{5}{16}-\frac{3}{4}$ **20.** $\frac{11}{32}+\frac{7}{64}-\frac{1}{8}+16\frac{2}{3}$

§ 3. MULTIPLICATION OF FRACTIONS

58. How to Multiply Fractions. A fraction may be multiplied in two ways:

(1) By multiplying its numerator,
(2) By dividing its denominator.

Thus

$$4\times\frac{3}{8}=\frac{12}{8}=\frac{3}{2}=1\tfrac{1}{2}\quad\text{or}\quad 4\times\frac{3}{\underset{2}{\cancel{8}}}=\frac{3}{2}=1\tfrac{1}{2}$$

The second method should be used when possible; otherwise multiply the numerator.

Thus

$$7\times\frac{3}{16}=\frac{21}{16}=1\tfrac{5}{16}$$

The numerator was multiplied because cancellation into the denominator was impossible.

Two or more fractions may be multiplied together by multiplying their numerators together, and their denominators together, cancelling from any numerator into any denominator when possible.

Thus

$$\frac{3}{4}\times\frac{8}{21}\times\frac{14}{29}=\frac{4}{29}$$

$$\frac{5}{8}\times\frac{1}{2}\times\frac{3}{7}=\frac{15}{112}$$

$$\overset{4}{\cancel{10}}\times\frac{5}{\underset{13}{\cancel{52}}}=\frac{20}{13}=1\tfrac{7}{13}$$

59. Examples in Multiplication of Fractions. Under paragraph number and title, perform the following indicated operations:

1. $4\times\frac{5}{9}$

2. $7\times\frac{15}{16}$

3. $8\times\frac{5}{64}$

4. $34\times\frac{5}{72}$

5. $3\times\frac{16}{81}$

6. $120\times\frac{12}{75}$

7. $\frac{3}{8}\times\frac{1}{6}$

8. $\frac{5}{9}\times\frac{18}{25}\times\frac{1}{2}$

9. $\frac{3}{16}\times\frac{5}{8}\times\frac{3}{4}$

10. $8\times\frac{3}{4}\times\frac{7}{36}\times\frac{1}{2}$

11. $\frac{22}{35}\times\frac{5}{44}\times\frac{3}{8}$

12. $\frac{7}{16}\times56\times\frac{5}{28}\times\frac{1}{15}$

13. $\frac{1}{9}\times\frac{27}{60}\times\frac{5}{8}\times\frac{3}{21}$

14. $\frac{13}{18}\times\frac{72}{15}\times\frac{1}{52}\times15$

15. $\frac{1}{12}\times1728\times\frac{3}{8}$

16. $300\times16.75\times\frac{3}{25}$

17. $\frac{5}{9}\times\frac{1080}{15}\times35$

18. $\frac{5}{16}\times\frac{2}{3}\times18\times\frac{126}{205}\times\frac{1}{9}$

§ 4. DIVISION OF FRACTIONS

60. How to Divide Fractions. A fraction may be divided in two ways:

(1) By multiplying its denominator,

(2) By dividing its numerator.

Thus

$$\frac{3}{8}\div2=\frac{3}{8\times2}=\frac{3}{16}$$

$$\frac{16}{45}\div8=\frac{16}{45\times8}=\frac{2}{45}$$

$$\frac{60}{109}\div45=\frac{60}{109\times45}=\frac{4}{327}$$

61. A Fractional Divisor. When the divisor is a fraction, it must be inverted and used as a multiplier.

Thus

$$\frac{1}{2}\div\frac{3}{8}=\frac{1}{\cancel{2}}\times\frac{\overset{4}{\cancel{8}}}{3}=\frac{4}{3}=1\frac{1}{3}$$

$$\frac{5}{16}\div\frac{15}{32}=\frac{\cancel{5}}{\cancel{16}}\times\frac{\overset{2}{\cancel{32}}}{\underset{3}{\cancel{15}}}=\frac{2}{3}$$

$$\frac{8}{50}\div\frac{3}{100}=\frac{8}{50}\times\frac{100}{3}=\frac{16}{3}=5\frac{1}{3}$$

62. Examples in Division of Fractions. Under the paragraph number and title, solve the following as shown in the preceding illustrations:

1. $\frac{3}{4}\div5$

2. $\frac{7}{8}\div14$

3. $\frac{9}{16}\div81$

4. $\frac{11}{18}\div4$

5. $\frac{1}{3}\div\frac{8}{9}$

6. $\frac{5}{2}\div\frac{10}{31}$

7. $\frac{3}{32}\times\frac{16}{27}\div\frac{81}{144}$

8. $\frac{1}{16}\div3\times\frac{32}{112}$

9. $\frac{3}{8}\times\frac{16}{33}\times\frac{1}{2}\div\frac{1}{16}$

10. $\frac{11}{30}\times\frac{5}{121}\div\frac{42}{58}$

11. $15\times\frac{14}{210}\div8\times\frac{7}{64}$

12. $5^2\times\frac{13}{75}\div\frac{39}{48}\times144$

13. $72^{-2}\times\frac{18}{5}\times\frac{144}{42}$

14. $\frac{3}{4}\times\frac{8}{51}\div\frac{27}{102}$

15. $\frac{7}{12}\times\frac{84}{147}\times\frac{15}{36}$

16. $\frac{55}{64}\div45\times\frac{3}{16}$

17. $\frac{5}{8}\div\frac{9}{2}\times\frac{7}{50}\times\frac{8}{32}$

18. $\frac{3}{4}\div 17\times\frac{9}{8}\div 42$

19. $14^2\div 96\times\frac{16}{50}\div\frac{15}{200}$

20. $\frac{1}{3.1416}\div\frac{1}{4}\times\frac{96}{120}\times 72$

63. How to take Half of an Odd Number and a Fraction. An odd number is a number not evenly divisible by 2.

Thus 3, 5, 7, 9, 21, etc., are odd numbers.

In computation it is often necessary to take half of a mixed number whose integral part is odd.

Thus

$$\frac{1}{2}\times 35\tfrac{7}{8} \quad \text{or} \quad 35\tfrac{7}{8}\div 2$$

In either case we think $\frac{1}{2}$ of 35 is 17 and 1 over.

Reducing the 1 to 8ths, we have

$$\frac{8}{8}+\frac{7}{8}=\frac{15}{8} \quad \text{and} \quad \frac{1}{2}\times\frac{15}{8}=\frac{15}{16}$$

Therefore

$$\frac{1}{2}\times 35\tfrac{7}{8}=17\tfrac{15}{16}$$

The same result would have been obtained by taking $\frac{1}{2}$ of 35 and by writing the sum of the numerator and denominator, over twice the denominator.

Thus

$$\frac{1}{2}\times 35\tfrac{7}{8}=17\tfrac{15}{16}$$

$$\frac{1}{2}\times 43\tfrac{3}{16}=21\tfrac{19}{32}$$

This method has been published frequently and should be in common use.

64. Problems. Under the paragraph number and title, take half of the following numbers, setting down the results by inspection by the method of the preceding paragraph:

1. $9\frac{3}{8}$ **2.** $7\frac{5}{16}$ **3.** $3\frac{11}{64}$
4. $5\frac{23}{32}$ **5.** $19\frac{1}{2}$ **6.** $15\frac{3}{4}$
7. $27\frac{1}{3}$ **8.** $43\frac{1}{6}$ **9.** $55\frac{3}{29}$
10. $67\frac{4}{5}$ **11.** $101\frac{3}{7}$ **12.** $391\frac{13}{64}$
13. $563\frac{7}{8}$ **14.** $97\frac{31}{32}$ **15.** $187\frac{61}{64}$
16. $371\frac{37}{64}$ **17.** $1121\frac{40}{53}$ **18.** $1373\frac{11}{15}$
19. $2903\frac{7}{8}$ **20.** $3789\frac{15}{32}$ **21.** $4293\frac{12}{17}$
22. $50007\frac{112}{129}$ **23.** $6187\frac{11}{49}$ **24.** $7833\frac{17}{24}$
25. $4541\frac{320}{631}$ **26.** $8725\frac{3}{8}$ **27.** $6045\frac{13}{64}$
28. $2883\frac{9}{16}$ **29.** $5173\frac{5}{16}$ **30.** $9349\frac{10}{37}$

§ 5. APPLIED PROBLEMS

65. Problems. Under the paragraph number and title and problem numbers and titles, solve the following problems:

1. Circle Computation. Rule the following table in the workbook, and formulate, compute and fill in the omitted entries.

Make your formulation from the following laws:

(1) Radius equals $\frac{1}{2}$ diameter.
(2) Circumference equals 3.1416 times diameter.
(3) Area equals 3.1416 times radius squared.

TABLE 11

DIAMETERS, RADII. CIRCUMFERENCES, AREAS

No.	Diam.	Rad.	Cir.	Area.	No.	Diam.	Rad.	Cir.	Area.
1	$7\frac{5}{16}$				8	$5\frac{3}{4}$			
2	$6\frac{3}{8}$				9	$13\frac{5}{32}$			
3	$11\frac{7}{32}$				10	$15\frac{7}{16}$			
4	$9\frac{1}{8}$				11	$17\frac{11}{32}$			
5	$19\frac{9}{16}$				12	$21\frac{13}{32}$			
6	$23\frac{15}{32}$				13	$22\frac{4}{16}$			
7	$8\frac{3}{4}$				14	$12\frac{1}{8}$			

2. Decimal Equivalents of the Fractional Parts of an Inch.

Rule the following table in the work-book and compute and fill in all the omitted decimal entries.

Rule the decimal columns wide enough for six figures.

TABLE 12

DECIMALS OF AN INCH FOR EACH 64TH

$\frac{1}{32}$d	$\frac{1}{64}$th	Fraction.	Decimal.	$\frac{1}{32}$d	$\frac{1}{64}$th	Fraction.	Decimal.
	1				33		
1	2			17	34		
	3				35		
2	4	$\frac{1}{16}$		18	36	$\frac{9}{16}$	
	5				37		
3	6			19	38		
	7				39		
4	8	$\frac{1}{8}$		20	40	$\frac{5}{8}$	
	9				41		
5	10			21	42		
	11				43		
6	12	$\frac{3}{16}$		22	44	$\frac{11}{16}$	
	13				45		
7	14			23	46		
	15				47		
8	16	$\frac{1}{4}$		24	48	$\frac{3}{4}$	
	17				49		
9	18			25	50		
	19				51		
10	20	$\frac{5}{16}$		26	52	$\frac{13}{16}$	
	21				53		
11	22			27	54		
	23				55		
12	24	$\frac{3}{8}$		28	56	$\frac{7}{8}$	
	25				57		
13	26			29	58		
	27				59		
14	28	$\frac{7}{16}$		30	60	$\frac{15}{16}$	
	29				61		
15	30			31	62		
	31				63		
16	32	$\frac{1}{2}$		32	64	1	

3. Safe Working Strength of Leather Belts. The ultimate strength of leather belting varies from 2800 to 6000 pounds per square inch of sectional area. The actual strength of a laced belt, however, is its strength at the lacing, where it varies from 1000 to 1500 pounds, giving an average of 1250 pounds.

If a factor of safety of about 4 were taken, the safe working strength would be about 300 pounds per square inch of sectional area.

Compute and fill in the omitted entries in the following table, using the following law:

Safe working strength of a belt per inch of width = 300 times thickness of belt.

In the table,

W = safe working strength per inch of width,
T = thickness of belt,
$W = 300 \times T$.

TABLE 13

SAFE WORKING STRENGTH OF BELTS PER INCH OF WIDTH

T	$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$
W									

4. The circumference of a circle is divided into 360 equal parts called degrees.

If C be used to denote the circumference, we have the formulas,

$$360° = C,$$

and

$$1° = \frac{1}{360} \times C.$$

Copy the two formulas, and in the following equations fill in the proper numbers in place of the interrogation points.

Rewrite each of the three equations with the fractions reduced to their lowest terms. Write again with the fractions reduced to a decimal of three places:

$$5^\circ = \frac{?}{360} \times C,$$

$$10^\circ = \frac{?}{360} \times C,$$

$$n^\circ = \frac{?}{360} \times C. \tag{1}$$

5. Rule the following table in the work-book and compute and fill in the omitted entries, using formula (1) of the preceding problem.

TABLE 14

FRACTIONS AND DECIMALS OF CIRCUMFERENCE

Arc in Degrees.	Fraction.	Decimal.	Arc in Degrees.	Fraction.	Decimal.
15			165		
30			180		
45			195		
60			210		
75			225		
90			240		
105			255		
120			270		
135			285		
150			300		

6. In Kent's Handbook, the tensile strength of single leather belting at the lacing is said to vary from 1000 to 1500 pounds per square inch of cross-sectional area.

Compute the average tensile strength.

7. In round numbers, compute the approximate average tensile strength per inch of width, of a single leather belt $\frac{5}{32}$ of an inch thick.

8. When a belt drives a machine, the tension on the tight side plus one-half the tension on the slack side, should equal 240 lbs. per square inch of cross-section for all speeds.

Determine by computation whether this is true in the following:

With a speed of 3000 ft. per minute, the tension on the tight side is 207 lbs., and the tension on the slack side is 68 lbs., per square inch of cross-section.

9. Mass. The mass of any substance or material equals its weight in pounds divided by the force of gravity.

Write the formula, denoting mass by m, weight by w, and gravity by g.

Compute the mass of a body weighing 1800 lbs., the force of gravity being equal to 32.2.

10. Kinetic Energy. The kinetic (moving) energy of a body is determined from the formula

$$K=\frac{m}{2}\times v^2,$$

in which K = kinetic energy;

m = mass of the moving body;

v = velocity in feet per second.

Copy the formula and write the law.

In place of m in the formula, write its value as formulated in Problem 9.

11. A.S.M.E. Standard Machine Screws. The threads per inch, of machine screws by the standard adopted by the American Society of Mechanical Engineers in 1907, may be computed by the formula,

$$T=\frac{6.5}{D+0.02},$$

in which T = threads per inch;

D = minimum outside diameter in inches.

Compute and fill in the omitted entries in Table 15:

TABLE 15

A.S.M.E. STANDARD MACHINE SCREWS

No.	D	T	No.	D	T
1	.070		6	.1338	
2	.0828		7	.1466	
3	.0955		8	.1596	
4	.1082		9	.1723	
5	.1210		10	.1852	

12. Dimensions of Standard Mandrels.

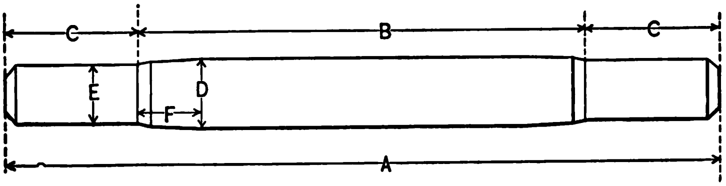


FIG. 10.

A = total length;
B = length of actual mandrel;
C = length of end for dog;
D = diameter of mandrel;
E = diameter of end for dog;
F = distance of size from small end.

The formulas below give the dimensions of mandrels within the size limits named:

Diameters $\frac{1}{2}''$ to $2''$.	Diameters $2\frac{1}{16}''$ to $4''$.
$A = 4D + 3''$	$A = 3D + 5''$
$B = 3D + 1\frac{5}{8}''$	$B = 2D + 3\frac{5}{8}''$
$C = \frac{D}{2} + \frac{11}{16}''$	$C = \frac{D}{2} + \frac{11}{16}''$
$E = \frac{7}{8}D$	$E = \frac{7}{8}D$
$F = \frac{3D}{4} + \frac{7}{16}''$	$F = \frac{D}{2} + \frac{15}{16}''$

Compute each dimension.

13. Diameter of Main Steam-pipe. In Kent's Pocketbook, the following formula is given for determining the diameter of main steam-pipes in marine engines:

$$D_s = D_c\sqrt{\frac{S}{V}},$$

in which D_s = diameter main steam-pipe in inches;

D_c = diameter of cylinder in inches;

S = piston speed in feet per minute;

V = mean velocity of steam in the pipe in feet per minute.

Copy the formula and write the law.

14. The diameter of the cylinder of a marine engine is 35 inches.

Compute the required diameter of the main steam-pipe, for a maximum piston speed of 860 ft. per minute, allowing a mean steam velocity of 8100 ft. per minute.

15. Breaking Load on Flagging. The approximate central concentrated load, under which stone flagging will break when resting on end supports, may be determined from the formula below and the table in Problem 16.

$$L = \frac{w \times t^2}{d} \times n,$$

in which L = breaking load in short tons (2000 lbs.);

w = width of stone in inches;

t = thickness of stone in inches;

* n = tabular number;

d = distance between supports in inches.

Copy the formula and write the law.

---

* See Table 16.

16. Compute and fill in the omitted entries in the table:

TABLE 16

BREAKING LOAD ON FLAGGING

No.	w	t	d	n	L	Material.
1	36	2	24	.744		Bluestone
2	60	3½	42	.624		Quincy Granite
3	84	4½	60	.312		Conn. Freestone
4	48	3	38	.576		L.F. Freestone
5	54	5	50	.264		Dor. Freestone

17. Cutting a Brass Tube. A brass tube 4 ft. 9 ins. long is to be cut into pieces 6½ ins. in length.

How many such pieces will it make, if there is a waste of $\frac{1}{16}$ of an inch on each piece in cutting?

18. Dimensions of Hand Reamers. The following formulas may be used to determine the dimensions of reamers within the diameter limits named:

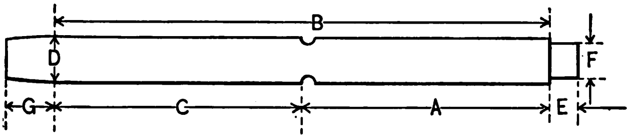


FIG. 11.

Diameters $\frac{1}{16}''$ to $1''$.	Diameters $1\frac{1}{16}''$ to $4''$.
$B=7D+1\frac{3}{4}''$	$B=3D+6''$
$C=4D+\frac{5}{8}''$	$C=1\frac{3}{4}D+3''$
$A=3D+1\frac{1}{8}''$	$A=1\frac{1}{4}D+3''$
$E=\frac{1}{2}D+\frac{1}{16}''$	$E=\frac{1}{2}D+\frac{3}{16}''$
$F=\frac{3}{4}D$	$F=\frac{3}{4}D$
$G=\frac{3}{4}D+\frac{1}{8}''$	$G=\frac{1}{2}D+\frac{3}{8}''$

Formulate and compute each dimension when $D=\frac{7}{8}''$; also when $D=1\frac{1}{2}''$.

19. Number of Wires within a Pipe. The number of wires of the same size which can be laid in a pipe or tube, may be computed by the folllowing approximate formula:

$$N=.907\left(\frac{D}{d}-.94\right)^2+3.7,$$

in which N = number of wires;
D = diameter of tube;
d = diameter of wire;

Copy the formula and notation, and show that the formula would give exactly the same results if written

$$N=.907\left(\frac{R}{r}-.94\right)^2+3.7,$$

in which R = radius of tube;
r = radius of wire.

20. Compute the number of $\frac{3}{8}$-inch wires which can be placed in a $3\frac{1}{2}$-inch pipe.

21. Ultimate Strength of Manila Rope. The breaking strength of a new manila rope in pounds may be computed by the following formula of Mr. T. Spencer Miller, as recorded in Engineering News of Dec. 6, 1890:

$$B=C^2\times c,$$

in which B = breaking strength in pounds,
C = circumference of rope in inches,
c = coefficient in table below.

TABLE 17

ULTIMATE STRENGTH OF MANILA ROPE

No.	Diam.	C	c	B	No.	Diam.	C	c	B
1	$\frac{1}{2}$	$1\frac{1}{2}$	900		7	$1\frac{1}{4}$	$3\frac{3}{4}$	760	
2	$\frac{5}{8}$	2	845		8	$1\frac{3}{8}$	$4\frac{1}{4}$	745	
3	$\frac{3}{4}$	$2\frac{1}{2}$	820		9	$1\frac{1}{2}$	$4\frac{1}{2}$	735	
4	$\frac{7}{8}$	$2\frac{3}{4}$	790		10	$1\frac{5}{8}$	5	725	
5	1	3	780		11	$1\frac{3}{4}$	$5\frac{1}{2}$	712	
6	$1\frac{1}{8}$	$3\frac{1}{2}$	765		12	2	6	700	

22. Painting Estimate. In House and Garden for September, 1911, Mr. George E. Walsh gives the following ways of estimating the amount of white lead in oil, required for a given area:

(1) First coat, 10 lbs. will cover 221 sq.ft; second coat, 10 lbs. will cover 324 sq.ft.

(2) Number of *gallons* required for two coats equals total area to be painted in square feet, divided by 200.

Formulate the law for each coat separately as given in (1), and for two coats as given in (2).

Compute the number of gallons required for two coats on 3000 sq.ft., by both methods, allowing 15¼ lbs. to the gallon.

23. The tension on the tight side of a belt which drives a machine is 173 lbs. The tension on the slack side is 134 lbs.

Show by computation whether these tensions fulfill the requirement of the law stated in Problem 8.

24. When a belt with an arc of contact of 180° drives a countershaft, the tension on the tight side plus one-half the tension on the slack side should equal 160 lbs. per square inch of cross-section.

Are the requirements of this law met when the tension on the tight side is 140 lbs. and on the slack side 41 lbs.? Why?

25. *Fuller's Formulas for Concrete.* If c, s, and g denote the number of parts of cement, sand, and gravel or broken stone in concrete; Q_c, the number of *barrels* of cement; Q_s and Q_g, the number of *cubic yards* of sand and gravel, respectively, the quantity of each which is required for 1 *cubic yard of concrete* may be determined from the formulas,

$$Q_c = \frac{11}{csg}; \; Q_s = \frac{3.8sQ_c}{27}; \; Q_g = \frac{3.8gQ_c}{27}.$$

Compute and tabulate the omitted entries below:

TABLE 17A

MATERIALS FOR ONE CUBIC YARD OF CONCRETE

Parts			Cement, bbl.	Sand, cu. yd.	Broken Stone or gravel, cu. yd.
Cement	Sand	Gravel			
1	2	3			
1	2	5			
1	3	7			
1	3	6			
1	1½	3			
1	2½	6			
1	2½	5			
1	4	6			

CHAPTER IV

WEIGHTS AND MEASURES

SECTION 1. HISTORICAL. SECTION 2. MEASURES OF LENGTH. SECTION 3. MEASURES OF AREA. SECTION 4. MEASURES OF VOLUME. SECTION 5. MEASURES OF WEIGHT. SECTION 6. MEASURES OF TIME. SECTION 7. MONEY. SECTION 8. APPLIED PROBLEMS.

§ 1. HISTORICAL

66. How Standards are Established. By the Constitution of the United States, the power to fix a standard of weights and measures is given to Congress. In the time of Washington, there were no standards and both he and the second president, John Adams, made several attempts to have Congress take action.

Finally in 1834 the Treasury Department adopted the following standards which were approved by Congress, two years later, and are standard throughout the United States:

The English yard = 36 inches;
*The wine gallon = 231 cubic inches;
The avoirdupois pound = 7000 grains;
*The Winchester bushel = 2150.42 cubic inches.

67. How the States Receive Standards. The same Congress which approved the four standards of the Treasury Department directed that the states of the Union be supplied with standards by the secretary of the Treasury, and by 1850 all states then admitted had received a full set of standard weights and measures. Since that time, each state has been supplied with standards, on admission to the Union.

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* No longer used in Great Britain.

All the states have also received the standard weights and measures of the Metric system, the use of which in the United States was legalized by Congress in 1866.

68. The Yard. The yard of 36 inches, when adopted, was the length between the twenty-seventh and the sixty-third inch marks on a brass bar eighty-two inches long, brought to the United States in 1813 and called from the London maker, the Troughton scale, and at 62° Fahrenheit presumably the exact length of the English standard yard of 1760 which was destroyed by the fire in the Houses of Parliament in 1834.

In 1856 the United States received from England two bars, copies of the British imperial yard which was legalized there in 1855, by both of which the yard of the Troughton scale was found to be .0087 of an inch too long. The bars were therefore adopted by the Office of Weights and Measures, as standards for the United States, in place of the Troughton scale.

By the act of 1893, the standards of the International Bureau of Weights and Measures, maintained in France by the principal nations, were made the basis of the common units in the United States. Our standard yard therefore since that time, has been $\frac{3600}{3937}$ of a meter.

69. The Gallon. The U.S. gallon of 231 cubic inches was the Winchester wine measure legally declared in 1689 as having a volume of 231 cubic inches.

Adopted by England in 1707 as a cylinder 7 inches in diameter and 6 inches high, it was used there until superseded in 1824 by the present imperial wine gallon of $277\frac{1}{4}$ cubic inches, approximately, or exactly 277.274 cubic inches.

70. The Troy Pound. The pound troy which is so named from its use in the city of Troyes as early as the beginning of the fifteenth century, was legalized in England by Edward III in 1497.

The avoirdupois and troy pound weights, made under Queen Elizabeth in 1588, were legal standards in England until 1824.

A standard troy pound weight, an exact copy of the imperial troy pound of Great Britain, was brought to the United States in 1827 and was made the standard of the Philadelphia Mint the following year.

71. The Avoirdupois Pound. This pound was introduced into England from Bayonne, France, about 1300, the word avoirdupois meaning " goods-of-weight."

The standard avoirdupois pound of the United States is $\frac{7000}{5760}$ of the pound troy.

72. The Bushel. The Winchester corn bushel of 2150.42 cubic inches, was the earliest recorded measure of capacity in England.

When superseded there in 1824 by the present imperial bushel of 2218.19 cubic inches, it had been in common use since the days of the Anglo-Saxons.

It takes its name from the fact that the ancient standard measure was kept in the town-hall of the city of Winchester.

73. The Metric System. Metric is the adjective form of the word meter which is a French word meaning measure. The metric system is therefore a measure system used in determining length, area, volume, and weight.

This system had its origin in a report to the French National Assembly in 1791. The object of this report was the establishment of a simple system, based on an absolutely fixed standard. French engineers were therefore employed who measured and computed the earth's quadrant (one-fourth circumference) through Paris, with very great care, their work being finished in 1799. One ten-millionth of this length was taken as the length of a meter.

As might be supposed, it was impossible to determine the quadrant with sufficient accuracy for a standard, and the result was, that the meter was finally declared by law to be 39.37 inches.

The use of the international metric system is now required in the following dependencies and departments of the United States:

In Porto Rico by proclamation of the military governor, 1899; in the Philippines by Act of Congress, 1901; in coinage, the international postal service, and the determination of electrical measures, by revised statutes; in the medical work of both war and navy departments by orders approved by the secretaries of the departments; and in the U.S. health and marine hospital service by regulations of the President, 1902.

§ 2. MEASURES OF LENGTH

74. Tables of Long Measure. Long measure is so named because used in determination of length. It is also called linear because of its use in the measurement of lines.

The following tables are used:

Linear measure,
Surveyor's long measure,
Engineer's measure,
Nautical measure,
Miscellaneous linear units,
Metric linear measure.

Table 18

UNITED STATES AND ENGLISH LINEAR MEASURE

Quantity.	Unit.	Symbol.
12 inches	1 foot	ft.
3 ft.	1 yard	yd.
5½ yd. 16½ ft.	1 rod 1 pole 1 perch	rd. P.
40 rd.	* 1 furlong	fur.
8 fur. 320 rd. 5280 ft.	1 mile	mi.

* Seldom used.

75. Problems in Linear Measure. By reference to the table, solve the following, expressing results fractionally, and to three places of decimals:

1. One inch is what decimal part of a foot?
2. How many inches in a mile?
3. How many yards in a mile?
4. Compute and fill in the omitted entries in the table below:

Table 19

LINEAR EQUIVALENTS

No.	Inches.	Decimal of Foot.	No.	Feet.	Decimal of Rod.
1	1		11	2	
2	3½		12	4	
3	4¼		13	5	
4	6		14	6	
5	8		15	7¼	
6	10		16	8⅓	
7	5¾		17	9	
8	7		18	12	
9	9		19	10	
10	1.5		20	14	

5. A rod is what decimal of a mile?

6. The boundaries of a forest measure 95 rods, 108 rods, 143 rods, and 84 rods.

Formulate and compute the distance around it in feet and in miles.

TABLE 20

SURVEYOR'S LONG MEASURE

Quantity.	Unit.	Symbol.
7.92 inches	1 link	li.
25 li.	1 rod	rd.
4 rd. 66 ft.	1 chain	ch.
80 ch.	1 mile	mi.

The chain in the above table is in general use for country surveys and is known as Gunter's chain. For other work, the engineer's chain or steel tape is used.

TABLE 21

ENGINEER'S MEASURE

Quantity.	Unit.	Symbol.
12 inches	1 link	li
100 links	1 chain	ch.

76. Problems in Surveyor's Measure. Solve the following in the work-book:

1. How many links in a Gunter's or surveyor's chain? How many inches?

2. The length of the surveyor's chain is what decimal of a mile?

3. The length of an engineer's chain is what decimal of a mile?

4. Reduce 360 chains to miles by the simplest method.

5. Write a formula for reducing chains to miles.

6. The measurements of the sides of a tract of land were as follows:

41 ch., 42 ch. 12 li., 54 ch. 84 li., 78 ch. 75 li.

Compute the number of rods in the boundary of the tract.

TABLE 22

NAUTICAL MEASURE

Quantity.	Unit.
6 feet	1 fathom
120 fathoms	1 cable-length
7⅓ cable-lengths 880 fathoms	1 statute mile
1.153 statute miles 1 minute of circumference of earth	1 geographic or nautical mile
3 geographic miles	1 league
20 leagues 60 geographic miles 69.16 statute miles	1 degree of earth's circumference
1 * knot	1 nautical mile per hour

* Observe that *a knot is* not distance, but *a rate of sailing.*

77. Problems in Nautical Measure. Solve the following in the work-book:

1. A statute mile is what is commonly known as a mile.

If 1.15273 statute miles equal 1 geographic mile (which is the length of a minute of the earth's circumference at the equator), formulate and compute the circumference of the earth in round numbers.

2. Taking the length of 1 minute of the circumference of the earth as given in the preceding problem, formulate and compute the number of statute miles in 1 *degree* of the earth's circumference.

3. A steamer which makes 21 knots goes how many statute miles per day of 24 hours?

Formulate.

4. Show by formula and computation how the rate of the steamer in Problem 3 compares with the speed of the Twentieth

Century Limited which makes the run of 978 miles from New York to Chicago in 18 hours.

TABLE 23

MISCELLANEOUS LINEAR UNITS.

Quantity.	Unit.
4 inches	1 hand
3 feet	1 pace
2⅓ feet	1 military pace
28 inches	
18 inches	1 cubit

5. A horse measures 15¾ hands in height. Formulate and compute the height in inches.

6. 300 ordinary paces equal how many military paces? This is what part of a mile?

How many rods?

7. A mile run is made in 4 minutes 42½ seconds. What time would it take to run the length of a city block, 200 feet, at the same rate?

8. In what time does an automobile running at a speed of 50 miles an hour cover a distance of 100 feet?

TABLE 24

METRIC LINEAR MEASURE

<table>
<tr><th colspan="5">1 meter = 39.37 inches.</th></tr>
<tr><th rowspan="2">Quantity.</th><th rowspan="2">Unit.</th><th rowspan="2">Symbol.</th><th colspan="2">Equivalent.</th></tr>
<tr><th>Inches.</th><th>Feet.</th></tr>
<tr><td>10 mm.*</td><td>1 centimeter</td><td>cm.</td><td>.3937</td><td></td></tr>
<tr><td>10 cm.</td><td>1 decimeter</td><td>dm.</td><td></td><td></td></tr>
<tr><td>10 dm.</td><td>1 meter</td><td>m.</td><td></td><td></td></tr>
<tr><td>10 m.</td><td>1 decameter</td><td>Dm.</td><td></td><td></td></tr>
<tr><td>10 Dm.</td><td>1 hectometer</td><td>Hm.</td><td></td><td></td></tr>
<tr><td>10 Hm.</td><td>1 kilometer</td><td>Km.</td><td></td><td></td></tr>
<tr><td>10 Km.</td><td>1 myriameter</td><td>Mm.</td><td></td><td></td></tr>
</table>

* mm. denotes millimeters.

A study of the table will show that the prefixes are multiples which have the following meaning:

$$\text{milli} = \frac{1}{1000} = .001,$$

$$\text{centi} = \frac{1}{100} = .01,$$

$$\text{deci} = \frac{1}{10} = .1,$$

$$\text{deca} = 10,$$

$$\text{hecto} = 100,$$

$$\text{kilo} = 1000.$$

78. Problems in Metric Linear Measure. Solve the following problems:

1. In the table 24, 1 centimeter $= \frac{1}{100}$ of a meter.

In the same form write the meanings of all the units in the table.

2. Compute and fill in the omitted entries in the columns headed feet and inches in the table.

3. Rule the table of metric linear measure in the work-book, with the headings rods and yards instead of inches and feet in the column of equivalents.

Compute and fill in the entries.

4. Express as meters, in one number,

29 Hm, 5 Dm, 7 m, 14 dm, 8 cm, 82 mm.

5. Place a meter stick and a yard stick side by side on the desk with the first graduation of each in exact alignment.

In the following table ruled in the work-book enter the reading from the meter stick for each inch on the yard-stick, as assigned.

TABLE 25

EQUIVALENTS OF INCHES IN CENTIMETERS

Inches	1	2	3	4	5	6	7	8	9	10	11	12
cm.												
Inches	13	14	15	16	17	18	19	20	21	22	23	24
cm.												
Inches	25	26	27	28	29	30	31	32	33	34	35	36
cm.												

6. Compute and tabulate in inches the following parts of a yard and of a meter.

$\frac{1}{16}, \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}.$

7. Express 346725.39 meters as kilometers; as myriameters; as centimeters; as hectometers; as decimeters; as decameters.

8. Formulate and compute 142.3 feet as meters; as centimeters; as kilometers; as decimeters.

9. Formulate and compute to four decimal places the number of miles in a 7 kilometer race.

10. With both meter stick and yard stick, measure the dimensions of the floor and check each measurement by computation.

Make a drawing of the floor to as large a scale as the page permits, showing both U. S. and metric dimensions on the drawing.

11. Measure the door, its panels and other parts, with yard and meter sticks, and make a drawing to scale showing both U. S. and metric dimensions.

12. Measure the floor area of the furnishings of the room (chairs, desk, etc.), and make a drawing to scale, showing the layout of the floor space, with both U. S. and metric dimensions.

§ 3. MEASURES OF AREA

79. Tables of Area. There are four tables used in the determination of surface or area:

Square measure,
Surveyor's square measure,
Metric square measure,
Metric land measure.

TABLE 26

SQUARE MEASURE

Quantity.	Unit.
144 square inches	1 square foot
9 sq. ft.	1 sq. yard
30¼ sq. yd. 272¼ sq. ft.	1 sq. rod
160 sq. rd. 43560 sq. ft.	1 acre

80. Problems in Square Measure. Solve the following problems in the work-book:

1. Formulate and compute the number of square inches in a square rod.

2. Formulate and compute the length in feet, of the side of a plot containing 1 square rod.

3. If an acre of land is fenced off in a square plot, what is the length of each side of the plot in feet? What is the length in rods?

Give formula.

TABLE 27

SURVEYOR'S SQUARE MEASURE

Quantity.	Unit.
625 sq. li.	1 sq. rd.
16 sq. rd.	1 sq. ch.
10 sq. ch.	1 acre
640 A.	1 sq. mile
36 sq. mi.	1 township (U. S. public lands)

TABLE 28

METRIC SQUARE MEASURE

Quantity.	Unit.	Sq. Inches.	Sq. Feet.
100 sq. mm.	1 sq. centimeter	.155	
100 sq. cm.	1 sq. decimeter	15.5	
100 sq. dc.	1 sq. meter		
100 sq. m.	1 sq. decameter		
100 sq. Dk.	1 sq. hektometer		
100 sq. Hk.	1 sq. kilometer		
100 sq. Km.	1 sq. myriameter		

4. A square meter is a square, measuring 1 meter on each side. Draw a dimensioned figure to illustrate a square meter, showing the length of the side in inches.

Formulate and compute the value of the square meter in square inches, to four decimal places, and enter the results in the table.

5. Formulate and compute the number of square feet in 1 square meter, using the result obtained in Problem 4, and make entry in the table.

6. By inspection of the entries in the table, fill in all the omitted entries.

7. There are three different ways of abbreviating or symbolizing the units of the preceding table:

1. As shown in the table,
2. By an exponent, as cm^2,
3. By s to denote square, as scm, meaning square centimeters.

In the table below, the numbers at the top of the column refer to these ways of abbreviating.

Rule the table in the work-book and fill in the symbols in the columns:

TABLE 29

METRIC AREA UNITS WITH ABBREVIATIONS

Unit.	Abbreviations.		
	1	2	3
Square millimeter	sq. mm.	mm.^{2}	smm.
Square centimeter			
Square decimeter			
Square meter			
Square dekameter			
Square hectometer			
Square kilometer			

In the following table the unit is the *are* (pronounced air) which is a square of land measuring 10 meters by 10 meters, or about one-fortieth of an acre. As shown the final vowel of the prefixes is not used.

The common units of land measure are the are and hekare.

TABLE 30

METRIC LAND MEASURE

1 Are = .02471 Acre.			
Quantity.	Unit.	Square Rods.	Acres.
10 ca.	1 decare		
10 da.	1 are		
10 a.	1 dekare		
10 Da.	1 hekare		

8. Formulate and compute the value of an *are* as a part of an acre to six decimal places and enter the result in the table.

9. Formulate in square rods the result obtained in Problem 7, compute, and enter in the table.

10. By inspection of the table fill in all the omitted entries.

§ 4. MEASURES OF VOLUME

81. Tables of Capacity. In the determination of volume or capacity the following tables are used:

Cubic measure,
Dry measure,
Liquid measure,
Apothecaries' fluid measure,
Common equivalents,
Metric cubic measure,
Metric wood measure,
Metric capacity measure.

The fundamental table used in the determination of volume or capacity is the table of cubic measure, so called because its units are the cube or third power of the linear units.

TABLE 31

CUBIC MEASURE

Quantity.	Unit.
1728 cubic inches	1 cu. ft.
27 cu. ft.	1 cu. yd.
* 24¾ cu. ft.	1 * perch (masonry)
40 cu. ft.	1 shipping ton
100 cu. ft.	1 register ton (tonnage of vessels)
16 cu. ft.	1 cord ft.
8 cd. ft.	1 cord of wood
128 cu. ft.	

---

* In some states. Not a legal standard.

A standard solid cord of wood measures 8 feet long, 4 feet wide, and 4 feet high.

TABLE 32

U. S. DRY MEASURE

Quantity.	Unit.	Cubic Inches.
2 pints	1 quart	
8 qt.	1 peck	
4 pk. 2150.42 cu. in.	1 bushel (struck)	
2688 cu. in.	1 bushel (heaped)	

The struck or Winchester bushel of 2150.42 cubic inches, in standard size 18½ inches in diameter and 8 inches deep, is used for measuring grain. It is called stricken or struck because, when filled, the top is struck off even with the edge by drawing a straight stick or board across the top of the measure.

The heaped bushel of 2688 cubic inches in theory is the Winchester bushel heaped in conical form to a height of 6 inches above the top of the measure. It is used in measuring vegetables and other coarse commodities.

In practice, the bushel is never heaped to the height of 6 inches and therefore has a capacity somewhat less than 2688 cubic inches.

The uncertainty of bushel measure has led the various state legislatures to pass laws as to the number of pounds of the various commodities which constitute a bushel. See page 101.

82. Problems in Capacity. Solve the following problems in the work-book.

1. Formulate, compute, and fill in each entry for the last column in the table of U. S. dry measure.

TABLE 33

U. S. LIQUID MEASURE

Quantity.	Unit.	Cubic Inches.
4 gills	1 pint	
2 pt.	1 quart	
4 qt. 231 cu. in.	1 gallon	
31½ gal.	1 barrel *	
2 bbl. 63 gal.	1 hogshead	
42 gal.	1 bbl. refined oil	

* The size of a barrel or cask varies to such an extent that the number of gallons capacity is sometimes stamped on the outside.

2. Formulate, compute, and fill in each entry for the last column in the table of U. S. liquid measure.

TABLE 34

U. S. APOTHECARIES' FLUID MEASURE.

1 M. =1 Minim =.00376 cubic inch.			
Quantity.	Unit.	Symbol.	Cubic Inches.
60 minims	1 fluid dram	fʒ	
8 fl. dr.	1 fluid ounce	f℥	1.8047
16 fluid ounces	1 pint	O	
8 pints	1 gallon	cong.	

3. Compute and fill in the omitted entries in the last column of the preceding table.

4. A 4-oz. bottle holds how many cubic inches? How many gills?

5. A 6-oz. bottle holds what part of a pint?

6. How many 12-oz. bottles can be filled from a gallon of liquid?

TABLE 35

COMMON EQUIVALENTS

1 Tablespoonful = ½ Fluid Ounce.

Quantity.	Unit (approx.)	Fluid Ounce.
2 saltspoonfuls	1 coffeespoonful	
2 coffeespoonfuls	1 teaspoonful	
45 to 60 drops		
3 teaspoonfuls (dry)	1 tablespoonful	
4 teaspoonfuls (liquid		
4 tablespoonfuls	1 wineglass	
2 wineglasses	1 gill	
2 gills	1 cup	
2 cups	1 pint	

7. A fluid ounce is approximately how many teaspoonfuls?

8. Compute and fill in the entries in the last column in the table.

9. A 16-oz. bottle is filled with medicine. If a patient takes three doses of 2 teaspoonfuls each, three times a day, how long will the medicine last?

TABLE 36

METRIC CUBIC MEASURE

1 Cubic Meter = 35.316 Cubic Feet.

Quantity.	Unit.	Symbol.	Cubic Inches.	Cubic Feet.
1000 cmm.*	1 cu. centimeter	c.c.		
1000 c.c.	1 cu. decimeter	cd.		
1000 cd.	1 cu. meter	cum.		
1000 cum.	1 cu. decameter	cD.		
1000 cD.	1 cu. hektometer	cH.		

* Cubic millimeter.

10. Formulate and compute each entry for the last two columns in the table.

Rule the table in the work-book with all entries complete.

In the following table the unit is the *stere*, pronounced stair, which is a cube measuring 1 meter on each edge, whose volume is therefore about one-fourth of a cord.

Table 37

METRIC CORD-WOOD MEASURE

Quantity.	Unit.	Cord Feet.	Cords.
10 decisteres	1 stere		

11. Formulate, compute, and fill in the entries for the last two columns, in a table ruled in the work-book.

12. A man delivers 128 cords of wood a distance of 5 miles. He sells the wood at $7.50 a cord and charges $1.00 per load of 2 cords for hauling.

Formulate and compute the amount of his bill, and the distance traveled, making all the computations, both in U. S. units and in Metric units.

Table 38

METRIC CAPACITY MEASURE

1 L. = 1 liter = 1 cubic decimeter = .908 quart, dry measure.
= 1.0567 quart, liquid measure.

Quantity.	Unit.	Liquid.			Dry.		
		Pt.	Qt.	Gal.	Qt.	Pk.	Bu.
10 mL.*	1 centiliter						
10 cL.	1 deciliter						
10 dL.	1 liter						
10 L.	1 decaliter						
10 DL.	1 hectoliter						
10 HL.	1 kiloliter †						

* Milliliters. † 1 kiloliter = 1 cubic meter.

13. Compute and fill in all the equivalents in the last six columns in the above table.

§ 5. MEASURES OF WEIGHT

83. Tables. The following tables are used in the determination of weight:

Avoirdupois weight,
Troy weight,
Assayer's weight,
Diamond weight,
Apothecaries' weight,
Metric weight.

The table in general use for the determination of the weight of commodities and other coarse articles is the table of avoirdupois weight.

TABLE 39

U. S. AVOIRDUPOIS WEIGHT

Quantity.	Unit.	Symbol.
16 drams	1 ounce	oz.
16 oz.	1 pound	lb.
100 lbs.	1 hundredweight	cwt.
20 cwt. 2000 lbs.	1 short ton (U. S.)	T.
2240 lbs.	1 long ton (Eng.)	

TABLE 40

TROY WEIGHT

Quantity.	Unit.
24 grains	1 pennyweight
20 dwt.	1 ounce
12 oz. 5760 grains	1 pound

TABLE 41

ASSAYERS' WEIGHT

Quantity.	Unit.
60 grains (troy)	1 carat grain
4 carat grains	1 carat
24 carats	1 pound

TABLE 42

DIAMOND WEIGHT (U. S.)

Quantity.	Unit.
16 parts	1 carat grain
3.2 grains	1 carat

The carats of the two preceding tables must not be confused with the carat signifying $\frac{1}{24}$th part, which is used to denote the fineness of gold.

Thus 14-carat gold is $\frac{14}{24}$ gold and $\frac{10}{24}$ alloy.

84. Problems in Measures of Weight. Solve the following problems in the work-book:

1. Compute the number of pennyweights in a carat grain.

2. Determine by reference to the tables and by computation whether the assayer's pound is a pound troy.

3. A carat in assayer's weight is what part of an ounce? This is how many pennyweights?

4. Compute the number of carat grains in an assayer's pound. This is what part of the number of troy grains?

TABLE 43

APOTHECARIES' WEIGHT.

Quantity.	Unit.	Symbol.
20 grains	1 scruple	℈
3 scr.	1 dram	ʒ
8 dr.	1 ounce	℥
12 oz. 5760 gr.	1 pound	lb.

The tables of apothecaries' weight and apothecaries' fluid measure, are used in compounding medicines in prescription work.

Following is the table of prescription symbols:

TABLE 44

PRESCRIPTION SYMBOLS

Symbol.	Meaning.	Symbol.	Meaning.
℞	take	Ft. Mixt.	let a mixture be made
P.	part	Ft. Haust.	let a draught be made
P. aeq.	equal parts	ad.	add to
aa	equal quantities	ad lib.	at pleasure
ss	half	M.	mix
gr	grains	Mac.	macerate
q.p.	as much as you please	Pulv.	powder
gtt.	drops	Pil.	pill
m.	minims	Solv.	dissolve
f ʒ	fluid dram	St.	let it stand
f ℥	fluid ounce	Sum.	to be taken
℈	scruples	D.	dose
O	pint	Dil.	dilute
cong.	gallon	Fil.	filter
aq.	water	Lot.	a wash
		Garg.	a gargle
℈xviij	18 scruples	Hor.Decub.	at bedtime
℥ixss	9½ ounces	Inj.	injection
Q.S.	as much as sufficient	Ess.	essence

As shown, numerals are expressed in the Roman notation with small letters, final i in a number being denoted by j. Sometimes dots are used in place of i.

Thus ℥..j means 3 oz.

5. Copy the following in the work-book and write the interpretation:

Capsicum	℈iij
Rhubarb	℥xijss
Citric acid	gr.ij
Aqua	O j
Alcohol	q.p.

6. ʒxvij is what part of cong. ijss?

7. A deciliter is what part of a gill? A centiliter is what part of a f ℥? A milliliter is what part of a f ʒ?

The standard of metric weight is the kilogram, which is the weight of 1 liter of distilled water at its maximum density.

By law, one kilogram = 2.204622 lbs. avoirdupois.

Following is the table of metric weight.

TABLE 45

METRIC WEIGHT *

1 Gram = 15,432 Grains.

Unit.	Symbol.	Grams.	Weight of What Volume of Water.	Avoirdupois Equivalent.
1 Millier or tonneau †	t	1,000,000	1 cubic meter	2204.6 lbs.
1 Quintal	q.	100,000	1 hectoliter	
1 myriagram	Mg.	10,000	10 liters	
1 Kilogram or kilo	Kg.	1,000	1 liter	
1 Hectogram	Hg.	100	1 deciliter	3.5274 oz.
1 Decagram	Dg.	10	10 cu.centimeters	
1 GRAM	g.	1	1 cu. centimeter	gr.
1 Decigram	dg.	$\frac{1}{10}$	$\frac{1}{10}$ cu. centimeter	
1 Centigram	cg.	$\frac{1}{100}$	10 cu. millimeters	
1 Milligram	mg.	$\frac{1}{1000}$	1 cu. millimeter	

* From Bulletin of Bureau of Standards. † 1 metric ton.

8. The U. S. gallon of 231 cubic inches is defined by law as the volume of 8.3388822 avoirdupois pounds of distilled water at its maximum density with the barometer at 30°.

Compute the volume in liters under the given conditions.

Compute the weight in kilograms under the given conditions.

9. Under the same conditions as in Problem 8, what is the weight of pure water per cubic inch and per cubic foot?

Write the formula for each unit.

85. Table of Equivalents. The following equivalents have been established by Congress.

Compute and fill in all omitted entries, ruling the tables in the work-book with the computed entries to four decimal places.

10. Long Measure.

Table 46

LONG MEASURE

U. S.	Metric.	Metric.	U. S.
1 inch	2.54 cm.	1 cm.	in.
1 foot	.3048 m.	1 dm.	ft.
1 yard	m.	1 m.	yd.
1 rod	5.029 m.	1 Dm.	rd.
1 mile	1.6093 km.	1 Hm.	mi.
		1 Km.	mi.

11. Square Measure.

Table 47

SQUARE MEASURE

U. S.	Metric.	Metric.	U. S.
1 sq. in.	6.452 scm.	1 scm.	sq. in.
1 sq. ft.	.0929 sm.	1 sdm.	sq. ft.
1 sq. yd.	sm.	1 sm.	sq. yd.
1 sq. rd.	25.293 sm.	1 are	sq. rd.
1 acre	40.47 ares	1 Ha.	A.
1 sq. mi.	259 Ha.	1 skm.	sq. mi.

12. Cubic Measure.

Table 48

CUBIC MEASURE

U. S.	Metric.	Metric.	U. S.
1 cu. in.	16.387 c.c.	1 c.c.	cu. in.
1 cu. ft.	28.317 cd.	1 cd.	cu. ft.
1 cu. yd.	.7645 cum.	1 cum.	cu. yd.
1 cord	3.626 steres	1 stere	cord

13. Capacity Measures.

TABLE 49

CAPACITY MEASURES

U. S.	Metric.	Metric.	U. S.
1 qt. (liq.)	.9463 L.	1 liter	qt. (liq.)
1 qt. (dry)	1.101 L.	1 L.	qt. (dry)
1 gal.	.3785 DL.	1 DL.	gal.
1 pk.	.381 DL.	1 DL.	pk.
1 bu.	.3524 HL.	1 HL.	bu.

14. Weight Measures.

TABLE 50

WEIGHT MEASURES

U. S.	Metric.	Metric.	U. S.
1 gr. troy	.0648 g.	1 gram	gr. troy
1 oz. troy		1 gram	oz. troy
1 oz. avoir.	28.35 g.	1 gram	oz. avoir.
1 lb. troy		1 kg.	lb. avoir.
1 lb. avoir.		1 kilo	lb. avoir.
1 ton (short)	.9072 tonneau	1 tonneau	T. (short).

15. In 1910, the importation of coal into the Philippines was 303117 long tons, valued at $972,341.

What was the importation in metric tons and what was the price per metric ton?

16. For the fiscal year ending June 30, 1911, the importation of coal into the Philippines was 403143 metric tons valued at $1,267,312.

Compute the weight in long tons and the cost per ton.

17. The avoirdupois pound is the weight of 27.7015 cubic inches of distilled water at its maximum density with the barometer at 30 inches. Compute this volume in liters.

86. Weights of Commodities. The tabulation of commodities commonly grown throughout the United States is selected from the Year Book of the Department of Agriculture for 1907.

TABLE 51

LEGAL WEIGHTS PER BUSHEL

(Pounds)

State.	Apples.	Beans.	Bran.	Buckwheat.	Clover Seed.	Coal.	Corn, Ear.	Corn, Shelled.	Corn Meal.	Flaxseed.	Oats.	Onions.	Potatoes.	Rye.	Timothy Seed.	Turnips.	Wheat.
Alabama....	..	60	..	..	..	..	70	56	46	..	32	..	60	56	..	..	..
Arizona.....	..	55	..	..	..	..	..	54	..	..	32	..	..	56	..	..	..
Arkansas ...	50	60	20	52	60	..	70	56	48	56	32	57	60	56	60	57	60
California ...	..	..	..	40	..	..	..	52	..	..	32	..	..	54	..	..	60
Colorado ...	..	60	..	52	60	80	70	56	50	..	32	57	60	56	45	..	60
Connecticut	48	60	20	48	60	80	..	56	..	55	32	52	60	56	..	50	60
Florida	..	60	20	..	..	..	70	56	48	..	32	56	60	56	..	54	60
Georgia	..	60	20	52	60	80	70	56	48	56	32	57	60	56	45	55	60
Hawaii	..	..	..	..	..	..	..	56	..	..	32	..	..	56	..	..	60
Idaho	45	..	..	42	60	..	..	56	..	56	36	..	60	56	..	..	60
Illinois	..	60	20	52	60	80	70	56	48	56	32	57	60	56	45	55	60
Indiana	..	60	..	50	60	80	68	56	50	..	32	48	60	56	45	55	60
Iowa	48	60	20	52	60	80	70	56	..	56	32	57	60	56	45	..	60
Kansas	48	60	20	50	60	80	70	56	50	56	32	57	60	56	45	55	60
Kentucky ...	..	60	20	56	60	76	70	56	50	56	32	57	60	56	45	60	60
Maine	44	60	..	48	..	..	..	56	50	..	32	52	60	50	..	50	60
Mass	48	60	20	48	60	..	..	56	50	55	32	52	60	56	45	..	60
Michigan ...	48	60	..	48	60	80	70	56	50	56	32	54	60	56	45	58	60
Minnesota...	50	60	..	50	60	80	70	56	..	..	32	52	60	56	45	..	60
Mississippi. .	..	60	20	48	60	80	70	56	48	56	32	57	60	56	45	55	60
Missouri.....	48	60	20	52	60	80	70	56	50	56	32	57	60	56	45	42	60
Montana....	45	60	20	52	60	76	70	56	50	56	32	57	60	56	45	50	60
Nebraska....	..	60	..	52	60	80	70	52	50	56	32	57	60	56	45	55	60
N.Hampshire	..	62	..	..	..	..	..	56	50	..	32	..	60	56	..	..	60
New Jersey..	50	60	..	50	64	..	..	56	..	55	30	57	60	56	..	..	60
New York...	48	60	20	48	60	..	..	56	50	55	32	57	60	56	45	..	60
N. Carolina .	..	..	..	50	60	..	..	56	..	55	32	..	..	56	..	..	60
N. Dakota...	50	60	20	42	60	80	70	56	..	56	32	52	60	56	45	60	60
Ohio.........	50	60	..	50	60	80	68	56	..	56	32	55	60	56	45	60	60
Oklahoma ...	..	60	20	42	60	80	70	56	..	56	32	52	60	56	..	..	60
Oregon......	45	..	..	42	60	..	..	56	..	..	32	..	60	56	..	..	60
Pennsylvania	..	..	..	48	60	76	..	56	..	..	32	50	56	56	..	..	60
Rhode Island	48	60	20	48	60	80	70	56	50	56	32	50	60	56	45	50	60
S. Dakota ...	..	60	20	42	60	80	70	56	..	56	32	52	60	56	42	60	60
Tennessee....	50	60	20	50	60	80	70	56	..	56	32	56	60	56	45	50	60
Texas........	45	60	20	42	60	80	70	56	..	56	32	57	60	56	45	55	60
Vermont	46	62	..	48	60	..	..	56	..	..	32	52	60	56	45	60	60
Virginia.....	..	60	..	52	60	80	70	56	..	56	30	57	56	56	45	55	60
Washington .	45	..	..	42	60	..	..	56	..	56	32	60	56	..	..	..	60
West Virginia	..	60	..	52	60	80	..	56	..	56	32	60	56	..	45	..	60
Wisconsin....	50	60	20	50	60	..	..	56	..	56	32	57	60	56	45	42	60

§ 6. MEASURES OF TIME

87. Circular Measure. A day is the time required for the earth to make one revolution. In that time it turns through 360°.

Following are the tables:

TABLE 52

CIRCLES

Quantity.	Unit.	Symbol.
60″ (seconds)	1 minute	′
60′	1 degree	°
30°	1 sign	
60°	1 sextant	
90°	1 quadrant	
180°	1 semi-circumference	
360°	1 circumference	

TABLE 53

TIME

Quantity.	Unit.	Symbol.
60″	1 minute	′
60′	1 hour	hr.
24 hr.	1 day	da.
7 da.	1 week	wk.
30 da.	1 month	mo.
52 weeks 12 months 365 days	1 year	yr.
366 da.	1 leap year	
10 yr.	1 decade	dec.
10 dec. 100 yr.	1 century	

Table 54

MONTHS

Name.	Ab.	No. Days.	Name.	Ab.	No. Days.
January	Jan.	31	July		31
February	Feb.	28*	August	Aug.	31
March	Mar.	31	September	Sept.	30
April	Apr.	30	October	Oct.	31
May		31	November	Nov.	30
June		30	December	Dec.	31

* 29 in a leap year.

88. The Year. A solar year is the time it takes the earth to make one complete circuit of its orbit or path around the sun.

The exact length of a solar year is 365 days, 5 hours, 48 minutes, and 49.7 seconds.

As the calendar year of 365 days is nearly 6 hours less, one day is added to the month of February every four years if evenly divisible by 4, except century years which must be evenly divisible by 400, otherwise a day is not added.

89. How to Subtract Dates. The method of determining the difference between two dates is illustrated below, the dates being August 19, 1863, and January 4, 1888.

$$\begin{array}{r} 11\ \ 30 \\ 1888-\ 1-\ 4 \\ 1863-\ 8-19 \\ \hline 24-\ 4-15 \end{array}$$

The year is first set down, then the number of the month, and the days.

As 8 months and 19 days were to be subtracted from 1 month and 4 days, 1 year was "borrowed" from 1888, making 11 months and 30 days as shown at the top of the columns.

90. U. S. Standard Time. In 1883, Congress adopted a system by which the United States is divided into four time belts, each 15° in width. The meridians by which these belts are determined are as follows:

75th, near Philadelphia,
90th, near St. Louis,
105th, near Virginia City, Mont.,
120th, near Carson City, Nev.

The numbers signify the number of degrees west of Greenwich, England.

Places within a limit of $7\frac{1}{2}$° on either side of one of these meridians, have the time of the meridian, the time being called Eastern, Central, Mountain, and Pacific.

From the fact that the earth turns through 360° in 24 hours, each of the belts corresponds to 1 hour of time. Therefore in traveling westward, watches are in theory set back 1 hour at each of these meridians. In practice the railroad companies in their time tables show at what station the time changes.

91. Longitude. The distance of a place, east or west from the meridian of Greenwich, Eng., is called its longitude. Thus the longitude of Los Angeles is 118° 13′ W., and of Manila 120° 58′ E.

TABLE 55

LONGITUDE OF CITIES

City.	Longitude.	City.	Longitude.
Albany	73° 44′ 56″ W.	Liverpool	3° 4′ W.
Berlin	13° 23′ 43″ E.	Madrid	3° 42′ W.
Bombay	72° 49′ E.	Moscow	37° 34′ E.
Boston	71° 4′ W.	New Orleans	90° 3′ W.
Brooklyn	73° 59′ W.	New York	74° 0′ 3″ W.
Buffalo	78° 55′ W.	Paris	2° 20′ 15″ E.
Calcutta	88° 23′ E.	Philadelphia	75° 9′ W.
Cambridge	0° 6′ E.	Rome	12° 29′ E.
Canton	113° 17′ E.	San Francisco	122° 24′ 32″ W.
Chicago	87° 36′ 42″ W.	St. Louis	90° 15′ W.
Denver	105° W.	St. Petersburg	30° 19′ E.
Edinburgh	3° 12′ W.	Vienna	16° 23′ E.
Glasgow	4° 18′ W.	Washington	77° 1′ W.
Hong Kong	114° 10′ E.	Yokohama	139° 36′ E.

92. Problems in Measures of Time. Solve the following problems in the work-book:

1. Compute the time at the cities which may be assigned from the preceding table, when it is 12 o'clock, noon, at Greenwich, England.

What is the formula?

2. A note bearing interest from March 21, 1907, was paid Nov. 8, 1910.

Compute the interest period.

3. Write the names of the cities in the table of longitudes, which have eastern, central, mountain, and Pacific time.

4. Using the data of Problem 1 in paragraph 77, how fast does a point on the earth's equator move per hour?

5. Compute the actual number of days from Sept. 27, 1911, to March 21, 1912.

6. Formulate and compute the number of seconds in an hour, and a day.

7. When one day is added to the month of February, making 29 days, the year is called a leap year.

Show by computation whether the following are leap years: 1858, 1900, 1800, 2000, 1908, 1904, 1912, 1906.

8. Compute the number of sextants in a circle. Sextant therefore means what?

9. Compute the number of quadrants in a circle. What therefore is the meaning of the word quadrant?

10. Was 1702 in the 17th century, or in the 18th? Give the figures which prove your answer.

11. The first flight across the United States was made by Calbraith P. Rodgers in a Wright biplane. Leaving Sheepshead Bay in the Borough of Brooklyn, New York, on Sept. 17, 1911, he landed at Pasadena, California, Nov. 5, 1911, having traveled 4231 miles in 4924 minutes of actual time in flying.

Formulate and compute his speed in miles per hour, and the actual number of days in the air.

93. International Date-line. In paragraph 90, it was explained why watches must be set back one hour for each 15° that one travels west. Therefore if one were to travel west from Greenwich, England, to the 180th meridian, or half way around the earth, and were to turn his watch back

1 hour at each 15°, the total number of hours it would be turned back including the final turn back at the 180th meridian would be $\frac{180}{15}=12$ hours.

Similarly if one were to travel east through 180°, his watch must be turned 12 hours ahead. The confusion which would result from such an arrangement has been avoided by the establishment of an International Date-line which differs but slightly from the 180th meridian, this meridian being selected because it crosses very few land areas.

On ships sailing west, one day is deducted when this line is crossed, and on ships sailing east one day is added.

§ 7. MEASURES OF MONEY

94. U. S. Money. The unit of United States money is the dollar, which was established by the Continental Congress in 1786 with the provision that it should be divided decimally.

TABLE 56

U. S. MONEY

Quantity.	Unit.	Symbol.
10 *mills	1 cent	c.
10 cents	1 dime	
10 dimes	1 dollar	$
10 dollars	1 †eagle	

* Used only in computation.
† $10 gold pieces only.

TABLE 57

U. S. COINS

Name.	Value.	Composition.	Total Weight.	Weight.	
				Metal.	Alloy.
Penny	1c.	Copper 95%	48 gr.		
Nickel	5c.	Nickel 25%	77.16 gr.		
Dime	10c.	Silver 90%	38.58 gr.		
Quarter	25c.	Silver 90%			
Half	50c.	Silver 90%			
Dollar	$1	Silver 371.25 gr.	412.5 gr.		
¼ Eagle	$2½	Gold 90%	64.5 gr.		
½ Eagle	$5	Gold 90%			
Eagle	$10	Gold 232.2 gr.			
Dbl. Eagle	$20	Gold 90%			

95. Foreign Money.

TABLE 58

ENGLISH MONEY

Quantity.	Unit.	Symbol.	U. S. Equivalent.
4 farthings	1 penny	*d.*	
12 pence	1 shilling	*s.*	
20 shillings	1 pound 1 sovereign	£	$4.8665
5 shillings	1 crown	*c.*	
21 shillings	1 *guinea	*g.*	

* Coinage stopped in 1816.

TABLE 59

FRENCH MONEY

Quantity.	Unit.	Symbol.	U. S. Equivalent.
10* millimes	1 centime	*ct.*	
10 centimes	1 decime	*dc.*	
10 decimes	1 franc	*fr.*	19.3c.

* Not coined.

TABLE 60

GERMAN MONEY

Quantity.	Unit.	Symbol.	U. S. Equivalent.
100 pfennige	1 mark	*RM.*	23.8c.

FOREIGN CURRENCY

CANADIAN.

Metal.	*Coin.*	*Value in U. S.*
Silver	50-cent,	$.462
	25-cent,	.231
	10-cent,	.092
	5-cent,	.046
Bronze.	1-cent,	.01

GERMAN.

Metal.	*Coin.*	*Value in U. S.*
Gold	20-mark,	$4.77
	10-mark,	2.385
	5-mark,	1.192
Silver	2-mark,	.477
	1-mark,	.2385
	20-pfennig,	.0477
Nickel	10-pfennig,	.02385
	5-pfennig,	012
Cop'r	2-pfennig,	.0048
	1-pfennig,	0024

ENGLISH.

Metal.	*Coin.*	*Value in U. S.*
Gold	Sovereign,*	$4.8665
	Half-sovereign,	2.433
	Guinea,	5.11
	Half-guinea,	2.555
Silver	Crown,	1.216
	Half-crown,	.608
	Florin,	.486
	Shilling,	.243
	Sixpence,	.12
	Fourpence,	.08
	Threepence,	.06
Cop'r	Penny,	.02
	Half-penny,	.01
	Farthing,	.005

FRENCH.

Metal.	*Coin.*	*Value in U. S.*
Gold	100-franc,	$19.30
	40-franc,	7.72
	20-franc,	3.86
	10-franc,	1.93
Silver	5-franc,	.965
	2-franc,	.386
	1-franc,	.193
	50-centime,	.096
	25-centime,	.048
Bronze	10-centime,	.02
	5-centime,	.01
	2-centime,	.00386
	1-centime,	.00193

PHILIPPINES, U. S.

Metal.	*Coin.*	*Value in U. S.*
Silver	Conant, (peso)	.50
	50-centavos,	.25
	20-centavos,	.10
	10-centavos,	.05
Cop'r.	1-centavo	.00½

* The English sovereign is 22 carats fine and weighs $123\frac{274}{623}$ grains.

96. Problems in Money. Solve the following problems in the work-book.

1. Compute and enter the omitted weights in the table of U. S. coins.

2. Compute and tabulate in the work-book, the number of grains of pure metal and of alloy in each of the U. S. coins.

3. Compute the weight of $1000 in each of the U. S. coins.

4. The monetary unit of England, the £ or sovereign has 113.0016 grains of pure gold.

By reference to the table of U. S. coins, determine the number of grains of pure gold in a $10 gold piece, and compute the value if £1 in U. S. money.

5. Compute and enter the U. S. equivalents in the table of English money.

6. Fill in the omitted entries in the table of French money.

7. A purchase of books is made in England, amounting to 35 shillings 9 pence. How much U. S. money should be sent in settlement?

8. Compute the value of 2000 francs in U. S. money.

9. Compute the value of 1800 marks in U. S. money.

10. A traveler has 8 50-pfennige pieces, 7 5-pfennige pieces, 30 5-mark pieces, 42 francs, 11 half-sovereigns, 12 crowns, and 18 quarters.

What is the total amount in

(1) U. S. money,
(2) English money,
(3) German money?

11. If one sends 50 c. in U. S. money to Germany, the International Money Order will be made out for how many marks and pfennige?

97. Miscellaneous Measures. The following tables are also used:

TABLE 61

COUNTING TABLE

Quantity.	Unit.	Symbol.
12 units	1 dozen	doz.
12 doz.	1 gross	gro.
12 gro.	1 great gross	gt. gro.
20 units	1 score	

TABLE 62

STATIONERS' TABLE

Quantity.	Unit.	Symbol.
24 sheets	1 quire	qr.
20 qr.	1 *ream	rm.
3 rm.	1 bundle	bdl.
5 bdl.	1 bale	bl.

* Paper manufacturers count 500 sheets to the ream.

§ 8. APPLIED PROBLEMS

98. Problems in Weights and Measures. Solve the following problems in the work-book:

1. The table below is compiled from the small tool catalogue of the Pratt & Whitney Co.

Compute and fill in the omitted entries as may be assigned.

The given pitches are in millimeters.

TABLE 63

MACHINISTS' HAND TAPS

Diam. Tap.		Pitch.			Diam. Tap.		Pitch.		
*M/m.	In.	French Std.	Inter'l. Std.	In.	M/m.	In.	French. Std.	Inter'l. Std.	In.
3	..	0.5	.....	..	11	..	.....	1.5	
4	..	0.75	.....	..	12	..	1.5	1.75	
5	..	0.75	.....	..	14	..	2.0	2.0	
6	..	1.0	1.0	..	16	..	2.0	2.0	
7	..	1.0	1.0	..	18	..	2.5	2.5	
8	..	1.0	1.25	..	20	..	2.5	2.5	
9	..	1.0	1.25	..	22	..	2.5	2.5	
10	..	1.5	1.5	..	24	..	3.0	3.0	

* M/m denotes millimeters.

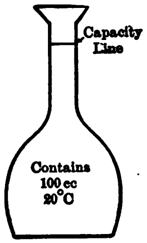


FIG. 12.

2. The drawing shows a glass flask used in volumetric analysis. It is marked to contain 100 c.c. at a temperature of 20° Centigrade.

This is how many cubic inches?

What part of a liter?

How many fluid ounces?

How many pints?

3. Compute and fill in the omitted entries in the following table:

Table 64
FLASKS

No.	Capacity.			No.	Capacity.		
	c.c.	Cu. In.	f℥		c.c.	Cu. In.	f℥.
1	25	.....	.....	7	1000		
2	30	.....	.....	8	2000		
3	50	.....	.....	9	3000		
4	200	.....	.....	10	4000		
5	300	.....	.....	11	5000		
6	500	.....	.....	12	6000		

4. Metric Boiler Formulas. Reuleaux gives the following metric formula for computing the stress in longitudinal seams of a boiler:

$$S=\frac{PD}{.2t},$$

in which S = stress in seams in kilograms per square millimeter;
P = pressure in atmospheres;
D = diameter of boiler in meters;
t = thickness of shell in millimeters;
$t=1.54PD+2.6.$

By substitution in these formulas, determine whether the t and S entries in the table are correct:

Table 65
BOILERS

No.	$P=$	4		7		10	
	D	t	S	t	S	t	S
1	.6	6.3	1.90	9.1	2.31	11.8	2.54
2	.8	7.5	2.13	11.2	2.50	14.9	2.70
3	1.0	8.8	2.27	13.4	2.61	18.0	2.78
4	1.5	11.8	2.54	18.8	2.79	25.7	2.92
5	2.0	14.9	2.68	24.2	2.89	33.4	2.99

5. Stress in Circumferential Boiler Seams. The stress in the circumferential seams of a boiler equals one-half the stress in the longitudinal seams.

By reference to Problem 4, formulate this law in terms of P, t, and D.

6. English Boiler Formulas. In the formulas below the notation is as follows:

S = stress in seams in pounds per square inch;
P = pressure in pounds per square inch;
D = diameter of boiler in inches;
t = thickness of shell in inches;
R = radius of curvature of head in inches;
t_h = thickness of head in inches;

$$t=0.0001PD+0.1;$$

$$t_h=\frac{R}{2}\times\frac{P}{S};$$

$$S=\frac{P}{2}\times\frac{D}{t};$$

$$R=D.$$

By substitution in these formulas, compute and fill in the omitted entries in the following table:

TABLE 66

BOILERS

No.	$P=$	60			105			150		
	D	t	S	t_h	t	S	t_h	t	S	t_h
1	24									
2	36									
3	42									
4	72									

7. Area of a Safety Valve. The area of a safety valve in square millimeters for each square meter of boiler-heating surface, equals 15 times the square root of the quotient of the volume of the steam in liters per kilogram, and the maximum boiler pressure in atmospheres.

Formulate this law and specify the notation in the same form as in Problem 4.

8. By the formula of the preceding problem, compute and fill in the omitted entries below:

TABLE 67

AREA OF SAFETY-VALVES

P	V	A		P	V	A	
		smm.	Sq. In.			smm.	Sq. In.
2	612	.....	.....	10	181		
4	378	.....	.....	11	167		
5	313	.....	.....	12	154		
7	244	.....	.....	13	144		
9	198	.....	.....	14	135		

9. Metric Equivalent of Pounds per Square Inch. Enter the following in the work-book and fill in the omitted entries, showing all computation in full:

1 kilo = ? lbs.
1 kilo per scm. = ? lbs. per scm.
1 inch = ? cm.
1 sq.in. = ? scm.
1 kilo per scm. = ? lbs. per sq.in., to 6 decimal places.

$$\text{Kg. per scm.} = \frac{?}{14.23}$$

Lbs. per sq.in. = Kg. per scm. $\times$?

10. Write a law for reducing kilograms per square centimeter, to pounds per square inch.

11. Write a law for reducing pounds per square inch to kilograms per square centimeter.

12. U. S. Equivalents of Parts of a Millimeter. In shops where the metric system is used, it is often necessary to reduce parts of a millimeter to inches.

Rule the table below in the work-book and compute all omitted entries to five decimal places.

TABLE 68
FIFTIETHS OF A MILLIMETER IN INCHES

mm.	Inches.	mm.	Inches.	mm.	Inches.	mm.	Inches.	mm.	Inches.
1	.....	11	.....	21	.....	31	.....	41	.....
2	.....	12	.....	22	.....	32	.....	42	.....
3	.....	13	.....	23	.....	33	.....	43	.....
4	.....	14	.....	24	.....	34	.....	44	.....
5	.....	15	.....	25	.....	35	.....	45	.....
6	.....	16	.....	26	.....	36	.....	46	.....
7	.....	17	......	27	.....	37	.....	47	.....
8	.....	18	.....	28	.....	38	.....	48	.....
9	.....	19	.....	29	.....	39	.....	49	.....
10	.....	20	.....	30	.....	40	.....	50	.....

13. Seller's Formula for Pitch. When a U. S. standard thread is to be cut on a screw, the pitch may be determined by the following formula:

$$P = .24\sqrt{D + .625} - .175,$$

in which P = pitch in inches;

D = diameter in inches;

Copy the formula and write the law.

14. Rule in the work-book as much of the following table as may be assigned, and compute and fill in the omitted entries.

TABLE 69
U. S. STANDARD SCREWS

D		P		D		P	
Frac.	Dec.	In.	mm.	Frac.	Dec.	In.	mm.
$\frac{5}{16}$				$1\frac{7}{8}$			
$\frac{9}{16}$				2			
$\frac{5}{8}$				$2\frac{1}{4}$			
$\frac{13}{16}$				$2\frac{1}{2}$			
$1\frac{3}{16}$				$2\frac{3}{4}$			
$1\frac{1}{4}$				3			
$1\frac{3}{8}$				$3\frac{1}{4}$			
$1\frac{1}{2}$				$3\frac{1}{2}$			
$1\frac{5}{8}$				$3\frac{3}{4}$			
$1\frac{3}{4}$				4			

15. Compute the numbers required in the following:

$$\$1.00 \text{ per yd.} = \left\{\begin{array}{l} ?\text{ francs per meter,} \\ ?\text{ marks per meter.} \end{array}\right.$$

$$\$.25 \text{ per lb. avoirdupois} = \left\{\begin{array}{l} ?\text{ marks per kilogram,} \\ ?\text{ francs per kilogram.} \end{array}\right.$$

$$\$.50 \text{ per U. S. gallon} = \left\{\begin{array}{l} ?\text{ francs per liter,} \\ ?\text{ marks per liter.} \end{array}\right.$$

16. Compute the number of U. S. gallons in 1 cu.ft. of water; also the number of liters.

17. If a U. S. gallon of pure water weighs 8.355 lbs., what is the weight of a liter?

18. A cubic foot of air weighs .08073 lb. at 32° F., when the barometer stands at 30″.

Compute the weight of 1 liter of air under the same conditions.

19. The A. L. A. M. standard screw has a body diameter .001″ less than the nominal diameter. This is how many mm.?

20. The size of pipe required to hold a number of wires of any diameter may be determined from the following approximate formula:

$$D = d\left(0.94 + \sqrt{\frac{N-3.7}{0.907}}\right),$$

in which

D = diameter of pipe in inches;

d = diameter of wire in inches;

N = number of wires.

How large a pipe is required to hold 25 wires 12 millimeters in diameter?

21. By use of the formula in Problem 20, compute and tabulate in the work-book such part of the following table as may be assigned.

TABLE 70

PIPE SIZES FOR WIRES

No.	d	N	D	No.	d	N	D
1	$\frac{1}{8}$	12		11	.162	42	
2	$\frac{3}{8}$	14		12	.324	80	
3	$\frac{1}{16}$	28		13	.460	56	
4	$\frac{5}{16}$	30		14	.409	70	
5	$\frac{3}{16}$	42		15	.307	87	
6	$\frac{5}{8}$	50		16	.225	49	
7	$\frac{1}{32}$	100		17	.177	75	
8	$\frac{3}{32}$	250		18	.148	90	
9	$\frac{3}{32}$	360		19	.105	20	
10	$\frac{3}{64}$	200		20	.018	51	

22. A pressure of 1 atmosphere, or 14.7 lbs. per sq.in., will sustain a column of mercury to a height of 29.962″ at 32° F.

Compute the height of this column in millimeters.

23. If an engine requires $\frac{7}{8}$ of a pint of kerosene per horsepower per hour, formulate and compute the cost of the kerosene used by a 4 H.P. engine in a 10-hour run, with oil at $3.15 per barrel.

24. A miner's inch is sometimes defined as the amount of water which will flow in 1 minute through a hole 1 inch square in a 2-inch plank, under a head 6 inches above the upper edge of the opening. If the flow under these conditions is 11.625 U. S. gallons per minute, this is how much less or more than the miner's inch of California, which is $1\frac{1}{2}$ cu.ft. per minute?

25. The imperial or English gallon contains 277.274 cu.in. If a cubic foot of water weighs 62.355 lbs., formulate and compute the weight of an imperial gallon of water.

26. If a U. S. gallon of pure water weighs 8.33111 lbs. at its maximum density at 39.1° F. with the barometer at 30″, formulate and compute the weight of a cubic foot of pure water under the same conditions.

27. Formulate and compute the weight of a barrel of refined petroleum, if the oil weighs $6\frac{1}{2}$ lbs. to the gallon.

28. If a boiler evaporates 8 lbs. of water per hour per pound of coal, compute the number of gallons evaporated in 5 hours by a boiler which used $2\frac{1}{2}$ tons of coal in that time.

29. Ordinary atmospheric pressure is 14.7 lbs. per sq.in. This is how many kilograms per square centimeter?

30. The weight of a cubic foot of steam under ordinary atmospheric pressure is .03800 lb.

Formulate and compute the number of cubic feet which will weigh 1 lb. under the same pressure.

31. If 13.817 cu.ft. of air weigh 1 lb. at a temperature of 32° with the barometer at 30″, the weight of steam under a pressure of 1 atmosphere equals how many thousandths of the weight of air?

32. The ratio of the weight of steam to the weight of the same volume of air is called the specific gravity of steam.

Tabulate the specific gravity of air and of steam, denoting the specific gravity of air by 1.

33. A cubic inch of mercury weighs 0.49086 lb. A pressure of 1 atmosphere will balance a column of mercury 29.9218 inches high.

Compute the pressure of 1 atmosphere in pounds per square inch, to five decimal places.

34. A pressure of 1 atmosphere under 29.92 inches of mercury, as computed in Problem 33, is how many kilograms per square centimeter under how many millimeters of mercury?

35. Under a pressure of 165 lbs. to the square inch, the mercury stands at 336 inches.

This is how many kilograms to the square centimeter, with how many mm. of mercury?

36. The bulk of steam under the conditions given in Problem 35 is 2.6873 cu.ft. to the pound.

What is the weight per cubic foot under these conditions?

37. Compute the weight of steam in kilograms per cubic meter, and the volume in cubic meters per kilogram, under the same conditions as in Problems 35 and 36.

38. In a test of a water meter, 10 cu.ft. as registered by the meter were drawn off and weighed, the weight being 650 lbs. A U. S. gallon of the same water at the same time weighed 8 lbs. 5¼ oz.

Was the meter fast or slow and how much?

39. If the charge for water is 25 cents per thousand gallons, what should a consumer pay who has used 8000 cu.ft.?

How many liters is this, and what is the weight of this amount of water in kilograms?

40. It is estimated that under 100 lbs. pressure, 1500 gallons of water will escape in 24 hours through a leak no larger than the lead in an ordinary pencil.

On an unmetered tap, what would be the loss to a water company under these conditions, with water at $2.50 per thousand cubic feet?

41. Compute and fill in the omitted entries in the table below:

Table 71

COST OF BRIDGES

Name.	Constructor.	Weight, tons.	Cost per Ton.	Total Cost.	
				Eng.	U.S.
Atabara bridge, Africa.........	Pencoyd Iron Works	622	£26.8.6		
Gokteik Viaduct, Burmah.......	Penn Steel Co.	4332	£41.10		
Uganda Viaducts, East Africa....	Amer. Bridge Co.	7000	£64.2.6		

42. The first Zeppelin air-ship had the following proportions:
Aluminum framework, 128 m. in length;
Internal diameter, 11.3 m.;
External diameter, 11.66 m.
Balloon capacity, 11300 cum.,
Total weight, about 102000 kilos;
2 Daimler 16 H.P. motors, weighing 450 kilos each;
Consumption of benzine, .375 kilogram per horse-power hour;
Capacity of tanks, 60 kilograms benzine.

(1) Determine the dimensions of the framework in feet,
(2) The weight in tons,
(3) Total weight of motors in pounds,
(4) Duration of run without replenishment of benzine.

43. By the amalgamation process of milling gold-bearing rock, 64687.7 oz. of gold were obtained from 57725 tons of ore at an expense of 10s. 1.2 d. per ton.

(1) How much did the value of the gold exceed the cost of milling, in both English and U. S. money?

(2) What was the troy weight of the gold and its metric weight in kilograms?

(3) How many ounces of ore were required for 1 ounce of gold?

44. At the Paris Exposition of 1900, the boilers were required to furnish 600 kilograms of steam per hour per square meter of grate. The exhibitors received 4 francs per 1000 kilos of steam produced, the daily evaporative capacity being 1500000 kilograms.

(1) Compute the number of pounds of steam required per hour per square foot of grate.

(2) How much did the exhibitors receive for the steam produced per day, in French, English, and U. S. money?

45. A winding drum for hoisting ore from a shaft 220 meters deep is 224 cm. in diameter and 85 cm. long. The maximum speed of hoisting is 8.5 m. per second and the load is as follows:

Cage	700	kilograms,
Wagons	275	"
Cable	345	"
Waste	1000	"
Coal	600	"

(1) Compute the dimensions of the drum in U. S. measure.

(2) Compute the depth of the shaft in feet,

(3) Compute the total load on the drum in U. S. tons,

(4) Compute the speed in feet per minute, and the number of minutes required to hoist the load from the bottom to the top of the shaft.

46. The data below are taken from the Engineering Magazine for May, 1905.

Compute and fill in the omitted entries.

TABLE 72

TUNNELS

Name.	Length.		Daily Advance.		Cost per Running Yd.	Years in Construction.	Total Cost
	Miles.	Km.	Feet.	M.			
Simplon....	11.9		17.0		$ 770		
St. Gothard.	9.25		9.7		700		
Arlberg.....	6.38		15.7		625		
Mt. Cenis...	7.9		5.1		1075		

47. The passenger locomotives, Type 10, on the Belgian State Railways, have the following specifications.

Compute and fill in the omitted entries.

TABLE 73

BELGIAN LOCOMOTIVES, TYPE 10

Specifications.	Dimensions.	
	Metric.	U. S.
Stroke	660 mm.	
Boiler pressure	14 kg. per sq. cm.	
Inside diameter, boiler	1.8 m.	
Outside diameter, boiler tubes	50 mm.	
Thickness, 390 boiler tubes	2.5 mm.	
Thickness, 51 boiler tubes	4.5 mm.	
Length of grate	2.5 m.	
Width of grate	2 m.	
Heating surface, firebox	20 sq. m.	
Heating surface, tubes	220 sq. m.	
Diameter driving wheels	1.980 m.	
Total weight, gross	102000 kg.	
Draw-bar pull	15160 kg.	
Diameter cylinders	500 mm.	
Grate area		

48. An air-compressor with a capacity of 7000 liters per minute, forces air into two reservoirs having a combined capacity of 142 cu.ft., the pressure maintained in the reservoirs being 94 lbs. per square inch.

Compute the capacity of each reservoir in liters and the pressure in kilograms per square centimeter.

49. The table below gives the length of the three lines of the Berlin Elevated and Underground Railway, and the running time on each. Compute and fill in the omitted entries.

TABLE 74

BERLIN ELEVATED AND UNDERGROUND RAILWAY

Distance.		Time in Minutes.	Speed per Hour.	
Meters.	Miles.		Miles.	Km.
9690		23		
6330		15		
5250		12		

TABLE 75

UNITED STATES AND METRIC EQUIVALENTS

Unit.	Equivalent.		Unit.	Equivalent.	
	Approximate.	Exact.		Approximate.	Exact.
1 acre	.40 hectare	.4047	1 mm.	.039 in.	.03937
1 bushel	35¼ liters	35.24	1 oz. (avoir.)	28⅓ g.	28.35
1 cm.	.39 in.	.3937	1 oz. (troy)	31 g.	31.10
1 c.c.	.06 cu. in.	.0610	1 peck	8.8 liter	8.809
1 cu. ft.	.028 cum.	.0283	1 pint	.47 l.	.4732
1 cu. in.	16.4 c.c.	16.387	1 pound	.45 kg.	.4536
1 cum.	35.3 cu. ft.	35.31	1 qt. (dry)	1.1 l.	1.101
1 cum.	1.3 cu. yd.	1.308	1 qt. (liq.)	.95 l.	.9464
1 cu. yd.	.76 cum.	.7645	1 scm.	.16 sq. in.	.1550
1 ft.	30½ cm.	30.48	1 sq. ft.	.093 sm.	.0929
1 gal.	3.8 liters	3.785	1 sq. in.	6.5 scm.	6.452
1 grain	.065 g.	.0648	1 sq. mile	260 Ha.	259
1 gram	15½ gr.	15.43	1 sm.	1.2 sq. yd.	1.196
1 hectare	2.5 acres	2.471	1 sm.	11 sq. ft.	10.76
1 inch	25 mm.	25.40	1 sq. rod.	25.3 sm.	25.293
1 kilo	2.2 lbs.	2.205	1 sq. yd.	.84 sm.	.8361
1 km.	.62 mile	.6214	1 ton (U. S.)	.91 m. ton	.9072
1 liter	.91 qt. (dry)	.9081	1 ton (Eng.)	1 m. ton	1.017
1 liter	1.1 qt. (liq.)	1.057	1 ton (metric)	1.1 t. (U. S.)	1.102
1 meter	3.3 ft.	3.281	1 ton (metric)	.98 t. (Eng.)	.9842
1 mile	1.6 km.	1.6093	1 yd.	.91 m.	.9144

CHAPTER V

MEASURING INSTRUMENTS

SECTION 1. THE MICROMETER. SECTION 2. THE VERNIER CALIPER. SECTION 3. THE PLANIMETER

§ 1. THE MICROMETER

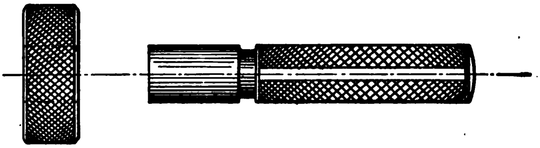
99. Description. The instruments commonly used for precise measurements in the shop are gages and calipers. Of all these instruments in their various forms, only two require explanation. These are the micrometer and the vernier calipers.

As the name indicates, the micrometer is an instrument which is used in taking small measurements, "micro" meaning small and "meter" meaning measure. The instruments in the form here shown are made in $\frac{1}{2}$-, 1-inch, 2-inch, and 3-inch sizes, the size being the opening between the spindle and the anvil. They are graduated both in U. S. and metric units, but the two are not combined in the same instrument owing to the difference in the screw.

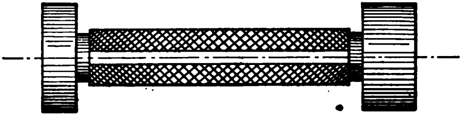
The parts of a micrometer are named as follows:

F, the frame,
A, the anvil,
S, the spindle,
B, the hub or barrel,
T, the thimble.

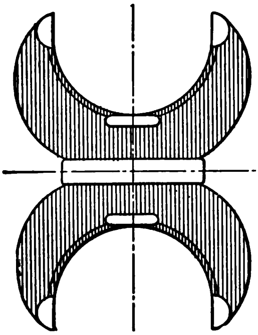
The spindle is attached to the thimble at the point *P*. The concealed portion of the spindle within the sleeve and

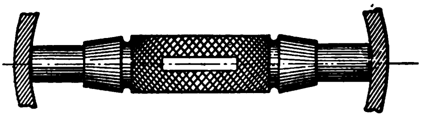


Standard cylindrical gage.



Internal limit gage.





End measuring rods with spherical ends.

External limit gage.

Fig. 13.

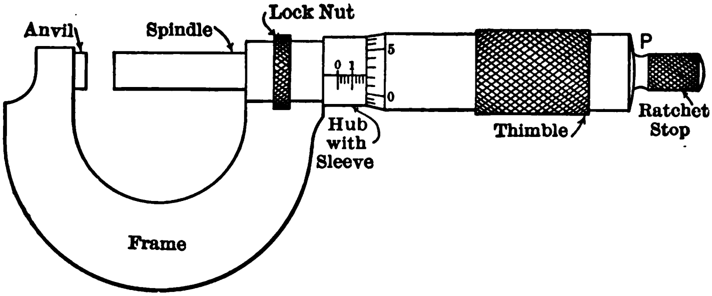


Fig. 14.

thimble is a screw having 40 threads to the inch, or, as it is called, a 40-pitch screw.

For purposes of adjustment, some makes of micrometer have a thin plate called a sleeve surrounding the hub or barrel. This sleeve carries the hub or barrel scale, so that when the parts become worn, a slight turn of the sleeve brings the base-line of the scale into proper adjustment with the thimble scale.

When the thimble is turned to the right or from the operator, the opening between the anvil and the spindle is decreased; when turned to the left, the opening is increased.

100. Problems. Solve the following problems, showing computations in full.

1. The hidden end of the spindle of a micrometer is a screw of what pitch? This means how many threads to the inch?

The distance between the threads is therefore what fractional part of an inch?

2. The distance between the threads on the concealed part of the spindle of the micrometer is what decimal part of an inch?

Show the computation.

3. With the micrometer held as shown in Fig. 15, the thimble is turned through exactly 1 revolution from the operator.

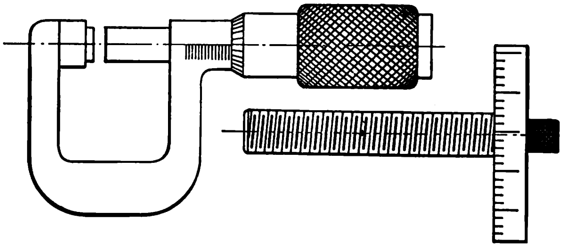


Fig. 15.

Is the opening between anvil and spindle increased or diminished? How much?

4. With the micrometer in the same position as in Problem 3, the thimble is turned toward the operator exactly one-half a revolution. Is the opening decreased or increased?

What fractional part of an inch?

What decimal part of an inch?

5. With the micrometer in the same position, the thimble is turned from the operator one-fifth of a revolution. Is the opening increased or decreased?

What fraction of an inch?

What decimal of an inch?

6. With the position of the micrometer unchanged, the thimble is turned from the operator one twenty-fifth of a revolution.

The spindle has moved through exactly $\frac{1}{25}$ of how many fortieths of an inch?

This is what fractional part of an inch?

This is what decimal part of an inch?

7. To move the spindle .025″, the thimble must be turned through how many revolutions?

8. To move the spindle .001″, the thimble must be turned through what part of a revolution?

How would you prove this to anyone by the use of .025″ and .001″?

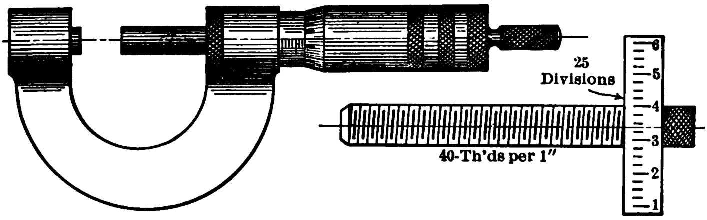


Fig. 16.

9. At each revolution the spindle moves how far?

At $\frac{1}{25}$ of a revolution the spindle moves how far?

10. The spindle rests against the anvil. Three turns are given toward the operator. What is the opening?

11. Seven turns are given toward the operator in addition to those in Problem 10.

What is the opening?

12. With the micrometer set as in Problem 11, $5\frac{1}{2}$ turns from the operator are given.

Compute the opening in two different ways.

101. Graduation of the Sleeve. It will be evident that it would be both inconvenient and difficult to measure pieces with the micrometer without some means of determining the number of revolutions and parts of a revolution from contact with the anvil to contact with the piece whose measurement is required. This difficulty is overcome in a very simple manner.

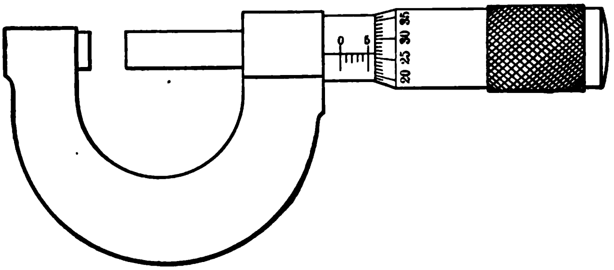


Fig. 17.

The scale on an instrument is called its graduation. Since the thimble moves along the sleeve a distance of exactly one inch, the position of the thimble's beveled or inner end, when the spindle of a 1-inch micrometer is in contact with the anvil, is denoted on the sleeve by a graduation marked zero.

When the micrometer is fully open, the position of the beveled end of the thimble is denoted by a graduation on the sleeve, marked 10. The intermediate graduations which are indicated by lines perpendicular to the axis of the sleeve, of the same length as the 0 and 10 graduations, are marked from 1 to 9 inclusive. Each of these ten spaces is divided into four equal parts by lines of equal length, as seen in the drawing and on the micrometer.

On a 2-inch micrometer the graduations extend along the sleeve a distance of exactly 1 inch, the same as on a 1-inch instrument, the graduation marked 10 denoting the position of the beveled end of the thimble when the opening is 2 inches.

102. Problems. Solve the following problems, showing all computations which it may be necessary to make.

1. The tapered end of the thimble stands at 10 on a 2-inch micrometer. What is the opening?

2. On a 2-inch micrometer the thimble stands at 0.
What is the opening?

3. On a 1-inch micrometer the thimble stands at 0.
What is the opening?

4. Write a statement explaining why the sleeve of a micrometer has forty graduations extending a distance of exactly 1 inch.

5. On a 1-inch micrometer the thimble is set at 8.
What is the opening?
Prove it by computation.

6. On a 2-inch micrometer the thimble is set at 3.
What is the opening?

7. Under the same conditions as in Problem 5 compute the opening in millimeters.

8. Under the same conditions as in Problem 6 compute the opening in millimeters.

9. The first *numbered* graduation following zero on the sleeve denotes what decimal of an inch?

Therefore the graduation numbered 3 denotes what decimal of an inch?

10. The first *unnumbered* graduation denotes what decimal of an inch?

Therefore the third denotes what decimal of an inch?

11. Tabulate the openings both in inches and millimeters for each numbered graduation increasing by 1 from 0 to 10 inclusive, on a 2-inch micrometer.

TABLE 76

READINGS OF MICROMETER

Grad.	Reading.		Grad.	Reading.	
	In.	mm.		In.	mm.
0			6		
1			7		
2			8		
3			9		
4			10		
5					

103. Graduation of the Thimble. As seen on the instrument, the beveled end of the thimble is graduated from 0 to 25, each fifth division being numbered.

When the micrometer is closed and when fully open, zero on the thimble is exactly over the line extending lengthwise on the sleeve.

If it is not, and if the micrometer cannot be adjusted, it should be discarded as a measuring instrument, although it may serve for demonstration purposes.

104. Problems. Solve the following problems, showing computations.

1. Each graduation of the thimble means a movement of the spindle, of what decimal of an inch?

2. The micrometer as graduated in U. S. measure gives readings to what part of an inch?

3. The sleeve reading is 7, the thimble reading 15. Compute the opening of a 1-inch micrometer.

4. Under the same conditions as in Problem 3 compute the opening of a 2-inch micrometer both in inches and millimeters.

5. Compute and fill in the omitted entries in the table below in which *B* denotes the sleeve reading, *T* the thimble reading, and *D* denotes the dimension of the piece which is held between the anvil and the spindle.

TABLE 77

MICROMETER READINGS

No.	*B*	*T*	*D*		No.	*B*	*T*	*D*	
			In.	mm.				In.	mm.
1					11				
2					12				
3					13				
4					14				
5					15				
6					16				
7					17				
8					18				
9					19				
10					20				

105. Ratchet Stop. The micrometer, while durable and reliable under proper use, is easily thrown out of adjustment or otherwise injured by rough handling. Until one has acquired by use of the instrument a feeling of the suitable pressure in taking a measurement, readings of the same piece will not agree and the instrument is liable to be strained by too great pressure upon the piece being measured.

This danger is entirely removed by the ratchet stop. The ratchet is attached to the outer end of the thimble which is therefore turned by it. The moment the spindle and anvil grip any piece with sufficient pressure, the thimble stops and the ratchet slips by the pawl, when the reading may be taken after two or three clicks of the pawl are heard.

106. Quick-adjusting Micrometer. A 40-pitch screw necessarily means a slow movement of the spindle and therefore loss of time in adjusting the instrument to the various dimensions of different pieces whose measurement may be required.

This has been obviated by the manufacture of a so-called quick-adjusting micrometer. In this instrument, a slight pressure on a plunger in the ratchet end of the thimble releases the nut so that the spindle may be instantly moved to and fro. On release of the pressure on the plunger, the screw immediately reengages the nut and the pressure of the spindle against the piece under measurement is secured by the ratchet.

107. Metric Micrometer. The full explanation which has been given in preceding paragraphs makes it possible to describe the metrically graduated micrometer in few words.

The sleeve of a metric micrometer is graduated for a distance of exactly 25 mm., every 5th division being numbered, so that the numbers run by 5's from 0 to 25 inclusive.

The thimble has 50 graduations, every 5th division being numbered, so that on the thimble the numbers run from 0 to 50 inclusive. See Fig. 17.

The screw has 2 threads to the millimeter.

The metric micrometer is also made with the quick-adjusting device and the ratchet stop.

108. Problems on the Metric Micrometer. Solve the following problems, showing all computations:

1. The drawing exhibits on an enlarged scale the first five graduations of the sleeve of a metric micrometer.

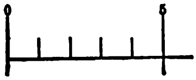


FIG. 18.

(1) Draw the graduation of the entire sleeve, in the work-book, using a scale of 5 to 1.

(2) Draw the graduation of the entire sleeve, also, the actual size it is on the instrument, or on a scale of 1 to 1.

2. The screw of a metric micrometer has how many threads to the 25 millimeters?

What, therefore, is the pitch?

3. The screw of a metric micrometer has how many threads to the inch?

4. In one revolution the spindle of a metric micrometer moves what part of a millimeter?

What part of an inch?

5. In how many turns will the thimble move the entire length of the graduation of the sleeve?

6. How is the thimble of a metric micrometer graduated?

What is the movement of the spindle in millimeters when the thimble is turned exactly one revolution?

Therefore how far does the spindle move when the thimble is turned exactly one-fiftieth of a revolution?

7. Each graduation mark on the beveled end of the thimble means a spindle movement of what decimal part of a millimeter?

8. The thimble reading is always the thimble graduation which is in coincidence with the line extending lengthwise of the sleeve.

Compute the size of a piece whose measurement is denoted by an 8 reading on the sleeve and a 13 reading on the thimble.

9. A piece under measurement gives readings of 21 sleeve, 36 thimble for one dimension, and 11 sleeve, 22 thimble for the other dimension.

What are the two dimensions?

10. With the U. S. micrometer, take three measurements of

as many of the wires on the desk, as may be assigned, turning them slightly each time so as to measure different diameters in case the wires are not perfectly round. Enter the measurements and the average of the three measurements in table 78 as indicated by the headings.

11. Reduce each average in table 78 from Problem 10, to 1000ths of a millimeter and enter in the table.

12. With the metric micrometer measure the same wires in the same way as in Problem 10 and enter in the table.

13. Reduce each metric average to 1000ths of an inch, and enter in the table.

TABLE 78

U. S. MICROMETER MEASUREMENT

METRIC MICROMETER MEASUREMENT

Wire.	Measurements.			Average.		Wire.	Measurements.			Average.	
	1	2	3	In.	mm.		1	2	3	mm.	In.
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
10						10					

109. The Ten-thousandths Micrometer. When readings are required to the 10-thousandths (.0001) of an inch, they may be obtained by the use of the so-called 10-thousandths micrometer.

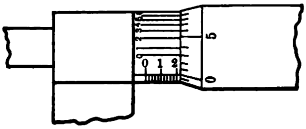


FIG. 19.

This instrument, as shown in the cut, is only the regular micrometer with an additional graduation whose lines extend lengthwise of the barrel or sleeve, and which may be called the ten-thousandths scale.

The graduation is as follows: The distance occupied by the ten graduations on the 10-thousandths scale is exactly the same as the distance occupied by *nine* graduations on the beveled end of the thimble.

Since the length of the 10 divisions of the 10-thousandths scale equals the length of 9 *thimble* divisions, the length of 1 division of the 10-thousandths scale equals $\frac{9}{10}$ of the length of 1 *thimble* division. Therefore the difference between the length of a 10-thousandths division and a thimble division is exactly $\frac{1}{10}$ of the length of a thimble division.

110. How to Read a Ten-thousandths Micrometer. The preceding problems illustrate two things:

(1) The 10-thousandths reading on any measurement is denoted by the number of the graduation on the 10-thousandths scale, which is in *coincidence* with a graduation of the thimble. In the figure the 10-thousandths reading is zero.

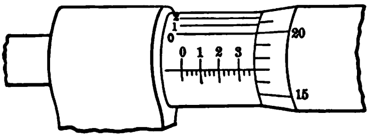


FIG. 20.

(2) When no graduation of the thimble is coincident with the base-line of the barrel or sleeve scale, *the thimble reading in thousandths* is denoted by the number of the thimble graduation which precedes the base-line of the hub or sleeve scale.

111. Problems. Solve the following and verify on the instrument:

1. Observe in Fig. 19 that when the thimble stands at zero, the graduation mark 3 on the thimble is in exact coincidence

with zero of the 10-thousandths graduation. Therefore graduation mark 4 on the thimble lacks what part of a thimble division, of coinciding with graduation mark 1 of the 10-thousandths scale?

Therefore 4 on the thimble can be brought into coincidence with 1 on the 10-thousandths scale by turning the thimble to the left, or backward, exactly what part of a thimble division?

2. When the thimble of a U. S. micrometer is turned through exactly one division, the spindle moves what decimal part of an inch?

Therefore when the thimble is turned through $\frac{1}{10}$ of a division, the spindle moves through $\frac{1}{10}$ of what part of an inch?

This equals how many thousandths of an inch?

3. When the thimble is set at zero, 5 on the thimble is exactly how many 10ths of a thimble division beyond 2 on the 10-thousandth scale?

To make 5 on thimble coincide with 2 on 10-thousandths scale, the thimble must be turned backward exactly how many 10ths of a thimble division?

But a thimble division means a spindle movement of what part of an inch?

Therefore when 5 on the thimble is in coincidence with 2 on the 10-thousandths scale, the spindle opening is what part of an inch more than when the thimble stands at zero?

4. The thimble is set at 4. What graduation mark on it is in exact coincidence with 0 on the 10-thousandths scale?

Draw the setting in the work-book.

5. Set the thimble as in Problem 4. Turn it backward until 9 on the thimble is in coincidence with zero on the 10-thousandths scale.

The opening is how many 10-thousandths of an inch *more* than when the thimble was set at 4?

6. In table 79 the notation is as follows:

B, hub, barrel, or sleeve reading,
T, thimble reading,
10, 10-thousandths reading,
D, distance between anvil and spindle.

Take each setting on the U. S. micrometer, and fill in the omitted entries.

7. Observe graduation line 1 on the 10-thousandths scale in Fig. 20. Is it correctly placed? Why? Illustrate by an enlarged figure.

TABLE 79

MICROMETER* READINGS

No.	B	T	10	D
1	5	0	2	
2	7	18	4	
3	4	3	1	
4	6	2	2	
5	8	1	0	
6	2	20	3	
7	9	16	7	
8	3	6	8	
9	1	21	0	
10	7	18	9	

* See problem 6, paragraph 111.

§ 2. THE VERNIER CALIPER

112. The Vernier. This instrument is named from its inventor. Its principle has already been explained in the account of the 10-thousandths micrometer.

On whatever instrument a vernier is placed, it always consists of a scale which contains *one more* graduation in a given length than the scale along which it slides, has in the same length.

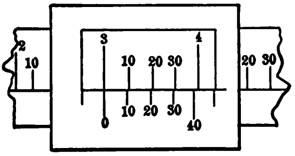


FIG. 21.

In the illustration, the bar or upper piece has 40 graduations to the inch. The vernier or lower piece is graduated so that its 25 graduations extend exactly the same length as 24 of the bar graduations.

Therefore the difference between the length of *one bar* graduation and *one vernier* graduation, equals exactly $\frac{1}{25}$ of the length of one bar graduation.

But one bar graduation $= \frac{1}{40}'' = .025''$.

Therefore the difference between the length of one bar graduation and one vernier graduation, equals $\frac{1}{25} \times \frac{1}{40}'' = .001''$.

113. How to Make a Vernier Reading. When an instrument is equipped with a vernier, the reading is always determined as follows:

(1) If zero of the vernier is in *conicidence* with a bar graduation, the reading of the instrument is that graduation of the bar which is in coincidence. What this reading is in inches can be determined from the number of graduations to the inch.

(2) If zero of the vernier is *not* in coincidence with a bar graduation, the reading of the instrument is the bar reading added to the vernier reading. The **bar** reading in such case, is that graduation of the bar which immediately precedes zero of the vernier; the **vernier** reading is that graduation of the vernier which is in coincidence with a bar graduation. If the vernier unit is not marked on the vernier, observe into how many graduations the vernier scale is divided. The vernier unit may then be determined as explained in paragraph 112. The number of the coincident vernier graduation denotes the number of vernier units to be added to the bar reading.

In figure 21 the vernier unit as explained in paragraph 112, and as shown, is .001″.

114. The Vernier Caliper. This instrument is designed both for inside and outside measurement.

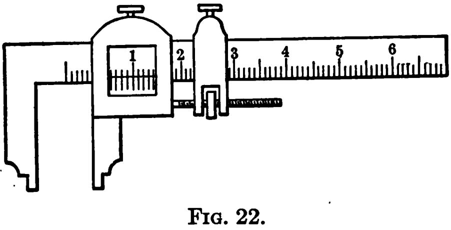


FIG. 22.

It consists, as illustrated, of a bar, at one end of which is attached at right angles a fixed piece called a jaw, and to which is clamped a movable jaw carrying a vernier.

§ 3. THE PLANIMETER

115. What the Planimeter is and How it is Used. The name of this instrument is formed from "planus" meaning flat, and "meter" meaning measure. The instrument is therefore by name, an area or surface measurer. It consists of four parts:

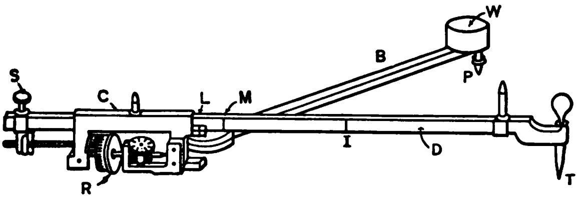


Fig. 23.

(1) A sliding piece *C*.

(2) A bar *B*, pivoted to the sliding piece *C*, and carrying at the outer end a steady weight *W*, and a sharp point *P* for pressing into the paper to hold the instrument while in use.

(3) A carrier bar *D*, which passes through and is secured to the sliding piece *C*, by the screw *S*. This bar has a fine steel point in the outer end, for tracing the boundary line of the diagram or area, and has two index marks denoted in the cut by *I* and *M*. These letters denote square inches and square centimeters respectively, square inches being recorded when the bar is clamped by the screw so that *I* is exactly in alignment with index *L* on the end of the sliding piece, and square centimeters being recorded when *M* is similarly aligned with *L*.

(4) A recording mechanism *R* on the under side of the sliding piece *C*. This mechanism as shown in the enlarged view consists of

(*a*) A spindle *E* turning in conical bearings.

(*b*) A hollow drum or wheel *F*. with vernier, rigidly attached to the spindle.

(*c*) A recording dial or counting disc *G*, geared to the spindle by a worm and worm wheel so as to rotate once for every ten revolutions of the drum.

The periphery of the drum is graduated into equal divisions, numbered from 1 to 0, and each numbered division is further graduated into ten equal parts.

The counting disc or dial has ten numbered graduations as shown in the cut.

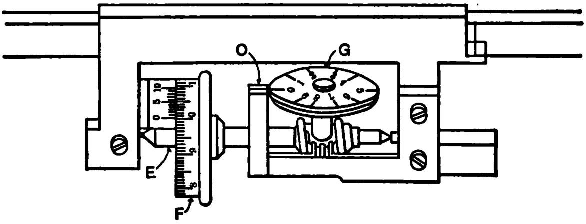


FIG. 24.

In tracing a diagram the instrument rests on the point *P* and the roller drum, the area being traced in a clockwise direction by the point *T*. When *T* has passed completely around the diagram, the reading is taken as follows:

1. Dial Reading. Each graduation mark on the dial denotes 10 units of area. Therefore there are ten times as many units of area as the number of units in the graduation mark just beyond the index 0 on the limb of the sliding piece. Thus the reading is 40 when the graduation mark 4 is just beyond the limb index 0.

2. Drum Reading. Each numbered graduation on the drum denotes 1 unit of area; each unnumbered graduation denotes .1 of a unit of area. Therefore the drum reading is the unnumbered graduation just beyond or coincident with the vernier zero.

Thus 5.8 units of area is the drum reading when the 8th unnumbered graduation following 5 is beyond the vernier zero.

3. Vernier Reading. Each line on the vernier denotes .01 of a unit of area. Therefore if line 7 of the vernier coincides with a line of the drum, the vernier reading is .07 unit of area.

116. Problems. In solving the problems below whenever a measurement is required, arrange the work as follows:

(1) Problem number and title.

(2) Number and name of measured piece.

(3) Name of instrument and the manufacturer.

(4) A carefully drawn sketch showing the instrument reading.

(5) Explanation of the reading by a statement of the bar, vernier or other graduations, computation of vernier unit, and addition of scale readings.

Any suitable instrument on the table may be used. If a piece is measured by more than one instrument, show sketch of reading on each.

Handle the instruments carefully, do not hit one against another or against the pieces to be measured, adjust them lightly to the piece, and hold them steady while measurement is taken.

1. Examine and sketch a planimeter. Write the names of its various parts as indicated by letters on the sketch.

2. Describe the drum graduation. Each numbered graduation denotes how many units of area?

Therefore each unnumbered graduation denotes what part of a unit of area?

3. The 5th unnumbered graduation following 4 is just beyond the zero of the vernier. What is the drum reading if the instrument is set so that I on the carrier bar, is aligned with L on the end of the sliding piece?

4. Set M on the carrier bar in alignment with L on the sliding piece. Turn the drum to record 6.3 units of area.

What is the unit?

5. In a sketch show a vernier reading of .03 of a unit of area.

6. Sketch a disc or dial reading of 50 square inches.

7. What is the area when the scales are set as follows:

Vernier, 2d line coincident,
Drum, 8 and 3d unnumbered beyond zero vernier,
Disc, 6 beyond limb index.

8. Determine the area of the diagram here shown, in square inches.

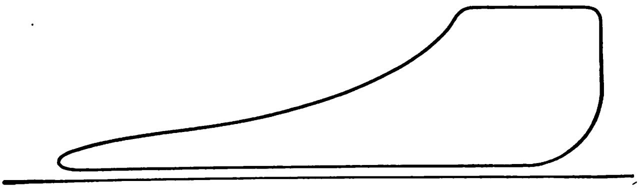


FIG. 25.

Reset the instrument and trace the area in metric units.

Check both readings by reducing the square inches to square centimeters, and by reducing the square centimeters as read, to square inches.

9. On the work-book paper trace the outlines of not less than five of the irregularly shaped models at the desk and determine and tabulate the areas both in U. S. and metric units.

10. With the micrometer and the vernier caliper measure and tabulate the dimensions of such machine tools or models on the desk as may be assigned.

CHAPTER VI

TAPER

117. Definition. Taper has been variously defined, but as used in this chapter it is the difference in diameters in turned work and the difference in widths or thicknesses in unturned work, for any given length measured along the axis of the work.

In the shop the problem of taper is always a question of how much the tail-stock of a lathe must be set over in order to turn a required taper.

Two things and two only, determine the set-over or poppet movement as it is sometimes called:

(1) The taper for any given unit of length,

(2) The length of the work over all.

Therefore in order to compute set-over, both these things must be known.

118. Formulas for Taper. Enter the following notation in the work-book:

S = set-over in inches,
t_i = taper per inch,
t_f = taper per foot,
T = total taper,
D = diameter at large end of taper,
d = diameter at small end of taper,
L_t = length of tapered part measured along the axis,
L_T = total length of the work.

(1) Formulate taper per inch in terms of taper per foot.
(2) Formulate taper per foot in terms of taper per inch.

(3) Formulate the total taper in terms of the diameters at the beginning and at the end of the taper.

(4) Formulate the taper per inch in terms of the total taper and the length of the tapered part in inches.

(5) Formulate the taper per inch in terms of the total taper and the length of the tapered part in feet.

119. Laws and Formulas for Set-over. There are two laws for the throw or set-over of the poppet head:

(1) The set-over in inches equals the taper per inch multiplied by one-half the total length of the work in inches.

(2) The set-over in inches equals one-half the difference of the diameters at the ends of the taper, multiplied by the total length of the piece in inches and divided by the total length of the taper in inches.

Formulate each of these laws, using the notation in paragraph 118.

120. Problems. Solve the following problems in the work-book, with a carefully drawn figure for each:

1. Standard Tapers. The following are the standard tapers:

American,	$\frac{9}{16}$" to 1 foot,
Brown & Sharpe,	$\frac{1}{2}$" to 1 foot, except No. 10, which is .5161 to 1 ft.
Jarno,	.6" to 1 foot.
Morse,	$\frac{5}{8}$" to 1 foot.

Tabulate the data, and formulate, compute, and tabulate the taper per inch of each standard.

2. Determine what standard is used in the piece here shown in section, and compute the diameter at the small end.

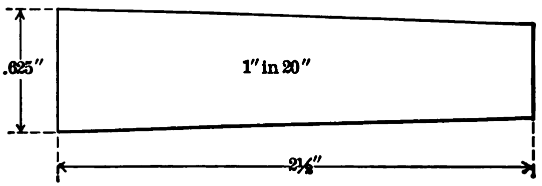


Fig. 26.

3. A Morse Taper. The drawings below show a Morse taper. With dimensions indicated, compute D, the diameter at the point where the taper begins.

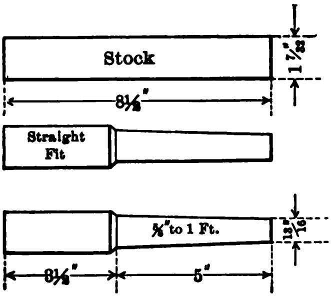


FIG. 27.

4. Formulate and compute the set-over in Problem 3.

5. Proportions of Jarno Taper. When the number of a Jarno taper is known, the proportions are determined by the following formulas, in which

N = number of the taper,
D = diameter large end in inches,
d = diameter small end in inches,
L = length of taper in inches,

$$D = \frac{N}{8}, \qquad d = \frac{N}{10}, \qquad L = \frac{N}{2}.$$

Enter the formulas with the notation in the work-book and compute and fill in the omitted entries below:

TABLE 80

PROPORTIONS OF JARNO TAPERS

N	D	d	L	N	D	d	L
1				11			
2				12			
3				13			
4				14			
5				15			
6				16			
7				17			
8				18			
9				19			
10				20			

6. A piece 5 inches long has ⅝ inch difference in diameters. Find the taper per foot.

7. The difference between the large and small ends of a piece 7⅜ inches long is ¾ inch.

Find the taper per foot.

8. Calculate the total taper in a piece 10 ins. long, whose taper is $\frac{3}{4}$ in. per foot.

9. In cutting a taper the entire length of a piece, the tail-stock must be set over approximately one-half the amount of the total taper.

Explain why ?

10. How far must the tail-stock be set over to taper a piece 13 ins. in length, with American taper?

11. Sellers' Taper. The Sellers taper is $\frac{3}{4}$ in. to the foot. Compute and tabulate the omitted entries in the table below in which d = diameter at small end of shank in inches, and the other letters denote the dimensions shown in the sketch. D represents the diameter at the left end of dimension L.

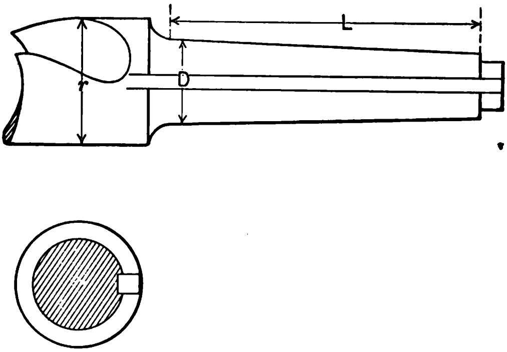


FIG. 28.

TABLE 81

SELLERS' TAPER DRILL SHANKS

No.	r	L	D	d
1	$\frac{1}{2}$	$2\frac{1}{4}$	$\frac{3}{16}$	
2	$\frac{11}{16}$	$2\frac{3}{4}$	$\frac{1}{4}$	
3	$\frac{7}{8}$	$3\frac{1}{2}$	$\frac{5}{16}$	
4	$1\frac{1}{8}$	$4\frac{1}{2}$	$\frac{3}{8}$	
5	$1\frac{5}{8}$	$6\frac{1}{2}$	$\frac{7}{16}$	

12. Compute the amount of set-over in the piece shown.

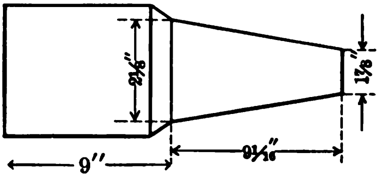


FIG. 29.

13. Determine the distance that the tail-stock should be set over to cut the following:

(*a*) A Morse taper on a piece 8¾ ins. long,

(*b*) A Brown & Sharpe taper on a piece 11⅛ ins. long,

(*c*) A Jarno taper on a piece 5⅜ ins. long.

14. Find the taper per foot in the tap shown below. Is it one of the tapers enumerated in Problem 1?

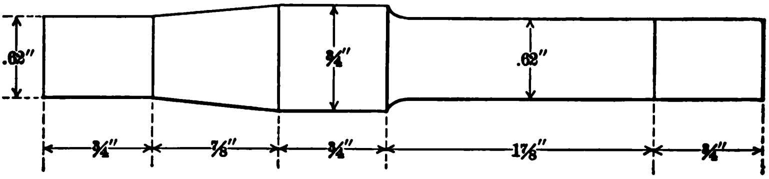


FIG. 30.

(*a*) What was the set-over to cut it?

(*b*) Compute the total length of the piece.

(*c*) The total length is how much more than six times the greatest diameter?

15. Brown & Sharpe Tapers. Calculate to three decimal places the entries for the last column of the table below and enter them in the table.

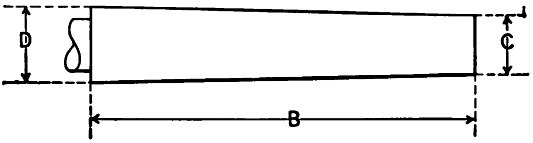


FIG. 31.

TABLE 82

BROWN & SHARPE TAPERS

No.	*B*	*C*	*D*	Taper in 12″.
1	2	.312	.3952	
2	$3\frac{9}{16}$	.75	.8985	
3	$5\frac{11}{16}$	1.0446	1.2888	
4	$8\frac{3}{4}$	2.25	2.5846	
5	$1\frac{3}{16}$	.25	.2991	
6	$1\frac{1}{4}$	.35	.4020	
7	$2\frac{3}{8}$	.50	.5989	
8	$6\frac{3}{4}$	1.25	1.5312	
9	$7\frac{3}{4}$	1.75	2.0521	
10	$9\frac{1}{4}$	2.50	2.8855	

16. Tapering Reamers. Determine to three decimal places the taper per inch in the reamers tabulated below and make the entries in the table.

TABLE 83

MORSE STANDARD TAPERS

No.	Small Diam.	Large Diam.	Length.	Taper per Inch
1	.369	.475	$2\frac{1}{8}$	
2	.572	.700	$2\frac{9}{16}$	
3	.778	.938	$3\frac{3}{16}$	
4	1.020	1.231	$4\frac{1}{16}$	
5	1.475	1.748	$5\frac{3}{16}$	
6	2.116	2.494	$7\frac{1}{4}$	

17. Taper of a Crank-pin. The crank-pin shown has an American taper.

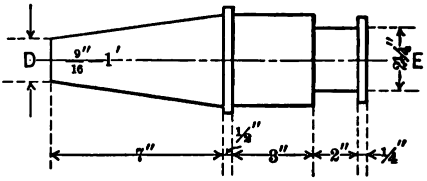


FIG. 32.

Required the total length of the pin, and the diameter at each end of the taper, the diameter at D being $\frac{1}{64}$ in. greater than at E.

What was the set-over for turning?

18. In order to taper a piece of work two notches were turned in it. The diameter at the bottom of the first notch was $1\frac{3}{16}$ ins. and at the bottom of the second $1\frac{9}{16}$ ins. The distance between the notches was $3\frac{15}{16}$ ins.

Determine the taper per foot, the taper being turned from the bottom of one notch to the bottom of the other.

19. By reading or otherwise, learn the different ways of turning a taper and write an account of each method stating its advantages and disadvantages.

20. A piece $11\frac{1}{2}$ ins. long is to be turned with a taper of .602 in. to the foot. The taper is to have a large diameter of $7\frac{1}{16}$ ins. and a length equal to $\frac{3}{4}$ the length of the piece.

Compute the small diameter.

21. A piece $18\frac{1}{2}$ ins. long is to be turned to a diameter of $6\frac{5}{8}$ ins. at the center, and to be tapered from the center to each end with a taper of 1 in. in 20 ins.

Determine

(1) The taper per foot,
(2) The end diameters,
(3) The set-over,
(4) The kind of taper.

22. In the piece shown formulate and compute B when the set-over is .1 in., using the notation below.

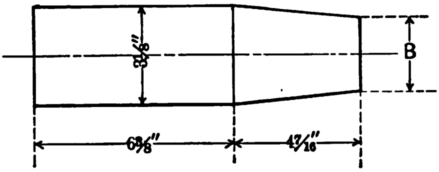


FIG. 33.

$A = 3\frac{1}{8}''$
$C = 6\frac{3}{8}''$
$D = 4\frac{7}{16}''$

23. The piece shown below is turned with two tapers.

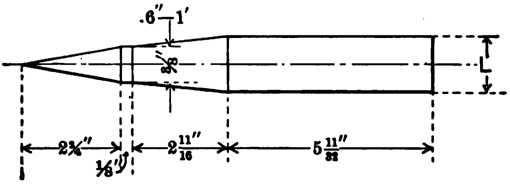


FIG. 34.

Determine

(1) Taper of the point,
(2) Diameter L,
(3) Total length,
(4) Set-over for both tapers.

24. Draw the piece in Problem 23 with the same dimensions, but with the second taper the same per inch as the taper of the point. · Compute the diameter L.

Draw the piece also with the following dimensions:

$$A = 3\tfrac{1}{16}'', \qquad C = 3\tfrac{17}{32}'',$$
$$B = 4\tfrac{5}{8}'', \qquad V = \tfrac{5}{16}''.$$

in which A, B, and C are the 1st, 3rd, and 4th horizontal dimensions, and V is the diameter between the tapers.

Determine

(1) Taper of point per foot,
(2) Diameter L when the second taper is the same as the first.
(3) The set-over.

25. On the step-cone pulley shown determine A, C, and L by measurement on some lathe in the shop, and compute the taper per inch.

26. Taper-turning Device. The set-overs or poppet movements as computed in the preceding problems are slightly in error owing to the fact that the lathe centers enter the work. This makes the distance between centers less than the length of the piece, and therefore affects the taper.

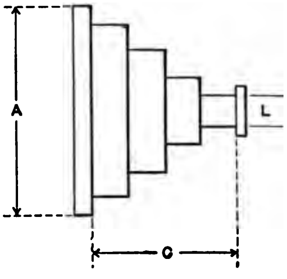


FIG. 35.

Does it make the set-over as computed, too much or too little? Why?

Write a short account accompanied by drawing or sketch, of some taper-turning device which you have used or of which you have read.

27. A fly-wheel shaft 17 ft. 5 ins. long and 21 ins. in diameter for 3 ft. in the center, tapers to a diameter of 16½ ins. at the bearings which are 18 ins. in length.

Compute the taper per inch.

28. The taper of a cotter usually varies from ⅜ in. to ¾ in. per foot measured along the axis. The large diameter in the drawing is $\frac{15}{32}$ in.

Determine the following:

(1) ⅜ in. per foot is 1 in how many inches?

(2) ¾ in. per foot is 1 in how many inches?

(3) What is the small diameter?

(4) What are the limits of variation in the large diameter with the small diameter as computed?

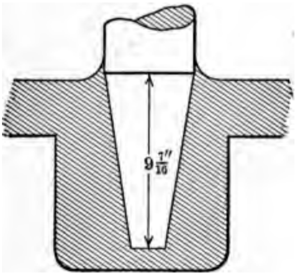


Fig. 36.

29. The greatest taper which can be given to a cotter depends on the coefficient of friction between the cotter and the surface into which it is fitted, and is generally 1 in 7.

When the taper is greater than 1 in 24 some means for locking the cotter should be employed.

Determine from the preceding

(1) The usual taper per foot,

(2) The greatest taper per foot before locking device is necessary.

30. On the spigot shown the taper per foot is ¾″. Compute the depth of the cut at A, in turning.

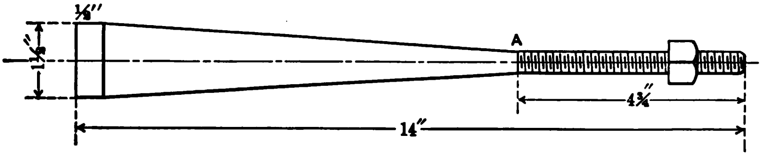


Fig. 37.

31. In the drawing the tooth-gearing tenons are .5″ in width at the shoulder *A* and .45″ at the point *L*.

The distance from the shoulder to the point *L*, measured along the axis, is 1.7″.

Compute the width of the tenon at *B*, and the taper per inch.

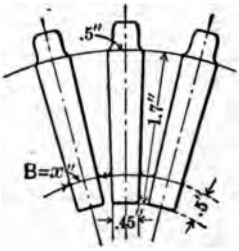


FIG. 38.

32. Using the dimensions as marked on the spindle of the tap-wrench shown in the drawing, compute (1) *A* when the taper is 2.3″ per foot; (2) the thickness at the point.

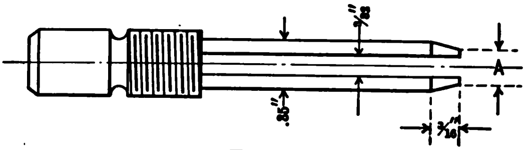


FIG. 39.

33. The drawing shows the shell of the same tap-wrench.

Determine whether the taper is the same as that of the jaws of the spindle above.

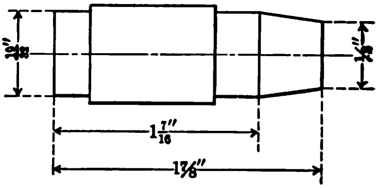


FIG. 40.

34. Required the taper per foot of the handle of the fork-wrench as dimensioned.

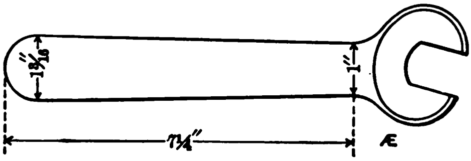


FIG. 41.

35. Compute the taper per inch of the cast-steel acorn below.

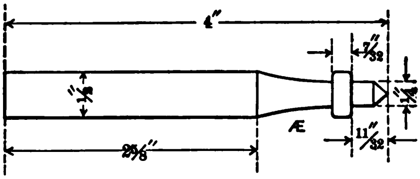


Fig. 42.

36. Compute the taper per foot of the spoke of the spur-gear whose section is shown.

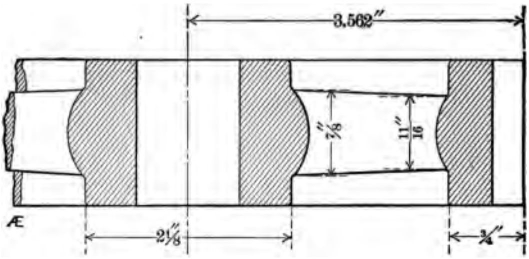


Fig. 43.

37. Determine by measurement the dimensions of the key here shown and calculate the taper per foot.

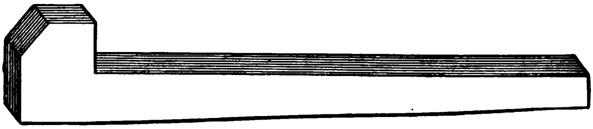


Fig. 44.

38. The drawing represents a piston-rod with dimensions as indicated. The depth of the shoulder at *V* is the same as at *C*. The taper is one-half inch per foot.

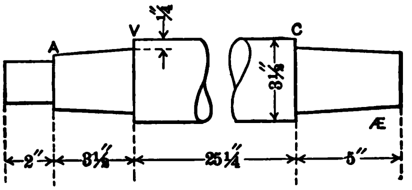


Fig. 45.

Compute

(1) The small diameter of both tapers,

(2) The depth of cut at *A* and *V*,

(3) The diameter *L*.

39. A connecting-rod 12′ 3″ long has a diameter of 3⅓″ at the cross-head end. If this diameter is .95 of the diameter at the center of the rod, compute the taper per foot.

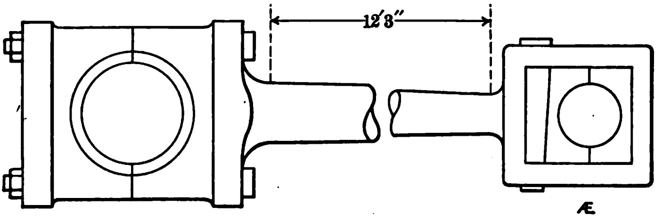


Fig. 46.

40. The cross-head end diameter of a connecting-rod is 2¾″. The stroke is 21″ and the length of the rod is 3.75 times the length of the stroke. The rod tapers uniformly form the ends to the center.

Find the diameter at the center, the taper being 1 in 20.

The diameter at the cross-head end equals what part of the diameter at the center?

41. The diameter of the connecting-rod in Problem 40, at the crank-pin end is .92 of the diameter at the center.

Determine

(1) The difference in diameters at the ends,

(2) The taper per foot from the center to the crank-pin end.

42. Long connecting-rods with equally heavy ends are usually barrel-shaped with end diameters ranging from ¾ to ⅞ of the diameter at the center of the rod. This corresponds to what range of tapers per foot in a rod 26′ 3″ long and having a diameter of 4⅛″ at the center?

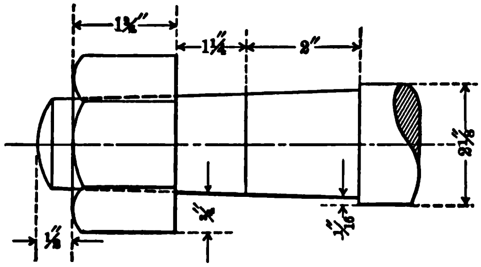


Fig. 47.

43. In the piece shown determine

(1) The taper per foot with American taper,

(2) The inner diameter of the nut.

44. Eccentric Strap. Determine the width at *R*.

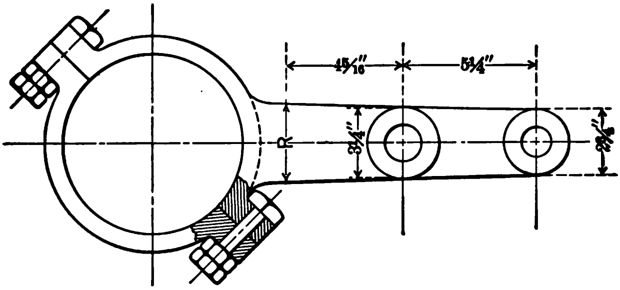


Fig. 48.

45. Starting Cock. Determine

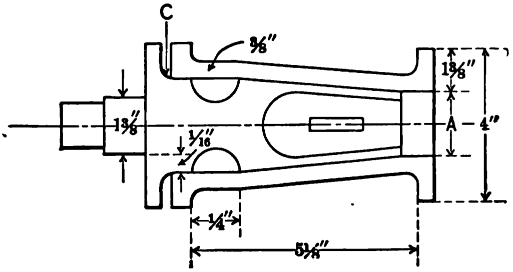


Fig. 49.

(1) The width at *A*,
(2) The taper per foot,
(3) The depth of the shoulder at *C*.

46. Throttle-valve Handle. Determine both tapers.

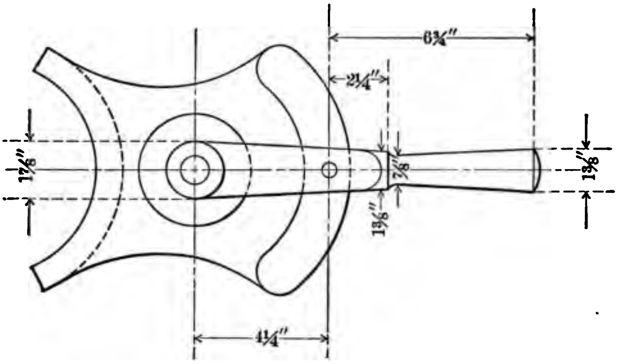


FIG. 50.

47. Jeweler's Vise and Cap Nut. Compute

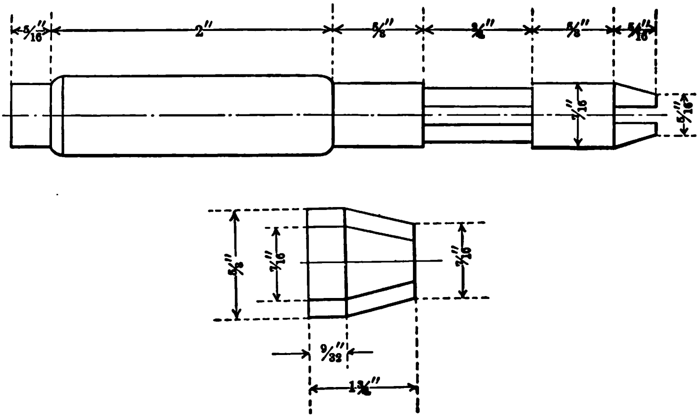


FIG. 51.

Required:
- (1) Length over all,
- (2) Taper per foot of jaws,
- (3) Depth of cut at the point,
- (4) External taper of cap nut per foot.

48. Drill-holder. Compute

(1) Taper per foot,

(2) Depth of cut at narrowest part of taper.

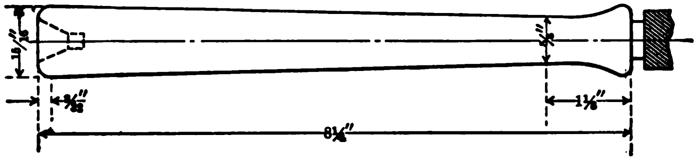


FIG. 52.

49. Thimble. Compute the taper per inch.

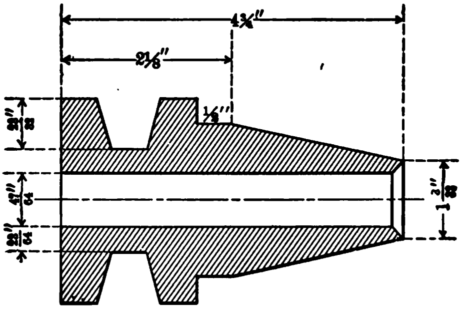


FIG. 53.

50. Flat-head Screw.

A = diameter of body,

$$B = 2A - 0.0052$$

$$C = \frac{A - 0.0052}{1.739}$$

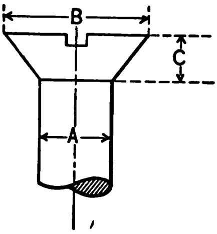


FIG. 54.

Compute and tabulate the taper per inch when A has the following values:

TABLE 84

SCREW TAPERS

A	B	C	Taper per Inch
.070			
.140			
.190			
.270			
.320			
.375			

51. Make the calculation necessary to determine the diameter D, Jarno taper being used.

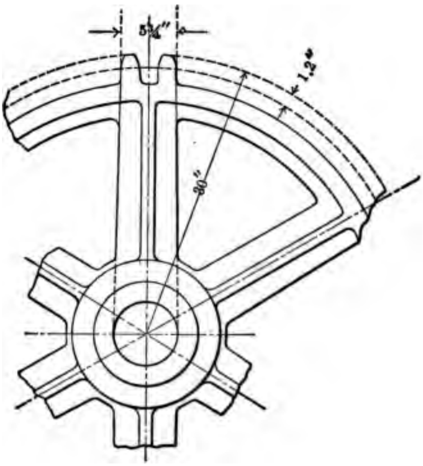


FIG. 55.

52. In the screw arbor shown in Fig 56, the small taper T_1 is B. & S.; the large taper T_2 is $1\frac{7}{8}''$ to the foot.

Compute the diameter d,

when $E=\frac{15}{32}''$, $F=3\frac{29}{32}''$,

$J=\frac{3}{8}''$, $C=\frac{9}{16}''$,

$H=.60''$.

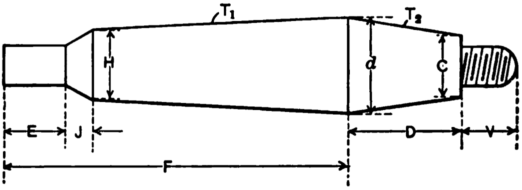
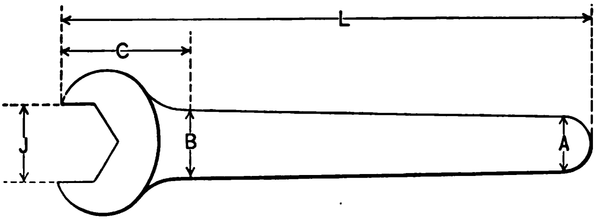


FIG. 56.

53. Using the result obtained in Problem 52 and the same data, compute D, the length of the large taper.

54. In the wrench here shown



$A=.65J$

$B=.8J$

$L=77J$

$C=1\frac{1}{4}J$.

FIG. 57.

Compute the taper of the handle per inch and per foot when $J=\frac{3}{4}''$.

55. The drawing shows a machine tap. Compute the taper per foot when

$L=6\frac{1}{16}''$. $A=3\frac{7}{8}''$,

$B=\frac{1}{2}''$, $C=\frac{1}{4}''$,

$D=\frac{3}{8}''$, $d=.015''$.

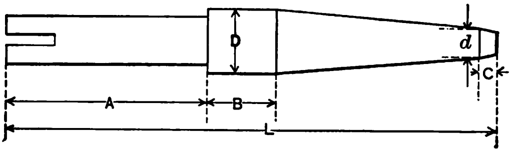


FIG. 58

CHAPTER VII

MENSURATION

SECTION 1. AREAS. SECTION 2. VOLUMES. SECTION 3. APPLIED PROBLEMS.

§ 1. AREAS

121. Definition and Outline. Mensuration is that part of mathematics which treats of the length of lines, the area of surfaces, and the volume of solids.

In this chapter we shall learn how to calculate the following flat surfaces:

Rectangle,
Parallelogram,
Triangle,
Trapezoid,
Regular polygon,
Circle,
Ellipse,
Irregular area.

Also the following solids:

Prism,
Cylinder,
Pyramid,
Cone,
Sphere.

It is necessary to know how to calculate these various forms because the materials of construction and the things which are constructed both in wood, iron, and other substances, have these different shapes singly or in combination.

122. The Rectangle. A rectangle as shown is a four-sided, square-cornered flat surface whose opposite sides are parallel straight lines.

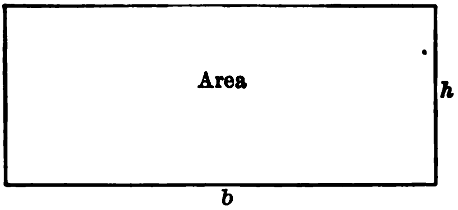


FIG. 59.

Breadth and length are called dimensions.

When the length equals the breadth the rectangle is called a square.

The figure below shows why area rectangle = length × breadth.

3 rows and 6 square feet in each row.

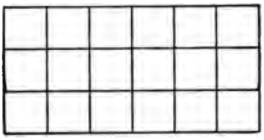


FIG. 60.

Total number square feet in rectangle $= 3 \times 6 = 18$.

$A =$ area of rectangle,
$b =$ length or base,
$h =$ breadth or height,
$A = bh$.

Divide both sides of this equation by h.

Therefore if A and h are known, how can b be found?

Divide both sides of the equation by b.

Therefore if A and b are known, how can h be found?

Under the heading draw a rectangle and letter the formulas for area, length, and height.

123. The Parallelogram. As shown, a parallelogram is the same as a rectangle except that it is not square-cornered.

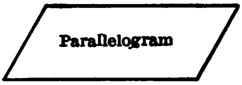


FIG. 61.

The figure below shows why

area parallelogram = length × height.

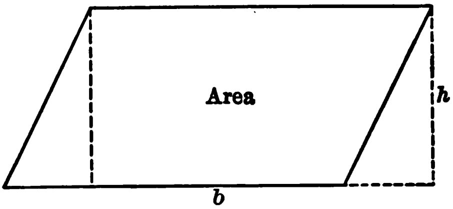


FIG. 62.

If the right triangle at the left were cut off and placed in the position of the triangle at the right, a rectangle would be formed of the length and height of the given parallelogram.

Under the paragraph heading, draw a parallelogram and letter the formulas for area, length, and height.

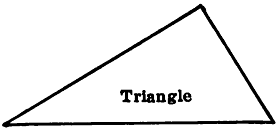


FIG. 63.

124. The Triangle. As shown, a triangle is a three-angled three-sided flat surface whose sides are straight.

The figure below shows why

area triangle = $\frac{1}{2}$ base × altitude.

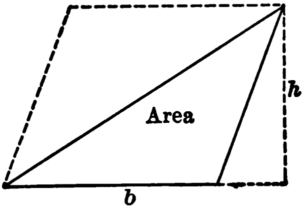


FIG. 64.

The triangle is half the parallelogram of the same base and height; therefore its area is half the area of the parallelogram.

Under the paragraph heading draw a triangle and letter the formulas for area, base and altitude.

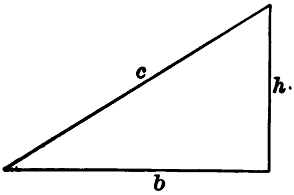


FIG. 65.

125. The Right Triangle. A right triangle as shown has *one* right angle.

The hypotenuse is the side opposite the right angle.

With great care draw a large right triangle, denoting the sides as shown.

Measure its hypotenuse and square the measurement.

Measure b and square it.

Measure h and square it.

State the relation of c^2 to b^2+h^2.

Therefore in a right triangle, the square of the hypotenuse equals what?

$$c=\sqrt{?+?}$$

$$b=$$

$$h=$$

Copy the formula, supplying the quantities which are denoted by interrogation marks under the radical.

Formulate also a and b.

126. The Trapezoid. A trapezoid as shown is a flat four-sided figure having only *two* sides parallel.

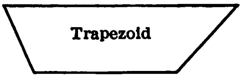


FIG. 66.

The formula may be determined from the figure as follows:

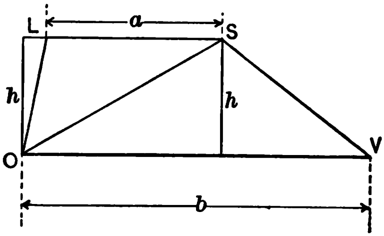


Fig. 67.

The diagonal *OS* divides the trapezoid into what kind of figures?

Therefore area *OLS* = what (taking *a* for base)?

And area *OSV* = what (taking *b* for base)?

Therefore area trapezoid = what?

Under the paragraph heading draw a trapezoid and letter the formula for area.

127. Problems. Solve the following problems in the work-book. In each show a carefully drawn figure which approximates the proportions indicated by the data.

All problems on machines in the shop must be accompanied by a dimensioned and lettered sketch with the name, number, and manufacturer of the machine from which the calculations were made.

1. Measure 3 rectangular and 3 triangular areas of parts from some of the machines in the shop, and compute the areas of each.

2. The drawing represents the gable end of a building.

Compute the length of the rafter *OT* and the area of *ROSLD*.

What is the rise of the rafter per foot of width of the building?

The total rise divided by the width of the building is called the pitch of the roof.

What is the pitch of the roof as dimensioned?

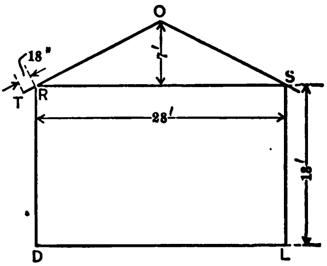


Fig. 68.

3. Determine the area of the cross-section in the cut.

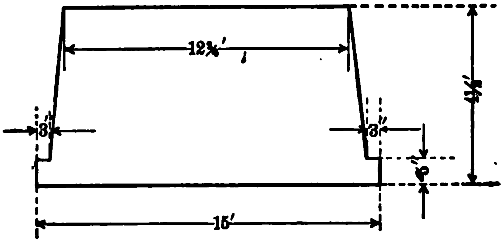


FIG. 69.

4. Measure and compute not less than two trapezoidal areas of some machine in the shop or in the room equipment.

5. In the figure, *a* is parallel and *h* is perpendicular to *b*.

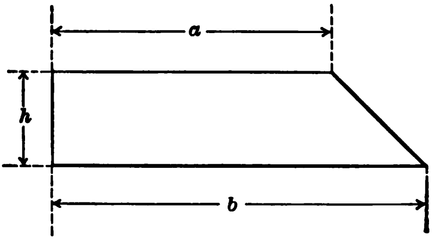


FIG. 70.

Compute the area when

$a = 14\frac{3}{4}''$,

$b = 19\frac{1}{2}''$,

$h = 5''$.

6. Compute the length of the rafters on each pitch and the total area of the entire gable end of the building as dimensioned.

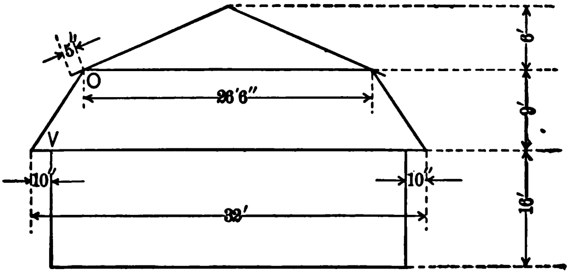


FIG. 71.

7. Measure the dimensions of the models on the desk, formulate and compute the areas, and tabulate as indicated below:

AREAS OF MODELS

No.	Name.	a	b	h	Formula.	Area.

8. The figure shown has a 90° angle at the lower right corner. Formulate and compute the area of the part marked I.

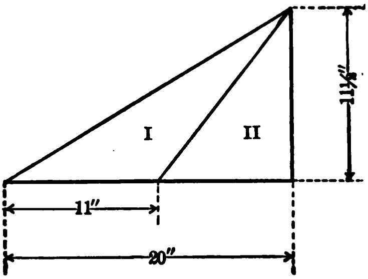


Fig. 72.

9. The cut shows a cross-section of a piece with a Brown & Sharpe taper.

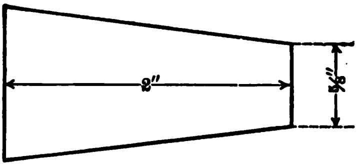


Fig. 73.

Draw the figures to scale and compute the cross-sectional area.

10. Formulate the area of each of the four parts of figure 74, denoting the areas by A_1, etc.

Combine these areas in a formula for A_T or total area.

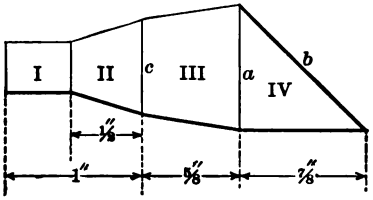


FIG. 74.

Compute A_T when

$$a = 3.1''$$
$$c = 2.1''$$

and II has a taper of 5″ to the foot.

128. The Regular Polygon. A polygon is a flat surface bounded by more than three straight sides.

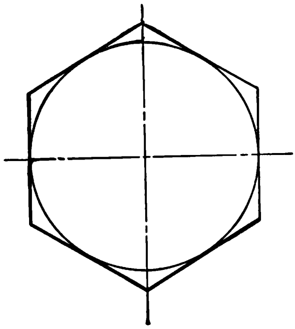


FIG. 75.

A *regular* polygon always has equal sides and equal angles.

The distance around a polygon is called the perimeter.

The perpendicular from the center to any side is called the apothem.

Regular polygons are named from the number of sides as shown in the table.

TABLE 85.

NAMES OF REGULAR POLYGONS

Sides.	Name.
5	pentagon or 5-gon
6	hexagon or 6-gon
7	heptagon or 7-gon
8	octagon or 8-gon
10	decagon or 10-gon

129. Area of a Regular Polygon. The area of a regular polygon may be determined in two ways:

(1) By perimeter and apothem,

(2) By tabular number.

In this paragraph only the first method will be considered.

The formula for perimeter and apothem is determined from the figure as follows:

When lines are drawn from the center of the polygon to each of the vertices, the polygon is divided into what kind of figures?

Are they all equal or are they unequal? Why?

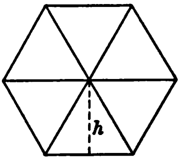


FIG. 76.

How does their number compare with the number of sides of the polygon?

If a 5-gon were divided as shown, there would be how many triangles?

If a 10-gon, how many?

If a n-gon, how many?

Formulate the area of the triangle in the figure having base s and altitude h.

The area of the hexagon or 6-gon is how many times the area of the triangle?

Formulate the area of the hexagon, A_6 in terms of s and h.

Formulate the area of a 12-gon, A_{12}, in terms of s and h.

Formulate the area of an n-gon, A_n, in terms of s and h.

The distance around a polygon is called its *perimeter*.

The perimeter of a 6-gon equals how many times s?

Formulate the perimeter of a 7-gon.

Formulate the perimeter of an n-gon.

The distance from the center of a regular polygon to any of the sides is called the apothem.

Write a law for the area of a regular polygon in terms of perimeter and apothem.

Formulate the law.

Measure the polygon in figure 76 and compute its area.

130. Area of a Polygon by Use of a Constant. Draw two regular hexagons, 1 and 2, one having a side equal to unity; the other, a side equal to $2\frac{1}{2}$.

Denote the side of 1 by s_1 and the side of 2 by s_2.

The areas of regular polygons having the same number of sides are proportional to the squares of the sides of the polygon.

Therefore $$\frac{A_1}{A_2}=\frac{s_1^2}{s_2^2}.$$

But $s_1 = \text{what?}$

Therefore $s_1^2 = \text{what?}$

Substitute this value of s_1^2 in the proportion and take the product of the means equal to the product of the extremes.

Write the following law:

The area of any regular polygon equals the square of one of its sides multiplied by the area of a similar polygon whose side is unity.

Denoting the area of any polygon by A, its side by s, and the area of a polygon whose side is unity by A_1, formulate the law.

Is the formula the same or different from the formula obtained by the solution of the preceding proportion?

Following is a tabulation of regular polygons whose sides equal unity, the constant in the last column denoting the area.

TABLE 86

AREA OF REGULAR POLYGONS. SIDE = 1

Number of Sides.	Constant.
3	.433013
5	1.720477
6	2.598076
7	3.633912
8	4.828427
9	6.181824
10	7.694209
11	9.365640
12	11.196152

131. Sum of Angles of a Regular Polygon. Draw a regular pentagon. From any point *O*, within the pentagon, draw a line to each vertex.

How many triangles are formed?

The sum of the angles of each triangle equals 180°.

Therefore the sum of the angles of all the triangles equals how many times 180°?

But the sum of the angles about the point 0 equals how many times 180°?

Therefore the sum of the angles of the pentagon equals the sum of the angles of the triangles *minus* how many times 180°?

Therefore the sum of the angles of the pentagon equals how many 180's minus how many 180's?

This is how many 180's?

And one angle of the pentagon equals what part of this?

Therefore each angle of a pentagon equals how many degrees?

The sum of the angles of a *hexagon* equal how many 180's?

This is how many 180's less than the number of sides of the hexagon? Why?

The sum of the angles of a *triangle* equals how many 180's?

This is how many 180's less than the number of sides?

Therefore the sum of the angles of an n-gon equals how many 180's less than the number of sides?

Is the following statement true?

$$\text{Sum} \angle s \text{ polygon} = (n-2)\ 180^\circ.$$

Write a law for the sum of the angles of a polygon.

132. Angle of a Regular Polygon. An angle of a regular octagon equals what part of the sum of its angles?

If

n = number of sides,

θ = number of degrees in angle of polygon,

does

$$\theta = (n-2)\frac{180}{n}?$$

Compute

θ when $n=5$.

133. Apothem of a Regular Polygon. The cut shows a regular polygon with apothem, the formula for which is as follows:

$$h = \frac{s}{2}\tan\frac{\theta}{2}.$$

The reason for this formula will be evident when problem 31 in Chapter XVI has been worked, in the paragraph containing problems on the right triangle.

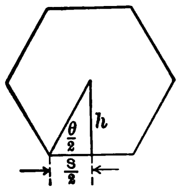


FIG. 77.

134. Formulas for Regular Polygon. Following are the formulas for a regular polygon in which

A = area in square inches,
P = perimeter in inches,
h = apothem in inches,
s = length of side in inches,
c = constant in the table in paragraph 130,
n = number of sides of the polygon,

θ = number of degrees in an angle of the polygon,
S = number of degrees in the sum of the angles of the polygon.

$$A = \tfrac{1}{2}Ph,$$
$$A = cs^2,$$
$$S = (n-2)180°,$$
$$\theta = (n-2)\frac{180°}{n},$$
$$h = \frac{s}{2}\tan\frac{\theta}{2}.$$

135. Problems in Regular Polygons. Solve the following problems by the formulas of the preceding paragraph:

1. The side of a regular octagon equals 15 inches.
Compute the area.

2. A regular decagon has a side measuring 18 inches. The apothem is approximately 27.7 inches.
Determine the area by two different formulas.

3. Compute and tabulate the entries below:

TABLE 87

ANGLES OF REGULAR POLYGONS

No.	Sds.	θ	$\frac{\theta}{2}$	No.	Sds.	θ	$\frac{\theta}{2}$
1	3			8	10		
2	4			9	11		
3	5			10	12		
4	6			11	13		
5	7			12	14		
6	8			13	15		
7	9			14	16		

4. Compute and fill in the omitted entries in the following table. $\tan\frac{\theta}{2}$ can be read in the table of tangents in the back part of the book. The symbols have the same meanings as in paragraph 134.

TABLE 88

APOTHEMS

No.	Sds.	$\frac{\theta}{2}$	s	h	A
1	3		2.51″		
2	4		4.75″		
3	5		5.4″		
4	6		6.3″		
5	7		6.92″		
6	8		7.42″		
7	9		8.00″		
8	10		8.9″		
9	11		9.34″		
10	12		12.00″		

5. A steel plate whose shape is a regular decagon measures 1$\frac{7}{8}$″ on each side.

Determine the following:

(1) Apothem,
(2) Radius,
(3) Area.

6. Compute the distance across the flats of a hexagonal nut, and the distance across the diagonally opposite corners.

7. Measure not less than two hexagonal or octagonal areas on some machine in the shop and compute the area.

136. The Circle. A circle as shown is a flat surface with a curved boundary which is everywhere the same distance from the center.

The Circle

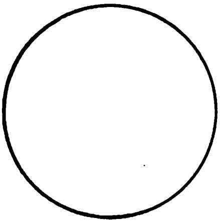


FIG. 78.

The circumference is the boundary line.

A diameter is a line through the center with its extremities in the circumference.

A radius is half of a diameter, or a line from center to circumference.

A chord is a line whose extremities are in the circumference. If

C = circumference,
R = radius,
D = diameter,
A = area,
$\pi = 3.1416$,

we have the formulas

$$C = 2\pi R,$$
$$C = \pi D$$
$$A = \tfrac{1}{2}CR,$$
$$A = \pi R^2,$$
$$A = \frac{\pi D^2}{4}.$$

137. Circle Computation. Rule the following table in the work-book.

Formulate circumference and area in terms of diameter, and compute and tabulate the omitted entries to four decimal places.

TABLE 89

AREAS AND CIRCUMFERENCES

No.	Diam.	Circum.	Area.	No.	Diam.	Circum.	Area.
1	.1			11	1.1		
2	.2			12	1.2		
3	.3			13	1.3		
4	.4			14	1.4		
5	.5			15	1.5		
6	.6			16	1.6		
7	.7			17	1.7		
8	.8			18	1.8		
9	.9			19	1.9		
10	1.0			20	2.0		

138. Sector and Segment. As shown, a segment is a portion of the circle bounded by a chord and the arc which the chord subtends.

By what is a *sector* bounded?

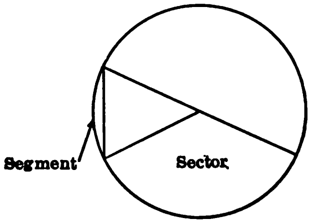


FIG. 79.

A formula for the area of a sector may be determined as follows:

The area of a sector of 1° equals how many 360ths of the area of the circle?

Therefore the area of a sector of 30° equals how many 360ths of the area of the circle?

Therefore the area of a sector of $n°$ equals how many 360ths of the area of the circle?

Formulate your answer to the last question, denoting area of the sector by A_s, and the area of the circle by A.

For A in the formula, substitute the different values of A as given in paragraph 136.

Look at the figure.

The area of a segment equals the area of the sector having the same arc, minus what area?

139. Problems. Solve these problems in the work-book.

1. Carefully draw a circle having a $\frac{3}{4}''$ radius with a sector and segment of 35°.

From the center of the circle draw a perpendicular to the chord which forms the segment.

Make such measurements as may be required in order to compute the areas of the circle, sector, and segment, and compute the areas.

2. Compute the area of a sector of 40° and $\frac{7}{8}$-inch radius.

3. In a $2\frac{1}{4}''$ circle lay off a segment of 60°.

From the center of the circle, draw a perpendicular to the chord of the segment.

By reference to one of the formulas in paragraph 134, write a formula for the perpendicular.

The chord of a 60° segment equals the radius of the circle.

What therefore is the length of the chord in inches?

Formulate and compute the area of the segment.

4. Measure the diameters and compute circumferences and areas of not less than 3 circular areas in the shop.

5. Measure and compute two areas of sectors or segments in the shop.

6. In a $16\frac{1}{2}''$ circle a chord is $4''$ from the center.

Formulate and compute the length of the chord.

7. A chord is 18 inches long in a 48-inch circle.

Formulate and compute its distance from the center.

140. Concentric and Eccentric Circles. Concentric circles have the same center, the word meaning "centers together."

Eccentric circles lie one within the other but have different centers.

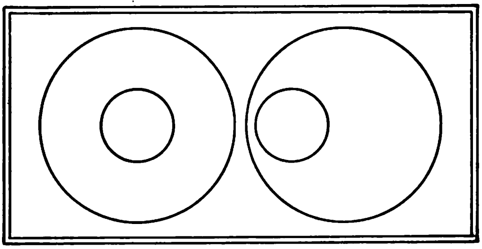


FIG. 80.

141. Problems. Solve the following:

1. Measure and compute the areas between the two circumferences in each figure of paragraph 140.

2. Measure and compute all the circumferences of 2 different step cones in the shop.

3. Measure at least one eccentric in the shop and calculate its circumference.

4. Formulate and compute the areas between the circumferences of three concentric circles with radii of 4, 7, and 11 inches.

5. Make a sketch of an eccentric on any machine which you have used or seen.

State how far the center of revolution is off center and explain the reason for it.

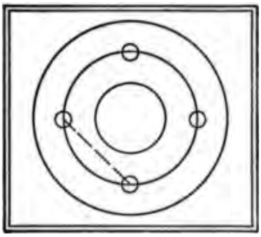


FIG. 81.

6. The drawing shows a flat ring whose inner radius $r=3''$, and whose outer radius $R=15''$. The circumference of the bolt circle is equally distant from the outer and inner circumferences of the ring. The four bolt holes are equally spaced.

Formulate and compute the distance between centers of the $1\frac{1}{8}''$ holes, both on the circumference and on the chord.

142. The Ellipse. An ellipse, as shown, is a flat surface with a curved boundary which is not everywhere equally distant from the center.

The lines OV and RS are perpendicular to each other, and are called the axes.

The long axis OV is called the major or transverse axis; the short axis RS is called the minor or conjugate axis.

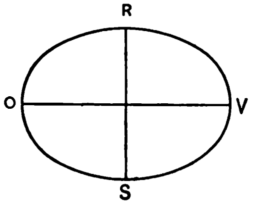


FIG. 82.

If

$a=\frac{1}{2}$ the major axis,
$b=\frac{1}{2}$ the minor axis,
$P=$ perimeter,
$A=$ area,

then

$$A=\pi ab,$$

$$P=\pi\sqrt{2(a^2+b^2)},$$

$$P=2\pi a\left\{.636+.634\left(\frac{b}{a}\right)^2\right\}. \qquad \text{(F. Lord's Formula.)}$$

$$P=\pi\left\{\frac{3}{2}(a+b)-\sqrt{ab}\right\}. \qquad \text{(Boussinesq's Formula.)}$$

The preceding formulas for perimeter are approximate, the exact formula below being too long for practical use without a table of values for the terms in the brace.

$$P=\pi(a+b)\left\{1+\frac{1}{4}\left(\frac{a-b}{a+b}\right)^2+\frac{1}{64}\left(\frac{a-b}{a+b}\right)^4+\frac{1}{256}\left(\frac{a-b}{a+b}\right)^6+\ldots\right\}$$

The dots at the end of the formula are read " and so on," meaning an indefinite number of other terms obtained from the law of development of the series. The numerical value of the unwritten terms is so small, however, as not materially to affect the value of the perimeter, so that the formula may be regarded as the exact formula for the perimeter of an ellipse.

143. Foci of an Ellipse. The foci are two points on the transverse or greater axis of an ellipse.

They may be determined on any ellipse as follows:

Take half the long axis in the compasses, and placing one leg on the extremity of the short axis, describe short arcs across both halves of the long axis.

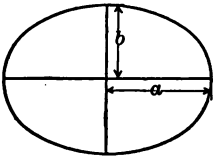


FIG. 83.

The points where these arcs cross the long axis are the foci.

144. Problems. Solve the problems below:

1. The axes of an ellipse are 6″ and 4″. Compute the area and the perimeter by each of the formulas.

2. Measure the axes of the figure in paragraph 142 and compute the perimeter by the simplest formula.

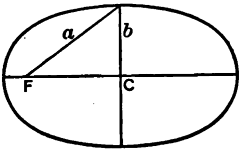


FIG. 84.

3. Draw the ellipse in paragraph 143, in the work-book and show the construction for the determination of the foci.

Denote the left focus on the figure by *F*.

Join *F* to the upper extremity of the

short axis. Refer to paragraph 142 and determine by what letter used in the notation of the ellipse the joining line should be denoted.

Dimension this letter on the figure.

Denote the center of the ellipse by C.

FC forms with a and b, what kind of a triangle?

Formulate FC in terms of a and b.

4. If $a = 10''$ and $b = 8''$, compute FC.

5. Using the same data as in problem 4, formulate and compute VF, which is the distance from the vertex to the focus.

6. Elliptic gears will mesh with each other when revolving about their foci.

At what distance from the end of the transverse axis should the hole for the center of revolution be drilled in an elliptic gear whose axes are $1\frac{1}{4}''$ and $2''$?

7. The eccentricity of an ellipse equals the distance of the focus from the center, divided by half the transverse axis.

Formulate eccentricity from Fig. 84, denoting eccentricity by e.

8. Compute the eccentricity of the gear in Problem 6.

9. Write an account of the use of any elliptic gears with which you are familiar.

10. Measure an elliptic piece in the shop and compute its area, perimeter, eccentricity, and location of focus.

145. The Earth's Orbit. The path of the earth around the sun is called its orbit. The work in the three preceding paragraphs should enable you to understand the law first expressed by the great astronomer Kepler, that the earth and the other planets move in ellipses with the sun at one focus.

This of course means that the earth is nearer the sun during a part of the year and therefore moves faster because of the increased attraction.

In a spare moment, you may be interested to read about this in a good astronomy, and to verify by computation some of the data there given, such as the eccentricity, perimeter or length of the orbit, average rate of motion per day, etc.

If this interest leads to further reading with regard to other bodies in the solar system, the time so spent will never be regretted.

146. Simpson's Rule for an Irregular Area. The area of an irregular figure may be accurately determined by the use of a planimeter, an instrument so constructed that the area of any figure whose boundary line has been traced by it, is read from the dial.

When this instrument is not available, or when great accuracy is not required, a sufficiently accurate computation may be made by the use of Simpson's Rule.

To apply this rule or law, a base line is drawn through the irregular area, and at equal distances along the line, beginning at each end, perpendiculars called ordinates are drawn, which terminate in the boundary of the figure as shown.

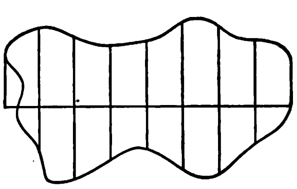


FIG. 85.

Any ordinate lying wholly outside the diagram therefore equals zero.

This process is known as the division of a figure by ordinates.

It is best to make an *even* number of divisions, and the greater the number the more closely will the result approximate the actual area.

SIMPSON'S RULE. Add together the first ordinate, the last ordinate, twice the sum of the other odd ordinates, and four times the sum of the even ordinates. Multiply this sum by the extreme length of the diagram and divide the result by three times the number of parts into which the diagram is divided.

Formulate this law, using the following notation:

A = area,
L = length of diagram,
y_1 = first ordinate,
y_L = last ordinate,

y_2, y_3, etc. = the other ordinates,

n = number of parts into which the diagram is divided.

147. Problems. Solve the following problems:

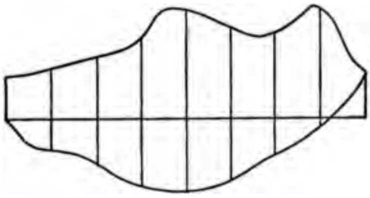


FIG. 86.

1. In the figure,

$y_1 = \frac{1}{4}''$, $y_L = 0''$,

$y_2 = 1\frac{3}{8}''$, $y_3 = 1\frac{1}{2}''$,

$y_4 = 1\frac{7}{8}''$, $y_5 = 1\frac{3}{16}''$,

$y_6 = 1\frac{1}{8}''$, $y_7 = 1\frac{1}{32}''$,

$y_8 = 1\frac{3}{4}''$, $L = 2\frac{3}{4}''$.

Formulate and compute the area.

2. Measure the figure in paragraph 146 and compute its area.

3. Apply Simpson's rule to Fig. 87 and formulate and compute the area.

4. Determine the area of such of the models on the desk as are assigned. Do not use pencil, but crayon, to divide the area by ordinates.

5. Measure and compute the area of two irregular flat surfaces on some machine in the shop.

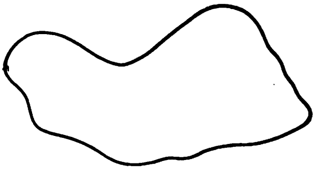


FIG. 87.

148. Irregular Area by the Ten Ordinates Method. To determine an area of irregular boundary by this method, the area is divided as shown into ten parts of equal width. An ordinate is then drawn through the center of each part, terminating in the boundary of the figure.

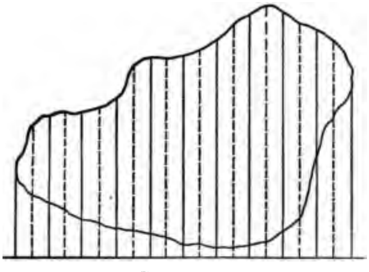


FIG. 88.

The area is obtained by multiplying the average length of the ten ordinates by the extreme length of the figure.

Observe that as there are ten ordinates, their average length equals the combined sum of the lengths of the ten ordinates with decimal point shifted one place to the left.

The sum of the lengths is best determined by taking the lengths of the ordinates in the dividers and by laying them off end to end on a straight line.

The combined length is then measured, the decimal point is moved one place to the left, and the area is obtained by multiplying the result by the extreme length of the diagram.

149. Problems. Solve the following problems, showing a carefully drawn figure with each:

1. Measure the figure in the preceding paragraph and compute its area by the 10-ordinates method.

2. Compute the area of the figure in Problem 1, by Simpson's rule.

3. Compute the area of the figure in Problem 1, by the planimeter.

If you don't know how to use this instrument, refer to paragraph 115.

4. Tabulate the results obtained in Problems 1, 2, and 3.

Which gave the more nearly correct result in this instance, the 10-ordinates method, or Simpson's rule?

5. By the three methods, determine and tabulate the areas of such of the diagrams on the chart as may be assigned.

In using the planimeter, slowly trace each diagram three times and take the average of the readings.

In the last column of the table, headed " Best Result," specify whether Simpson's or the 10-ordinates rule gave the more nearly correct area in each case.

6. With a finely sharpened pencil, trace on a sheet of paper such of the irregular models on the desk as may be assigned and compute and tabulate the areas as in Problem 4.

7. Obtain the outline of some irregular flat surface in the shop by laying a sheet of paper on it and by bending the paper over the edges of the piece. Carefully trace the outline with a soft pencil and take an impression of it by pressing the paper face downward on another smooth sheet.

Determine the area by each of the three methods.

§ 2. VOLUMES

150. The Rectangular Solid. As shown in the cut, a rectangular solid is box-shaped, all faces being rectangles.

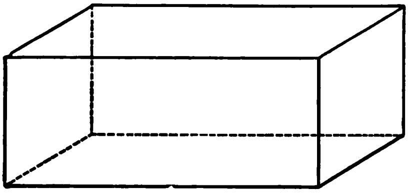


FIG. 89.

The base is the side on which it stands, and the altitude, which is denoted by h, is its height.

The three dimensions denoted by a, b, and h, in a material solid are commonly called length, width and depth.

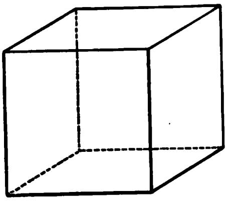


FIG. 90.
One Cubic Inch.

When these three dimensions are equal, the solid is called a cube.

A cubic inch as shown, is a cube whose length, breadth, and height, each measure 1 inch.

Volume is expressed in cubic measure, that is, in cubic inches, cubic feet, or whatever unit may be chosen.

Volume, therefore, is the cubical contents of anything having three dimensions.

The word cubic is used because the calculation of volume may be regarded as the determination of the number of *unit cubes* in any solid, a unit cube being a cube each of whose three dimensions measures one unit, that is, 1 inch, 1 foot, 1 yard, or some other unit.

Another reason why volume is called *cubic contents*, is that the volume of a cube is obtained by using the numerical measure of its edge three times as a factor. Hence the product obtained by using any number three times as a factor is said to be a cube.

The following explains why the area of the base multiplied by the height equals the volume of a rectangular solid.

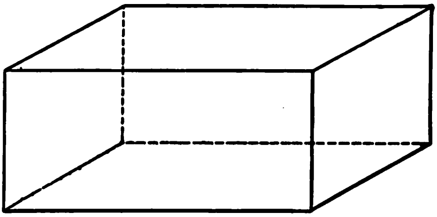


FIG. 91.

Since the base is a rectangle measuring 31″×22″, it contains 22×31 square inches.

Therefore in the height of 9 inches there are 9 layers of 682 cubic inches in each layer, making a total volume of 9×682 cubic inches.

Draw a rectangular solid with dimensions *a*, *b*, and *h*, and write the formula for the volume.

151. The Prism. A prism as shown is a solid whose sides are parallelograms and whose top and bottom are parallel to each other.

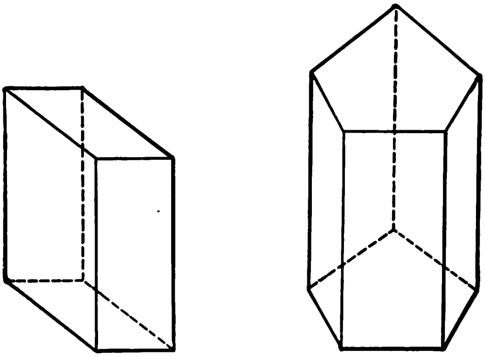


FIG. 92.

The top and bottom are called upper and lower **bases** respectively, which in shape may be a triangle, parallelogram or other four-sided figure, or a polygon of any number of sides.

Rectangular solids are therefore prisms.

The basal edges are the sides of the bases.

The lateral edges, as shown, are the lines in which the sides intersect each other.

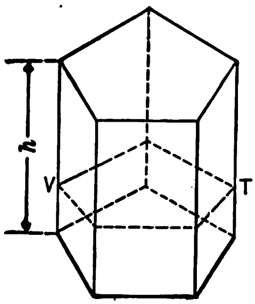


Fig. 93.

The perimeter of the base is the sum of the basal edges.

In this figure the edges are oblique to the bases, *VT* being a plane perpendicular to the edge and therefore a ***right*** section.

When the edges are perpendicular to the bases, the perimeter of the base is the same as the perimeter of the right section because the bases are themselves right sections.

Following is the notation:

A_t = total area,

A_b = area of base,

A_L = area sides, or lateral area,

P = perimeter of a right section,

V = volume,

h = length or height,

$V = A_b h$,

$A_L = Ph$.

152. Problems. Solve the following:

1. A prism whose base is a regular pentagon with a side of $9\frac{1}{4}$ inches, is $23\frac{1}{2}$ inches long.

Formulate and compute the total area.

2. Under the same conditions as in Problem 1, formulate and compute the volume of the prism.

3. A rectangular tank measures on the inside $11\frac{1}{2}'' \times 13\frac{1}{2}'' \times 9''$.

Formulate and compute the number of gallons which the tank contains when filled within an inch and a half of the top.

4. Measure such of the rectangular and prismatic models on the desk as may be assigned, and formulate and compute the volume and the total surface.

5. Measure and compute lateral areas and volumes of three pieces of material or machine parts whose shapes are prisms, each with a differently shaped base.

Make your calculations from a dimensioned sketch.

153. The Cylinder. A cylinder as shown is a solid whose lateral surface is curved and whose bases are parallel to each other.

Every section parallel to the base of the cylinder is a circle.

A_b = area of base;

h = length or height;

S_L = lateral surface;

S_T = total surface;

V = volume;

R = radius of base;

C = circumference of base.

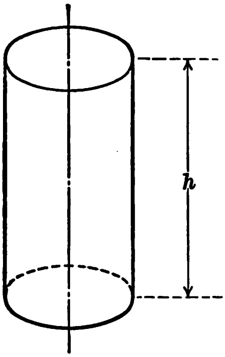


FIG. 94.

Under the heading draw a cylinder in the work-book and letter formulas for A_b, S_L, S_T, and V.

154. Problems. Solve the following in the work-book:

1. Formulate and compute the omitted entries in the table below, as may be assigned, tabulating both data and results.

TABLE 90

CYLINDERS

No.	D	h	C	A_b	S_L	S_T	V
1	4″	$3\frac{1}{2}$′					
2	5″	$2\frac{3}{4}$′					
3	16″	32″					
4	2.7″	19″					
5	1.54″	29″					
6	$\frac{3}{8}$″	$15\frac{3}{4}$″					

2. Measure, formulate and compute the curved surface and the volume of the assigned cylindrical models at the desk, tabulating both measurements and results.

3. Measure and compute the curved surface of not less than two machine parts in the shop.

155. The Pyramid. A pyramid as shown is a solid whose sides are triangles and whose base may be any figure bounded by straight lines.

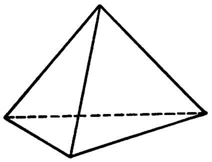


FIG. 95.

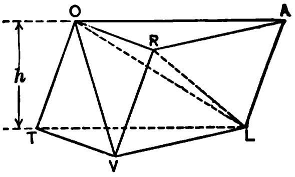


FIG. 96.

These figures show how the volume of a pyramid compares with the volume of a prism having the same base and height.

$O-TVL$ is the pyramid on whose base TVL, has been constructed a prism $ORATVL$.

The prism in Fig. 96 is here shown divided into three equal pyramids.

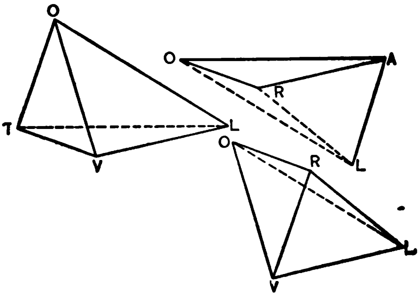


Fig. 97.

The volume of the pyramid $O-TVL$ is therefore equal to one-third the volume of the prism.

But the volume of a prism equals the height times the area of the base.

Therefore volume pyramid $= \frac{1}{3}$ altitude times area base.

156. Formulas for Pyramid. A regular pyramid has a regular polygon for its base and a perpendicular at the center of the base will pass through its apex.

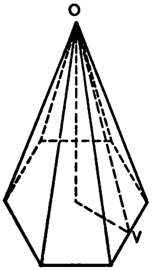


Fig. 98.

It will be seen from the figure that the slant height OV is the hypotenuse of the right triangle whose other sides are the apothem and the altitude.

Draw a large regular pyramid and write formulas for slant-height, apothem, lateral area, basal area, volume, and perimeter, using the following notation:

S = slant-height;
h = apothem;
a = altitude or height of pyramid;
A_L = lateral area;

P = perimeter of base;
A_b = area of base;
V = volume;
b = length of side of base;
n = number of sides;
θ = angle of basal edges with each other.

Number the formulas in succession.

157. Problems. Solve the following problems:

1. By the use of the formula of paragraph 133, compute the apothems of the base of the pyramids in the table below, in which

b = length of side of base;
a = altitude of pyramid,
h = apothem;
S = slant-height.

TABLE 91

REGULAR PYRAMIDS

No.	Sds.	b	a	h	S
1	3	1.2″	5.45″		
2	4	10.5″	3′ 2″		
3	5	.95″	4′ 7″		
4	6	14″	7½′		
5	7	21″	8¾′		
6	8	23″	15′		
7	9	17″	16.3′		
8	10	24″	17.1′		
9	11	16.1″	18.2′		
10	12	30.5″	19.4′		

2. Formulate and compute the slant-heights in the table of Problem 1, and fill in the omitted entries.

3. Measure one of the pyramids on the table, and compute area and volume.

4. Compute the apothem of the same pyramid, and check by measurement.

5. Compute the altitude of the same pyramid, and check by measurement.

6. Compute the slant-height of the same pyramid, and check by measurement.

7. Complete the following table in which

b = length of side of base

and

n = number of sides.

TABLE 92

REGULAR PYRAMIDS

No.	b	n	a	S	P	A_b	A_L	A_T	V
1	4.5 ″	5	19″						
2	5.34″	9	70″						
3	6.72″	9	41.5″						
4	7.29″	13	82″						
5	8.12″	12	78.6″						
6	8.65″	11	93″						
7	9″	10	114.1″						
8	12″	8	128″						

8. Measure some machine piece of pyramid shape in the shop, and compute its volume.

158. Frustum of a Regular Pyramid. As shown, the frustum of a pyramid is what remains when the top is cut off by a plane parallel to the base.

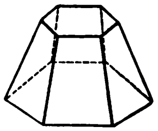


FIG. 36.

Each lateral face is therefore a trapezoid.

If a frustum is regular, the slant-height of all faces is the same, and the size and shape are the same.

Following are the notation and formulas:

h_B = apothem lower base;
h_b = apothem upper base;
A_L = lateral area;
A_B = area lower base;
A_b = area upper base;

V_F = volume;
A_T = area total surface;
S_F = slant-height;
a_F = altitude
P_B = perimeter lower base;
P_b = perimeter upper base;
L_B = length side lower base;
L_b = length upper base;

$$A_L = \frac{S_F}{2}(P_B + P_b);$$

$$A_T = ?$$

$$V_F = \frac{a_F}{3}(A_B + \sqrt{A_B A_b} + A_b);$$

$$a_F = \sqrt{S_F^2 - (h_B - h_b)^2}.$$

159. Problems.

1. Draw a large frustum of a regular pyramid, dimension the notation on the drawing, and write the three formulas.

2. Measure a piece whose shape is the frustum of a regular pyramid, and compute the volume.

3. Complete the following table as may be assigned:

TABLE 93

FRUSTUMS OF REGULAR PYRAMIDS.

No.	Sds.	L_B	L_b	S_F	h	a_F	A_L	A_T	V_F
1	3	3.46″	$\frac{1}{2}L_B$	8″					
2	4	4.51″		8.5″					
3	5	5.91″		8.7″					
4	6	6.84″		9″					
5	7	7.29″		13″					
6	8	8.4″		15″					

4. Measure a frustum of some regular pyramid in the shop and compute the total area and volume.

160. The Cone. A cone is a solid whose base is a circle and whose sides taper uniformly to a point directly over the center of the base, as shown in the cut.

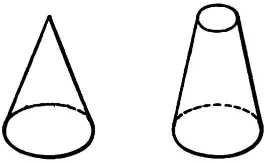

Cone. Frustum of cone.

FIG. 100.

The point is called the apex of the cone, and a perpendicular from the apex to the base is called the axis.

The formulas for the cone and the frustum of a cone are the same as for the regular pyramid and its frustum, the only difference being a difference in notation due to the fact that the base is a circle instead of a polygon.

Formulas for cone:

$$A_L = \pi r S;$$

$$V = \frac{a}{3}\pi r^2;$$

in which

A_L = area of curved surface of cone;
S = slant height;
a = altitude;
V = volume;
r = radius of base.

Formulas for frustum of cone:

$$A_F = \pi S_F(R+r);$$

$$V_F = \frac{\pi a_F}{3}(R^2+Rr+r^2).$$

161. Problems.

1. Draw a large cone with dimensioned notation and write formulas corresponding to those for the pyramid.

2. Draw a large frustum of a cone with dimensioned notation and write formulas corresponding to those for the frustum of a pyramid in paragraph 158.

3. Measure such of the models as may be assigned and apply the formulas of Problems 1 and 2, tabulating measurements and results.

4. On one of the models, take the measurements necessary to determine the radius of the base from the formula, and compute the radius.

5. Solve such of the following problems as may be assigned, tabulating both data and results.

TABLE 94

THE CONE

No.	r	S	a	A_L	A_T	V
1	2.25″	9″				
2	4.64″	11″				
3	5.75″	13″				
4	6.84″	15″				
5	7″	17″				
6	8.96″	19″				
7	12.4″	21″				
8	15.6″	23″				
9	17.3″	25″				
10	18.7″	27″				

6. Draw a figure similar to this, and write a formula for the altitude a in terms of the two radii and the slant-height.

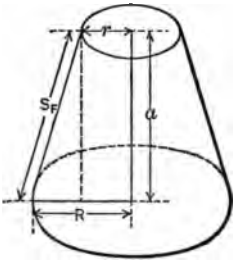


FIG. 101.

7. Complete the following table, as assigned:

TABLE 95

FRUSTUMS OF CONES

No.	R	r	S_F	a_F	A_B	A_b	A_L	A_T	V
1	4″	3″	8″						
2	4.5″	3.6″	7″						
3	6.8″	4.62″	8.5″						
4	7″	4.9″	15″						
5	7.8″	5.93″	19.4″						
6	8.2″	6.5″	19.9″						
7	9.7″	7.4″	20″						
8	10.3″	8.4″	21.3″						
9	13.3″	10.5″	29″						
10	63″	44″	20″						

8. Measure three conical pieces in the shop and compute lateral surface and volume.

9. Take the measurements necessary to determine altitude from the formula and compute the altitude.

10. Take the measurements necessary to determine slant-height from the formula and compute the slant-height.

Check by measurement of the slant-height.

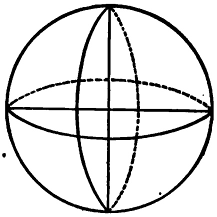
162. The Sphere. A sphere is a solid all points of whose surface are equally distant from the center.

It is a solid which would be marked out by one complete revolution of a circle about its diameter.

The surface of a sphere is its boundary of uniform curvature.

The diameter of a sphere is a line through the center terminating in the surface.

The radius is a line from the center to the surface.



$S = 4\pi R^2$, $V = \frac{4}{3}\pi R^3$.

FIG. 102.

A *great* circle of a sphere is a section formed by a plane through the center.

A small circle of a sphere is a section formed by a plane *not* through the center.

A sphere is inscribed in a cylinder, and the cylinder is circumscribed, when the sides, top, and bottom of the cylinder just touch (are tangent to) the sphere.

Following are the notation and formulas:

V_C = volume of circumscribed cylinder;

V = volume of sphere;

S = surface or area of sphere;

D = diameter;

R = radius;

A = area of great circle;

$V = \frac{2}{3} V_C;$

$S = 4A.$

Under the heading copy these formulas in the workbook and complete them by substituting for V_C from the formula for the volume of a cylinder, and for A from the formula for area of a circle.

163. Problems.

1. Draw a large sphere and write formulas for surface and volume in terms of radius; also in terms of diameter.

2. Measure the sphere in paragraph 162, and compute its area and volume.

3. Measure 5 spherical pieces in the shop and compute surface and volume.

4. If the first piece measured in Problem 3 had been submerged slowly in a vessel even full of water, how many cubic inches of water would have been forced out of the vessel?

The amount forced out is called the amount displaced or the displacement.

If each of the other four pieces measured in Problem 3, had been similarly submerged in a vessel even full of water, what would have been the displacement for each in cubic inches?

Therefore the total displacement of the five pieces equals how many gallons? (If not even, express to 3 decimal places.)

5. 1 cubic foot of fresh water weighs approximately 62.425 lbs.

Compute the weight of the total displacement of the five pieces.

6. Take or make a can with vertical sides and a flat circular bottom, and cut it off so that the depth is exactly equal to the diameter.

Make or procure a sphere which will just go into the can.

Fill the can even full of water, place it in a larger vessel and carefully submerge the sphere until it touches the bottom of the can.

Measure as accurately as possible the water displaced into the larger vessel.

By the formula for the volume of a sphere, the volume of water displaced should equal what part of the volume of water in the can when even full?

The volume of water remaining in the can should equal what part of the volume displaced?

Determine whether the amount displaced and the amount remaining are in this proportion, in three different ways:

1. Measurement,
2. Weighing,
3. Calculation of cubic inches.

7. Take or make a can with vertical sides and flat circular bottom, and tall enough so that when filled with water to a depth equalling its diameter, no water will overflow when a sphere of the same diameter is submerged.

Make a mark on the inside of the can at a height equal to the diameter.

Fill the can with water just to the mark.

Submerge the sphere and make a mark on the can just even with the surface of the water.

Compute

(1) The volume when filled to the lower mark;
(2) The volume between the two marks;
(3) Determine what part approximately, the second is of the first;
(4) State how the results compare with those obtained by the formula.

8. **Complete the following table, as assigned:**

TABLE 96

SPHERES

No.	D	S	V	No.	D	S	V
1	5″			11	31″		
2	7.3″			12	32″		
3	8.75″			13	33″		
4	9″			14	34″		
5	12.5″			15	35″		
6	10.7″			16	36″		
7	3.28″			17	37″		
8	2.7″			18	38″		
9	15.8″			19	39″		
10	21″			20	51″		

164. A Spherical Segment. The cut shows a segment of a sphere with height h and radius r.

Observe that when a sphere is cut in two by a straight cut, each piece is a segment. If the cut goes through the center, the two segments are hemispheres, *hemi* meaning "half." If the cut is at one side of the center of the sphere, one segment is greater than a hemisphere. The base of a segment is its flat surface.

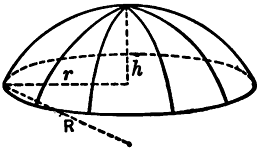


FIG. 103.

In the formulas below the notation is as follows:

A = area of curved surface of segment;

V = volume of segment;

h = height of segment;

r = radius of segment;

R = radius of sphere;

$$A = 2\pi Rh;$$

$$V = \frac{\pi}{6}(3r^2h + h^3) = \frac{\pi h}{6}(3r^2 + h^2);$$

$$V = \frac{\pi}{3}(3Rh^2 - h^3) = \pi h^2\left(R - \frac{h}{3}\right).$$

The formulas apply to segments less than a hemisphere. If the segment is greater than a hemisphere, the area or volume may be determined by subtracting the area or volume of the missing part, from the area or volume of the sphere.

165. A Zone. If two parallel cuts are made through a sphere, the portion of the sphere between the cuts is called a *zone*.

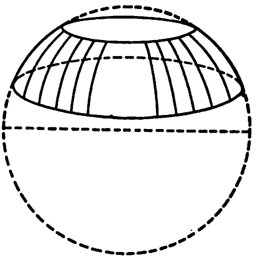


Fig. 104.

The formulas are as follows:

$$A = 2\pi Rh,$$

$$V = \frac{\pi}{6}[\tfrac{3}{4}h(d_1{}^2 + d_2{}^2) + h^3]$$

$$V = \frac{\pi h}{6}[\tfrac{3}{4}(d_1{}^2 + d_2{}^2) + h^2]$$

$$V = \frac{\pi h}{2}\left(r_1{}^2 + r_2{}^2 + \frac{h^2}{3}\right),$$

in which

R = radius of sphere,
h = height or thickness of zone,
V = volume of zone,
d_1 and d_2 = diameters of zone,
r_1 and r_2 = radii of zone.

Observe that the formula for the area of a zone is exactly the same as for the area of a spherical segment.

166. Volume of a Hollow Sphere. When a sphere is hollow it is sometimes called a spherical shell. Its volume is the volume of a sphere of the outside diameter minus the volume of a sphere of the inside diameter.

If

V = volume of hollow sphere;

V_o = volume of sphere of outside diameter;

V_i = volume of sphere of inside diameter;

D = outside diameter;

d = inside diameter;

R = outside radius;

r = inside radius;

Then

$$V = V_o - V_i,$$

$$V = \tfrac{4}{3}\pi(R^3 - r^3),$$

$$V = \frac{\pi}{6}(D^3 - d^3).$$

167. Problems.

1. Under suitable headings, copy the formulas of paragraphs 164, 165, and 166.

2. In the formulas for the volume of a sphere, compute $\frac{\pi}{3}$ and $\frac{\pi}{6}$ to four decimal places, and substitute the results in the formulas.

3. In the formula for the volume of a hollow sphere, compute $\frac{4}{3}\pi$ to three decimal places and substitute the result.

4. The outside diameter of a spherical shell is 12 inches, the inside is $10\frac{1}{2}$ inches.

Formulate and compute the volume.

5. The short diameter of a zone is $5\frac{1}{2}$ inches, the long diameter is 9 inches, and the height is 2 inches.

Formulate and compute the volume.

6. A cylindrical tank is 10 feet long and $2\frac{1}{2}$ feet in diameter. The ends are spherical segments whose center of curvature projects 5 inches beyond the base of the segment.

Formulate and compute the volume and the curved surface in cubic feet.

7. Draw the following table in the work-book and below it formulate and compute the following zones, as may be assigned:

TABLE 97

SPHERICAL ZONES

No.	d_L	d_s	h	A	V
1	5	3½	1½		
2	8	5¼	2		
3	13	10½	3		
4	22	18	5½		
5	41	30	10¼		

8. Formulate and compute the area and the volume of the following spherical segments as may be assigned:

TABLE 98

SPHERICAL SEGMENTS

No.	R	h	r	A	V
1	12	2½			
2	18½	5			
3		4¼	13		
4		7⅛	25		
5	32	15			

9. Formulate and compute the volume of the following spherical shells as may be assigned:

TABLE 99

SPHERICAL SHELLS

No.	D	d	V
1	11	10½	
2	14	13	
3	8	7½	
4	16	15½	
5	20	19	

10. Measure, formulate, and compute, such of the model spheres, shells, segments, and zones on the desk, as may be assigned. Tabulate both measurements and results and write the formulas below the table.

11. Measure and compute area and volume of a sphere, spherical shell, spherical segment, and zone, on some machine or material in the shop, as may be assigned.

Tabulate data and results and write formulas below the table.

168. Volume of a Ring. When a ring is of uniform section, its volume equals the *mean* circumference or perimeter times the cross-sectional area.

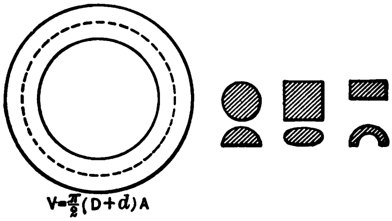


FIG. 105.

The mean circumference is one-half the sum of the outer and inner circumferences, as shown by the dash line in the cut.

If a ring is elliptical instead of circular, the mean perimeter P, in the formula for volume, may be determined by one of the formulas in paragraph 142.

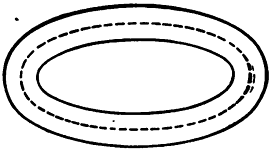


FIG. 106.

The sectional views in Fig. 105 show some of the shapes commonly used in rings.

By substituting for P and A from the formulas in the paragraphs treating of these shapes, their volume is easily determined from the general formula

$$V=PA,$$

in which $P=$ mean perimeter,

and $A=$ area of cross-section.

169. Problems. In the first five problems in this list, be careful to letter every formula so that it can easily be referred to, in computation.

Under every figure, letter the general formula, $V = PA$.

1. Draw a circular ring with a circular cross-section.
Write the formulas for volume in terms of mean diameter.
Formulate also in terms of mean radius.

2. Draw a circular ring with a square cross-section, and formulate volume.

3. Draw a circular ring with an elliptical cross-section, and formulate volume in terms of Lord's formula in paragraph 142, using a and b to denote one-half the mean axes.

4. Under carefully drawn figures, formulate the volume of circular rings of the following sections:

(1) Rectangular,
(2) Semi-circular,
(3) Concentric semicircular.

5. Under headings and carefully drawn figures, formulate the volumes of the following sections of an elliptical ring:

(1) Square;
(2) Rectangular;
(3) Circular;
(4) Semicircular;
(5) Concentric semicircular:
(6) Elliptical.

6. Formulate, compute, and fill in the omitted entries in the following table of a circular ring or torus in which

r = inner radius of ring;
w = width of rings of all sections;
t = thickness of rings of rectangular or elliptical section;
P = mean perimeter of ring;
A = area of cross-section;
V_c = volume circular section;
V_r = volume rectangular section;
V_e = volume elliptical section.

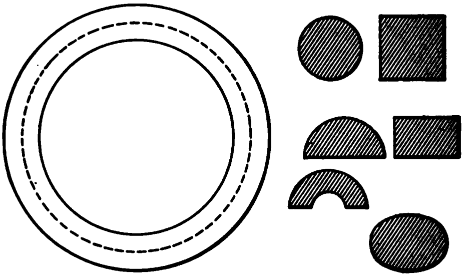


FIG. 107.

TABLE 100

CIRCULAR RINGS

No.	r''	w''	t	P	A	V_c	V_s	V_e
1	8	$1\frac{1}{4}$	1					
2	12	$\frac{3}{4}$	$\frac{1}{2}$					
3	$4\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{16}$					
4	6	$\frac{7}{16}$	$\frac{1}{4}$					
5	15	$1\frac{1}{8}$	$\frac{3}{4}$					
6	21	$1\frac{1}{2}$	1					

7. Write the formula for the volume of an elliptical ring.

Compute and tabulate the entries called for in the following table in which

$a_i = \frac{1}{2}$ inner major axis,
$b_i = \frac{1}{2}$ inner minor axis,

the other notation being the same as in Problem 6.

TABLE 101

ELLIPTIC RINGS

No.	a_i	b_i	w	t	A	P	V_c	V_e
1	6	4	2	1				
2	8	5	3	2				
3	9	5	2	$1\frac{1}{4}$				
4	10	7	3	$1\frac{1}{2}$				

170. Sector of a Sphere. Observe that the sector consists of a segment and a cone whose bases are coincident, the apex of the cone being at the center of the sphere.

The volume of the sector therefore equals the volume of the segment plus the volume of the cone, which gives the formula

$$V=\frac{\pi}{6}[r^2(h+2R)+h^3],$$

in which the notation is the same as in paragraph 164.

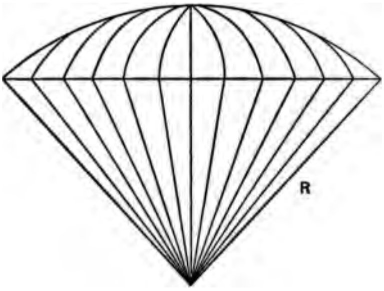


FIG. 108.

Measure the sector in the cut, and compute its volume.

171. The Ellipsoid. If an ellipse is rotated through a complete revolution about the major or minor axis, its perimeter will trace out a solid called an ellipsoid.

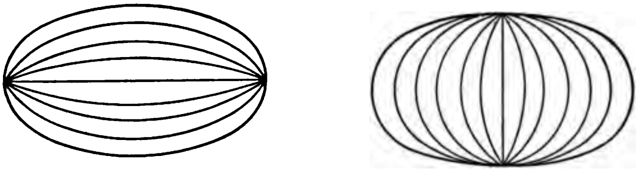


FIG. 109.

In the left figure, the ellipsoid has been generated by revolution about the major axis; in the right figure, by revolution about the minor axis.

LAW FOR VOLUME. The volume of an ellipsoid equals $\frac{\pi}{6}$ times the fixed axis, times the square of the revolving axis.

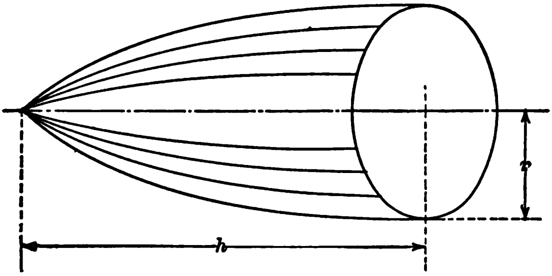
Draw two large figures in the work-book and formulate this law in the following notation:

V = volume of ellipsoid;
d_f = length of fixed axis;
d_r = length of revolving axis.

Under the figure in the work-book, copy the formula, compute the value of $\frac{\pi}{6}$ to four decimal places, and substitute.

172. The Paraboloid. If a parabola is revolved through one complete revolution about its axis, it traces out a solid, called the paraboloid.

The formula for volume is given under the cut.



$$V = \tfrac{1}{2}\pi r^2 h$$

FIG. 110.

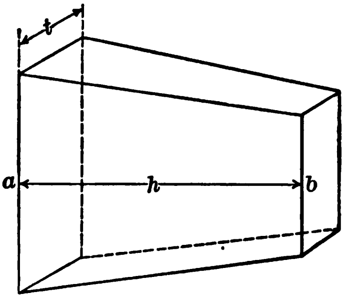
Draw a large figure in the work-book, copy the formula, compute $\frac{\pi}{2}$ to four decimal places, and substitute in the formula.

173. The Frustum of a Wedge. The formula for the volume of a frustum of a wedge as given under the figure, is obtained as follows:

The front face of the wedge is a trapezoid whose parallel sides are a and b, and whose altitude is h.

The area of this trapezoid is therefore expressed by the formula

$$A=\frac{h}{2}\ (a+b).$$

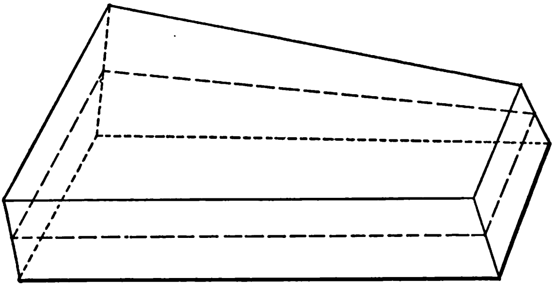


$V=\frac{ht}{2}(a+b)$

FIG. 111.

Draw a large figure, formulate V in terms of A and t, and substitute for A.

174. The Prismoidal Formula. In addition to the solids of the preceding paragraphs, there are irregularly shaped solids having five or more flat or plane faces two of which are parallel. The parallel faces are called top and bottom or bases of the solid, and the other faces are called the sides. These solids are known as prismoids, the word meaning



$V=\frac{h}{6}(A_b+A_t+4A_m)$

FIG. 112.

"prism-like." The formula may be used not only for the volume of a prismoid, but for the volume of any of the regular solids, also. Since its use to determine the volumes of the regular solids usually makes more computation than the regular formulas, it is used in computation on the regular solids only as a checking formula.

In the formula for the prismoid, the notation is as follows:

V = volume of prismoid;
h = height or perpendicular distance between the parallel faces;
A_b = area of base;
A_t = area of top;
A_m = area middle section or section half way between top and bottom.

175. Problems.

1. By reference to paragraph 170, formulate and compute the volume of a spherical sector when $h = 5\frac{1}{2}''$ and $R = 15''$.

2. Measure the ellipsoids in paragraph 171 and compute their volumes.

3. Measure the paraboloid in paragraph 172 and compute its volume.

4. Measure the frustum of the wedge in paragraph 173 and compute its volume.

5. Measure such of the models on the desk as may be assigned, and under dimensioned sketch, formulate and compute their volumes.

6. Compute and fill in the omitted entries below:

TABLE 102

SPHERICAL SECTORS

No.	R	h	V	No.	R	h	V
1	4	$1\frac{1}{2}$		6	7	$2\frac{1}{4}$	
2	6	2		7	11	3	
3	8.2	3.6		8	14	$4\frac{1}{2}$	
4	9.3	$2\frac{1}{2}$		9	21	5	
5	5	$1\frac{1}{8}$		10	32	$7\frac{1}{4}$	

7. Compute and tabulate the volumes below:

TABLE 103

ELLIPSOIDS

No.	d_r	d_f	V	No.	d_r	d_f	V
1	4	3		5	3	4	
2	5	$3\frac{1}{2}$		6	$3\frac{1}{2}$	5	
3	8	$5\frac{1}{4}$		7	$5\frac{1}{4}$	8	
4	9	7		8	7	9	

8. Formulate and compute the volumes of the paraboloids below, and tabulate the results:

TABLE 104

PARABALOIDS

No.	r	h	V
1	5	$11\frac{1}{2}$	
2	$7\frac{1}{4}$	15	
3	$9\frac{3}{8}$	20	
4	12	28	
5	21	$50\frac{1}{8}$	

9. In the following table, the notation is the same as in the figure of paragraph 173.

Formulate, compute, and tabulate the omitted entries.

TABLE 105

WEDGE FRUSTUMS

No.	a	b	h	t	V
1	7	5	21	3	
2	9	$5\frac{1}{4}$	24	$3\frac{1}{2}$	
3	13	7.4	28	4	
4	15	9	32	$3\frac{1}{8}$	

10. Measure the prismoid in paragraph 174 and formulate and compute its volume.

11. Measure the model prismoids on the table, as assigned; dimension the measurements on a neatly drawn figure, and compute and tabulate measurements and results.

§ 3. APPLIED PROBLEMS

176. Problems in Mensuration. In the solution of the following problems, show a carefully drawn and dimensioned figure for each.

Double-underline every result with a straight-edge and specify exactly what the result denotes.

1. The cylinder of an engine is 50 inches long, $\frac{13}{16}$ of an inch thick, and has an internal diameter of 32 inches.

Compute the number of *square* feet of felt required to cover its outside surface.

2. A flywheel is 96 inches in diameter. Compute the velocity in feet per second in round numbers, of a point on the face of the rim when the flywheel is making 180 revolutions per minute.

3. A boiler has 278 tubes, each 20 feet long and $2\frac{3}{4}$ inches in diameter.

Find the total heating surface of the tubes.

4. Measure, formulate, and compute the capacity in gallons of not less than 5 different cans for kerosene oil. If any of the cans measured do not hold a gallon, what illegitimate profit does the dealer make at 12 cents a gallon on 1000 gallons of oil when distributed in these cans?

Tabulate measurements and results in the following form in the work-book:

TABLE 106

CAPACITY OF OIL CANS

No.	Shape.	Dimensions Bottom.	Height.	Area, Bottom.	Cu. In.	Gallons.	Shortage.
1							
2							
3							
4							
5							

5. Compute the volume both in quarts and cubic inches of three different pails which are not of the same diameter at top and bottom.

6. A casting when submerged in a cylindrical vessel of water 10 inches in diameter, raised the level of the water 7 inches.

Compute the volume of the casting.

7. Computations for Steel Bars. Under proper headings, formulate each item called for in the following table, and solve three of the sixteen problems, as assigned. Tabulate the results, expressing weight per linear foot to two decimal places, and all other results to four decimal places.

One cubic inch of steel weighs .28$\frac{1}{3}$ pound. The weight required in the table is weight per linear foot.

TABLE 107
STEEL BARS

Prob. No.	Size in Inches.	Area □ bar.	Weight □ Bar.	Area ○ Bar.	Weight ○ Bar.	Circum. ○ Bar.
1	$\frac{1}{16}$					
2	$\frac{1}{8}$					
3	$\frac{1}{4}$					
4	$\frac{1}{2}$					
5	$\frac{17}{32}$					
6	$\frac{45}{64}$					
7	$\frac{3}{4}$					
8	$\frac{7}{8}$					
9	1					
10	1$\frac{3}{16}$					
11	2					
12	2$\frac{5}{8}$					
13	3					
14	4$\frac{5}{16}$					
15	5$\frac{9}{16}$					
16	6					

8. A horizontal, cylindrical boiler 66 inches in diameter and 18 feet long, has 54 horizontal tubes in it.

Compute the total length of the tubes and the number of square feet of steel required for the boiler exclusive of the ends, tubes, and lap of the plates.

9. A boiler has 275 tubes, each 19 feet 3 inches long and 2$\frac{5}{8}$ inches in diameter.

What is the total heating surface of the tubes?

10. The number of teeth in a spur-wheel is 120; the pitch (distance from center cf one tooth to the center of the next) is $1\frac{1}{4}$ inches.

Required the diameter of the wheel measured to the center of the teeth.

11. The sketch represents a cross-section of a 30-foot girder. Compute its cost at current prices per ton of 2000 lbs.

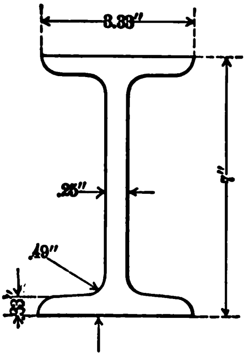


FIG. 113.

12. A $2\frac{1}{2}$-inch rainfall on a roof 40 feet by 24 feet discharges into a cistern 6 feet long and 4 feet wide. What will be the depth of the water in the cistern, if it contained . 100 gallons before the rainfall?

13. A crib, the inside dimensions of which are 15 feet by 7 feet 4 inches by 8 feet, is full of corn in the ear.

What is the value of the whole when shelled, at $.92 a bushel?

Given: 2 bushels of ears make 1 bushel of shelled corn.

14. The piston of a steam engine is 18 inches in diameter, and the mean effective pressure of steam is 150 pounds per square inch.

Required the total pressure on the piston in tons.

15. How many cubic yards of earth must be removed in digging a trench for the walls of a building, if the building is 40 by 100 feet, and the trench is 5 feet deep, 3 feet wide, and is to be dug 6 inches outside the building dimensions.

16. The length of a rectangle is 81 feet; its width is $5\frac{1}{3}$ feet.

Compute the length of a square whose area equals the area of the rectangle.

17. A room measures 18 feet by 14 by 9 feet. What will be the cost of three coats of paint for the walls and ceiling, at 7 cents per square yard for each coat, allowing 25 square feet for openings?

18. Water is flowing at the rate of 500 feet per minute, through a pipe 16 inches in diameter, into a rectangular reservoir.

In what time will the surface of the water be raised 3 inches, if the reservoir measures 600 feet by 250 feet?

19. A bar of copper 1 foot long, 9 inches wide, and $\frac{5}{8}$ of an inch thick, is rolled into a plate 6 feet by 4 feet.

What is the thickness of the plate?

20. The mean pressure on the crank pin in the direction of its motion is $\frac{1}{1.5708}$ of the mean pressure on the piston.

Compute the mean pressure on the crank pin of a direct-acting engine, the diameter of whose cylinder is 17 inches, when the mean pressure of the steam is 60 lbs. per sq. in.

21. Formulate and compute the approximate capacity in gallons, of each size of can in the table below, in which G denotes the number of gallons; h, height of conical tap; D, diameter; H, height of circular part of can:

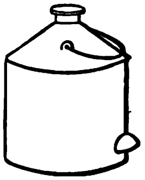


Fig. 114.

Table 108

CAPACITY OF OIL CANS

No.	1	2	3	5	10
D	6¼	8	9	10¼	14
H	7	9	10	12	15½
h	2½	2¾	3	3½	3½
G					

Entries in table are in inches.

22. An engine fly-wheel, 80 inches in diameter, makes 160 revolutions per minute. What is the velocity in feet per second of a point on the rim?

23. Compute the cost of lining a box with sheet lead, the dimensions being as follows: length 3 feet 3 inches, width 2 feet 6 inches, and depth 1 foot 9 inches. The sheet lead is furnished at 67.5 cents per square yard.

24. The diameter of a safety-valve on a boiler is 3 inches.

Compute the total pressure tending to raise the valve when the steam-gage stands at 120.

25. A circular plot of ground 100 yards in diameter is bordered by a walk 10 feet wide.

Compute the cost of the walk at $2.75 per square yard.

26. A room 16 feet by 20 feet 6 inches by 10 feet has two doors, each 3 feet by 7 feet, and two windows each 5 feet 3 inches by 6 feet. At 24 cents per square yard, determine the cost of plastering the walls and ceiling, deducting for the openings (doors and windows).

27. A casting when submerged in a cylindrical vessel of water $10\frac{1}{2}$ inches in diameter raised the level of the water $7\frac{1}{4}$ inches.

Compute the volume of the casting.

28. A gas tank is 60 feet in diameter, and when full of gas stands 50 feet high. How many cubic feet of gas have been used when the tank stands 5 feet high?

29. The arc of contact of the belt on a $10\frac{1}{2}$-inch pulley, is $5\frac{1}{2}°$ more than the semi-circumference both on the top and bottom of the pulley.

Compute the length of belt contact.

30. A cistern measuring $8\frac{1}{2}$ feet by $5\frac{3}{4}$ feet, and having 6 inches of water, received the water from a 3.52 rainfall on a roof whose dimensions are 20 feet by 12 feet 3 inches.

Compute the depth of the water in the cistern after the rain.

31. A boiler tube is to be made 3 inches inside diameter, $\frac{1}{16}$ of an inch thick, and 10 feet long.

Compute the length of a block of brass 3 inches square from which it can be manufactured.

32. Compute the number of square feet of felt required to cover the external curved surface of a cylinder whose internal diameter is 33 inches, outside length 48 inches, and thickness $\frac{7}{8}$ inch.

33. If the stroke of the piston in Problem 32 is 40 inches, and the piston $3\frac{1}{2}$ inches thick, what is the rubbing surface between piston and cylinder?

34. A water tower 160 feet high and 55 feet in diameter is represented in a drawing as 12 inches high.

What should be the represented diameter?

Compute the capacity of the tower in gallons.

35. How many cubic feet of zinc $\frac{1}{16}$ of an inch thick will be required to cover the sides and bottom of a cistern 10 feet long, 6 feet wide, and 7 feet deep?

36. A plate of metal measuring 100.5 inches by 15.6 inches by $2\frac{1}{2}$ inches is melted and cast into a cube.

Compute the edge of the cube, making no allowance for loss.

37. A rectangular tank $8\frac{3}{8}$ feet long and $5\frac{1}{4}$ feet wide has 3 feet of water in it. If 100 gallons are drawn off, how much is the surface lowered?

38. What should be the area of the opening of a cold-air box for a furnace to supply one hot-air pipe 1 foot in diameter, and 5 hot-air pipes 8 inches in diameter, it being necessary that the cross-sectional area of the cold-air box be $\frac{3}{4}$ that of the hot-air pipes together.

39. A train is drawn by a locomotive with drive-wheels 56 inches in diameter. How many strokes per minute must the piston make in order that the train may run at the rate of 45 miles per hour, with no allowance for slipping.

40. A steam cylinder $1\frac{3}{4}$ inches thick has an external diameter of 38 inches. The piston rod is $6\frac{1}{2}$ inches in diameter.

What is the pressure on the piston in tons when the steam-gage stands at 180?

41. An emery wheel is 9 inches in diameter. How many revolutions per minute are necessary in order that the speed may be 5500 feet per minute?

42. Compute the capacity of an engine cylinder 15 inches in diameter, piston stroke 25 inches, allowing an addition of 7 per cent for clearance space.

43. Rectification of a Circumference. In the American Machinist for May 20, 1909, is the following method for laying off a straight line approximately equal to the circumference:

Draw a diameter and a radius *VT* making an angle of 30° with it.

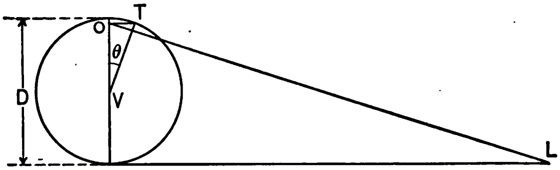


FIG. 115.

From the extremity of the radius draw a perpendicular to the diameter, *TO* in the figure.

From the other extremity of the diameter draw a tangent equal to 3 times the diameter terminating at *L*.

OL is the circumference.

The error in the preceding construction is said to be $\frac{1}{20000}$ of the circumference.

Compute the error when the following diameters are used:

5″, 12″, 20″, 35″.

44. The American Machinist for March 18, 1909, gives a method of rectifying a circumference, entirely different from that in Problem 43. Read up this method in the library, and carefully make the layout specified in the reference.

Compute the error when the same diameters are used as in the preceding problem.

Which method gives the better result in each instance?

45. The formula below for the perimeter of an ellipse is from Carpentry and Building for January, 1908:

$$P = 1.82D + 1.315d,$$

in which P = perimeter of ellipse;
D = major axis;
d = minor axis.

By this formula compute the perimeter of the ellipse in Problem 1, paragraph 144.

46. In Trautwine's Handbook the following formula is given for the perimeter of an ellipse:

$$P = \pi\sqrt{\frac{D^2+d^2}{2} - \frac{(D-d)^2}{C}}$$

in which C = constant in the following table;
D = large axis;
d = short axis.

Table 109

CONSTANTS FOR TRAUTWINE'S FORMULA FOR PERIMETER OF AN ELLIPSE

D	C	D	C
5d or less	8.8	20d	9.8
6d	9.	25d	9.87
7d	9.2	30d	9.92
8d	9.3	40d	9.98
9d	9.35	50d	10.04
10d	9.4	60d	10.10
12d	9.5	70d	10.17
14d	9.6	80d	10.23
16d	9.68	100d	10.35
18d	9.75		

By Trautwine's formula, compute the perimeter of the ellipse in Problem 1, paragraph 144.

47. In a thousand (1000) horse-power chain drive, the sprockets have 61 teeth and 83 teeth.

When the large wheel is making 70 revolutions per minute, the chain runs 1448 feet per minute.

Compute the diameter of both wheels.

48. Formulate and compute the capacity in bushels of the ash cans tabulated below:

TABLE 110

ASH CANS

No.	3	5	6
Size Bush.	15×26	18×26	20×26

49. The milk-can shown in the cut has the following dimensions:

$D = 12\frac{1}{8}''$;
$d = 7''$;
$h = 16\frac{1}{2}''$
$r = \frac{3}{4}''$;
$t = 4''$.

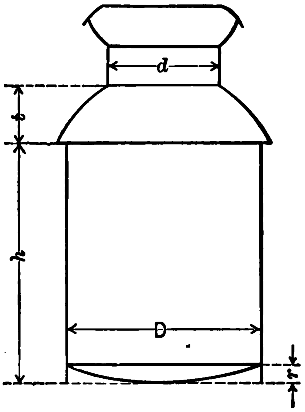


FIG. 116.

Formulate and compute its capacity in quarts.

50. The number of quarts of milk in a can is determined by a "measuring stick."

These are made of steel coated with tin, and are graduated in quarts. When placed in the can so that the point touches the center of the bottom, the number of quarts may be read directly on the stick, after removal from the can.

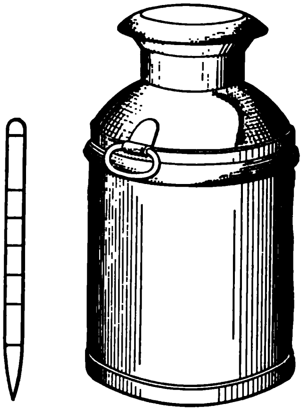


Fig. 117.

In the table below, h denotes the distance of the graduations in inches from the point of the stick.

Measure these distances on the stick at the desk, and enter the measurements in the table.

Are the graduations correctly placed for a can having the same dimensions as in Problem 49?

Table 111

MEASURING-STICK GRADUATIONS

Quarts.	h	Quarts.	h
5		25	
10	5″	30	
15		35	16″
20		40	$19\frac{3}{8}$″

51. The drawing represents an elliptical gasolene tank for an automobile.

Determine the dimensions of a tank of this shape on some machine and formulate and compute its capacity in gallons.

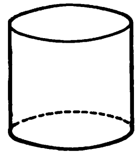


Fig. 118.

52. Below are tabulated some of the standard dimensions for U. S. dry measures.

Determine by computation whether the one assigned in each size has the specified capacity:

Table 112
STANDARD DRY MEASURES

No.	½ Bushel.		1 Peck.		½ Peck.		1 Quart.	
	Diam.	Height.	Diam.	Height.	Diam.	Height.	Diam.	Height.
1	$13\frac{1}{4}$	$7\frac{3}{4}$	8	$10\frac{1}{4}$	$6\frac{1}{4}$	$8\frac{1}{4}$	$3\frac{1}{2}$	7
2	$13\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{4}$	10	$6\frac{1}{2}$	$8\frac{1}{8}$	$3\frac{1}{4}$	$6\frac{1}{2}$
3	$13\frac{3}{4}$	$7\frac{1}{4}$	$8\frac{1}{2}$	$9\frac{1}{2}$	$6\frac{3}{4}$	$7\frac{1}{2}$	4	$5\frac{3}{8}$
4	14	7	$8\frac{3}{4}$	9	7	7	$4\frac{1}{4}$	$4\frac{3}{4}$
5	$14\frac{1}{4}$	$6\frac{3}{4}$	10	$6\frac{7}{8}$	$7\frac{1}{4}$	$6\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{1}{4}$
6	$14\frac{1}{2}$	$6\frac{1}{2}$	$10\frac{1}{2}$	$6\frac{1}{4}$	$7\frac{1}{2}$	$6\frac{1}{8}$	$4\frac{3}{4}$	$3\frac{3}{4}$
7	$14\frac{3}{4}$	$6\frac{1}{4}$	11	$5\frac{5}{8}$	$7\frac{3}{4}$	$5\frac{11}{16}$	5	$3\frac{3}{8}$
8			$9\frac{1}{4}$	8			$5\frac{1}{4}$	$3\frac{1}{8}$

53. The circumference of the reel used by a sealer of weights and measures is exactly 1 yard. Compute its diameter.

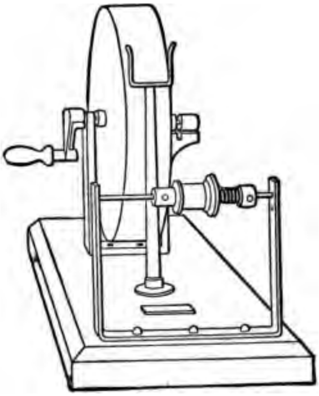


Fig. 119.

54. The drawing shows a cross-section of a drain tile resting on a concrete support.

Formulate and compute the number of cubic feet of concrete required for 200 feet of pipe $\frac{3}{4}$ of an inch thick with the following data:

$$a=4', \qquad L=4'\,4'', \qquad b=18'',$$
$$h=9'', \qquad d=6''.$$

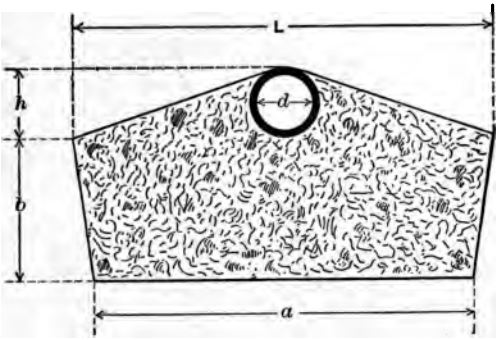


FIG. 120.

55. The foundation walls for a building are 9 feet in depth and $3\frac{1}{2}$ feet in thickness. The building measures 34 feet by 50 feet.

If the foundation projects 3 inches outside the building, formulate and compute the number of cubic yards in the foundation.

56. A passenger coach is equipped with two gas tanks, each $9'\,6''$ long and $20\frac{1}{2}''$ in diameter. By compression they may be filled so that each cubic foot of tank capacity will contain 10 cubic feet of gas.

Compute the number of cubic feet of gas in the two tanks when filled under compression.

57. In order not to lose time in taking water, a track tank between the rails is employed from which the tender of a locomotive is filled by means of a scoop.

How many gallons of water are contained in a tank 1300 feet long, 19 inches wide and 7 inches deep, if the water averages $2\frac{1}{2}''$ below the top of the tank?

58. In 58 minutes a boy chips a flat surface $3\frac{1}{2}'' \times 2\frac{1}{4}''$ and $\frac{3}{16}''$ deep, in a wrought-iron piece.

How many cubic inches of metal does he remove in 30 minutes?

59. Cone pulleys are used in order that different speeds may be obtained.

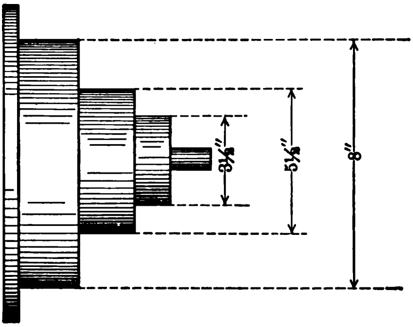


FIG. 121.

Using the dimensions shown in the cut, or those obtained by measurement of one of the step-cones in the shop, formulate and compute the areas of the sides of the steps.

60. When the size of an engine cylinder is given as 24"×30", the first number is the diameter of the cylinder, the second is the stroke of the piston. A double-acting engine is one in which the steam is admitted to each side of the piston alternately.

Compute the pressure on each side of the piston of a double-acting 24"×30" engine having a piston *rod* 3¾" in diameter.

61. A cylindrical tank with flat ends lies on its side in a horizontal position. The tank is 16 feet 4 inches long and 5½ feet in diameter.

Compute the number of gallons in it when the water stands at a level of 3 feet 9 inches.

62. A stand-pipe 100 feet high and 35 feet in diameter has a gage on the side showing the depth of the water at any time.

If both depth and gallons were marked on the gage, every half foot would represent how many U.S. gallons?

When the gage stands at the 25-foot mark, how many U. S. gallons are in the tower?

How many kiloliters?

63. An iron rod measures .4218 inch in diameter. This is about how many 16ths of an inch?

64. The 100-foot smokestack for a factory is to be held in place by 5 guy wires attached ¾ the way up the stack and fastened to concrete blocks on a level with the bottom of the stack, each 70 feet distant from it.

Allowing a total of 30 feet for fastening, how many feet of wire will be required?

65. With a flat chisel a machine shop apprentice chipped a wrought-iron surface $8'' \times 2\frac{1}{8}'' \times \frac{1}{8}''$ in 1 hour and 40 minutes.

If a cape chisel cuts twice as fast as a flat chisel, in what time would the job have been done, by using the cape chisel to remove the first quarter of the metal, and the flat chisel for the remaining three quarters?

66. A pump has a water cylinder $5\frac{1}{2}$ inches in diameter. The stroke is 14 inches. How many gallons are pumped per hour at an average of 100 strokes per minute, with no allowance for leakage.

67. The mean effective pressure on the piston of the cylinder of a steam engine may be roughly taken as 40 per cent of the boiler gage pressure, and the effective power at the draw-bar as 45 per cent of the power developed in the cylinders.

The draw-bar pull is what per cent of the boiler gage pressure?

68. Assuming that you are the Peoples' Water Co., write out a bill for the amount due from a customer who has used 8000 cubic feet at 25 cents per 1000 gallons.

69. On a certain planer the ratio of the cutting speed to the return speed is as 1 to 2.75.

What is the return speed when the cutting speed is 18 feet per minute?

70. Under a load of 80.53 lbs. a wire 107 inches long and .0463 inches in diameter lengthened .272 inches.

What was the elongation per foot to five decimal places?

71. When a house is heated by a furnace the cross-sectional area of the cold air box should equal $\frac{3}{4}$ the total cross-sectional area of the hot air pipes supplying the rooms.

What should be the area of the cold air box for 6 hot air pipes, 2 being $9\frac{1}{2}$ inches in diameter, 3 being 8 inches in diameter, and the remainder $7\frac{1}{4}$ inches in diameter?

CHAPTER VIII

WEIGHT

177. What Weight is. Everyone is familiar with the idea of weight, which is commonly regarded as one of the properties of matter, like hardness, elasticity, and malleability. But weight is not inherent in matter and is therefore not one of its properties. All substances have weight solely because they are pulled or attracted by the earth, as a magnet attracts iron.

Weight, then, is the earth-pull on a body. If a piece of timber or iron, say a beam or an iron girder, could be placed in space sufficiently far to be beyond the pull of the earth and every other body, it would have no weight whatever, yet it would still be hard or soft, rough or smooth, elastic or non-elastic. Having no weight, if at rest it would remain at rest; if in motion it would remain in motion. It could neither fall nor move in any other direction unless set in motion.

Weight, therefore, is the amount that the earth attracts.

178. How Weight is Determined. It will now be evident that if the weight hanging from the scale-beam in Fig. 122 balances the beam when in a notch marked 35 lbs., the earth pull on a block lying on the platform of the scale or in the scoop, is 35 lbs. A platform scale, a balance, a steelyard, or other weighing machine, is therefore a device for determining how much the earth attracts any material or combination of materials.

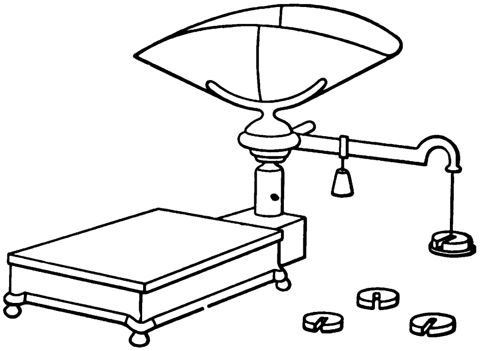


Fig. 122.

At this point it may be asked, how does a weighing-scale enable us to determine the weight of anything? If the scale is the kind shown in the figure below, in which the beam is supported at the center, it is evident that a 1-lb. weight at one end will balance 1 lb. of material at the other.

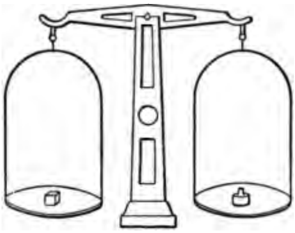
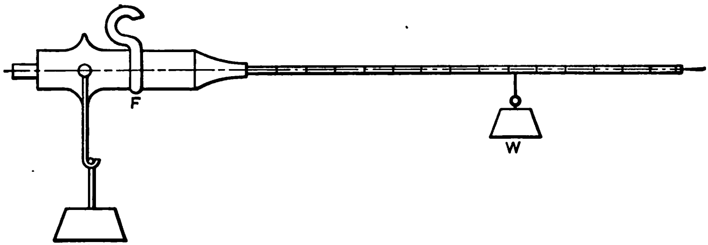


Fig. 123.

In order, therefore, to determine a pound of nails, a pound weight is placed in one pan of the scale and enough nails are placed in the other to make the beam horizontal. The principle involved is that the beam balances when the amount of material in one pan exactly equals the amount of material in the other pan; in this instance, when the earth pull on the nails is the same as the earth pull on the pound weight.

179. Law of Weight. When the weight arm is longer than the material arm, the beam will balance when the weight multiplied by the distance between the weight and the point on which the beam turns equals the material multiplied by the distance between the point at which the material is attached to the beam and the point on which the beam turns.



$W \times D = M \times B$

Fig. 124.

In the cut F is the fulcrum, or point on which the beam turns:

W is the weight;

M is the piece to be weighed;

D is the weight arm or distance between F and the point at which W is attached;

B is the material-arm or distance between F and point at which material is attached.

Draw the figure and place these letters on it.

The law of weight is frequently stated in the following form, and is known in mechanics as the

Law of Moments. The force times the force arm equals the weight times the weight arm.

180. Problems. Solve the following problems by substitution in the formula of paragraph 179:

1. Compute M when

$$W = 1\tfrac{1}{4} \text{ lbs.};$$
$$B = \tfrac{5}{8} \text{ inches};$$
$$D = 12\tfrac{1}{2} \text{ inches}.$$

2. Complete the following table in which W and M are pounds and B and D are inches:

TABLE 113

WEIGHT COMPUTATION

No.	W	B	D	M
1	$\frac{3}{8}$	$\frac{5}{8}$	$4\frac{3}{16}$	
2	$\frac{1}{2}$	$\frac{5}{8}$	$3\frac{1}{8}$	
3	$\frac{3}{4}$	$\frac{5}{8}$	$11\frac{3}{8}$	
4	$\frac{5}{16}$	$\frac{5}{8}$	$11\frac{3}{8}$	
5	$\frac{5}{8}$	$\frac{5}{8}$	$13\frac{1}{2}$	
6	$\frac{3}{16}$	$\frac{5}{16}$	$15\frac{5}{16}$	
7	$\frac{5}{16}$	$\frac{5}{16}$	$7\frac{1}{8}$	
8	$\frac{7}{16}$	$\frac{5}{16}$	$9\frac{5}{64}$	
9	$\frac{9}{16}$	$\frac{5}{16}$	$12\frac{3}{64}$	
10	$\frac{11}{16}$	$\frac{5}{16}$	$8\frac{5}{8}$	

3. Complete the following table, and make drawing to scale of a scale-beam graduated and marked as per entries in the table.

Determine B by measuring a scale-beam.

TABLE 114

GRADUATION OF SCALE-BEAM

No.	W	B	D	M	No.	W	B	D	M
1	$\frac{7}{8}$			1	11	$\frac{7}{8}$			11
2	$\frac{7}{8}$			2	12	$\frac{7}{8}$			12
3	$\frac{7}{8}$			3	13	$\frac{7}{8}$			13
4	$\frac{7}{8}$			4	14	$\frac{7}{8}$			14
5	$\frac{7}{8}$			5	15	$\frac{7}{8}$			15
6	$\frac{7}{8}$			6	16	$\frac{7}{8}$			16
7	$\frac{7}{8}$			7	17	$\frac{7}{8}$			17
8	$\frac{7}{8}$			8	18	$\frac{7}{8}$			18
9	$\frac{7}{8}$			9	19	$\frac{7}{8}$			19
10	$\frac{7}{8}$			10	20	$\frac{7}{8}$			20

4. Determine whether the beams of the weighing scales used in the shops are correctly graduated and marked, and whether the various weights are correctly marked.

181. Density. The drawing represents a tank whose capacity is 1 cubic foot.

If a tank of this size were even full of water it would not weigh the same as when filled with oil or some other liquid.

A cubic foot of iron is not of the same weight as a cubic foot of gold.

As will be determined in a subsequent paragraph, a cubic foot of cast iron, of wrought iron, and of steel, all differ in weight. Not only do equal volumes of different kinds of wood differ in weight, but equal volumes of the same wood differ, such as oak that is quarter-sawed and oak sawed in the ordinary way.

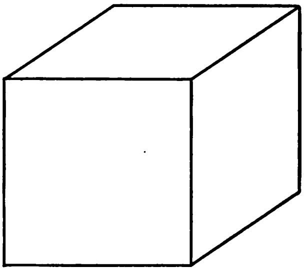


FIG. 125.

This difference in the weights of equal volumes of different materials and of the same material, is due to a difference in density or closeness of particles. Oak is heavier than pine because of greater density.

Materials of less density than water will float; those of greater density will sink.

182. Problems. In doing the work required in this paragraph use great care to secure accurate results.

Use at least two different pieces of the material under test.

In every instance determine the volume of the pieces, as follows:

(1) By computation from careful measurement of their dimensions.

(2) By weighing the water which overflows when a piece is sub-

merged in a vessel even full of water, and by calculation of the volume of the displaced water.

(3) By submergence of the piece in a vessel having a mark at the water level, and by computation of the volume of water between the levels before and after submergence.

In case of discrepancy, if the results are of apparently equal merit use the average value. If any result is evidently incorrect repeat the test.

Tabulate all tests as follows:

TABLE 115

No.	Material.	Shape.	Measurements.	Volume.			Weight.
				(1)	(2)	(3)	

1. Using a 2-gallon cup determine with the greatest accuracy the weight of one cubic foot of water.

Check, using a quart cup.

Check, using some other vessel.

Carry each result to three decimal places and find the average of the three results.

2. Using a piece of cast iron of any convenient size, determine the weight of one cubic foot of cast iron.

Check by calculating from a piece entirely different in shape.

3. Determine the weight of a cubic foot of steel, using any convenient piece.

Which has the greater density, cast iron, or steel?

4. Determine the weight of a cubic foot of wrought iron; the weight of a cubic inch.

5. Determine the weight of a cubic foot of copper; the weight of a cubic inch; the number of cubic feet to the ton.

6. Determine the weight of a cubic inch of brass; of a cubic foot.

7. Determine the weight of 1000 board feet of mahogany; of a cubic foot.

8. Make the same calculation as in Problem 7, using hard maple and white pine.

9. Calculate the weight of common brick per M; the weight per cubic foot.

10. Calculate the weight of a cubic foot of sand; of a cubic foot of cement; of a cubic foot of stone. Specify the kind of each.

11. Calculate the weight of a cubic foot of concrete, using several different specimens if possible. Taking the average of the various results, determine the number of cubic feet to the ton.

12. Complete the following table:

TABLE 116

WEIGHTS PER CUBIC FOOT

No.	Material.	Weight 1 Cu. Ft.	Times Heavier than Water.
1	Water		
2	Cast iron		
3	Steel		
4	Wrought iron		
5	Copper		
6	Brass		
7	Mahogany		
8	Hard maple		
9	White pine		
10	Brick, common		
11	Sand		
12	Cement		

183. Specific Gravity. It will be evident that the table furnishes an easy means of computing the weight of any piece of material of the kind given in the table. For example, if the weight of a steel shaft is desired it may be found by two different methods:

(1) By multiplying its volume in cubic feet by the weight of steel per cubic foot.

(2) By computing the weight of the same volume of water and multiplying the result by the number of times that steel is heavier than water.

The specific gravity of a solid or liquid is the number of times it is heavier than the same bulk of water. It is therefore the ratio of the weight of the substance, to the weight of the same volume of water.

Therefore:

$$\text{Specific gravity} = \frac{\text{weight of solid or liquid}}{\text{weight of equal vol. of water}}.$$

By reference to the table in Problem 12 in the preceding paragraph, fill in the omitted entries below:

Table 117

SPECIFIC GRAVITY

No.	Material.	Specific Gravity.	No.	Material.	Specific Gravity.
1	Brass		7	Mahogany	
2	Brick (common)		8	Maple (hard)	
3	Cement		9	Pine (white)	
4	Copper		10	Sand	
5	Iron (cast)		11	Steel (cast)	
6	Iron (wrought)		12	Water	

184. Formula for Weight. The formula below is expressed in the following notation:

W = weight of material;
C = weight of 1 cubic foot of water;
S = specific gravity of the material;
V = number of cubic feet of material;
$W = CSV$.

Copy the formula and the notation in the work-book.

185. Problems. Show a dimensioned sketch for each of the following problems and make the computation specified in each instance. Take weights from the table in the back part of the book.

1. By substitution in the formula, compute the weight of 3.89 cubic feet of steel; of 7.93 cubic feet of cast iron.

2. A bin measuring $34' \times 22' \times 11'$ is filled with sand weighing 100 lbs. to the cubic foot.

Compute the weight of the sand.

3. A cylindrical tank 8¾ feet in diameter and 28 feet long is ⅔ full of petroleum.

Compute the weight of the petroleum.

Given specific gravity of petroleum = .7.

4. Formulate and compute the weight of a cast-iron ball 13 inches in diameter. (Iron weighs .26 pound per cu. in.)

5. The steel plate in the sketch is $\frac{5}{16}$ of an inch thick, with other dimensions as marked.

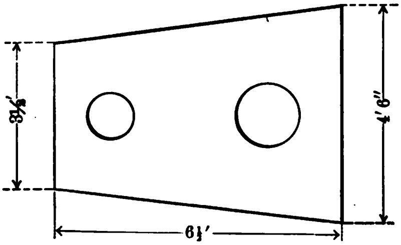


FIG. 126.

The diameters of the two circular holes cut in it are 1 foot 6 inches, and 2 feet respectively.

Required the weight of the plate.

6. Formulate and compute the weight of a piece of steel shafting 10 feet long and 2 inches in diameter.

7. A brass tube 8 feet long has an outside diameter of 3.2 inches and an inside diameter of 2.9 inches.

Compute the weight of the tube.

8. Compute the weight of two elliptic gear blanks of the same size, each $\frac{9}{16}$ of an inch thick, whose axes measure 9 inches and 6 inches. (See paragraph 142.)

9. Find the weight of a box with a lid, made of wood ¾ of an inch thick, measuring externally 4 feet × 3 feet × 3 feet, the material weighing 38.4 pounds per cubic foot.

10. A zinc tank, open at the top, is 30 inches long, 20 inches wide, and 16 inches deep, outside measure. It is made of zinc ¼ of an inch thick.

Find its weight when half full of water, zinc weighing 7.2 times as much as its own bulk of water.

11. A brass plate 1½ inches thick, in the shape of a semicircle with a radius of 12 inches, has 4 holes through it, each ¾ of an inch in diameter.

If the brass weighs .3031 pound per cubic inch, compute the weight of the plate.

12. A cubic foot of copper is drawn into No. 12 wire, B. & S. gage.

Compute the length of the wire and its weight per mile. (Determine the diameter of a No. 12 wire with a micrometer and compare the reading with that given in the table of wire sizes.)

13. An iron tank is 7 feet long, 5 feet wide, 6½ feet deep, and ¾ of an inch thick.

Find its cost at 7 cents per pound, and its weight when filled with water.

14. A hollow cast iron column 13 feet long, has an outside diameter of 10 inches and is 1⅜ inches thick. The weight of cast iron averages .262 pound per cubic inch, and the average weight of concrete is 130 pounds per cubic foot.

Compute the weight of the column when filled with concrete, assuming both the concrete and the column to be of average weight.

15. Calculate what weight of sheet lead 0.1″ thick, will be required to line a wooden tank, the internal dimensions of which are 6′×4′×3′.

Weight of 1″ thick sheet lead per square foot = 59 pounds.

16. The table below gives the number of cubic feet of coal of different sizes, in a long or English ton of 2240 pounds.

Copy the table in the work-book, and formulate and compute the number of cubic feet of the various sizes, in a short or U.S. ton of 2000 pounds.

Table 118

CUBIC FEET OF COAL IN 1 TON

No.	Kind of Coal.	Cubic Feet in Ton.	
		2240 Lbs.	2000 Lbs.
1	Egg	38.6	
2	Stove	39.2	
3	Nut	39.8	
4	Pea	40.5	
5	Buckwheat	41.1	
6	Average	39.8	
7	Broken	39.4	

17. A rectangular box without a cover, made of boards 1 inch thick, has the following external dimensions: length 8 feet 8 inches, width 5 feet, depth 4 feet.

What will be the cost of lining it with lead weighing 7 pounds to the square foot and costing 12 cents per pound?

18. The sketch shows a cross-section of a trapezoidal cast-iron beam 9′ in length.

Determine its weight.

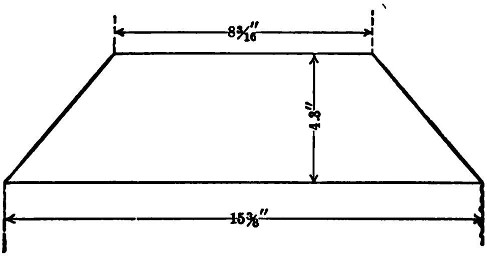


FIG. 127.

19. What is the weight of a brass tube 9.6 feet long, the outside diameter being $3\frac{1}{8}$ inches and the thickness being $\frac{3}{16}$ inch?

20. A cast-iron pipe of 18-inch bore is $1\frac{1}{8}$ inches thick and 12 feet long.

Determine its weight; also the diameter of a solid cast-iron column of the same length and weight.

21. Determine the weight of a cable 500 feet long formed of a single copper wire having a cross-section of .26 square inch, with an insulation covering .15 inch thick, the insulation weighing .038 pound per cubic inch.

22. In the sketch each side of the square rectangular block measures 19 inches, with a thickness of $3\frac{1}{8}$ inches.

The penetrating cylinder which passes half-way through the block has a bore of 10 inches, a thickness of $2\frac{3}{16}$ inches, and a length of $9\frac{3}{32}$ inches.

Compute the total weight of the two pieces, the block being cut away to the depth to which the cylinder penetrates.

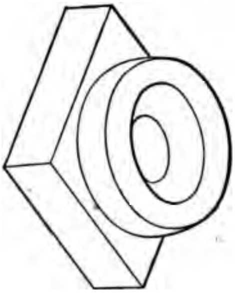


FIG. 128.

23. A steel shaft, $12\frac{1}{2}$ feet long and 5 inches in diameter, has a taper 28 inches long at both ends, the taper being 1.2 inches per foot.

Compute the weight of the shaft in pounds.

24. A hollow spherical cast-iron shell has an inside diameter of $6\frac{3}{32}''$ and is $1\frac{3}{64}''$ thick.

Compute its weight.

25. A lathe hand cuts a round bar of steel into 11 pieces each 16″ long, the waste from the cutting tool being $\frac{3}{16}''$ at each cut.

Determine the weight of the bar and the loss in weight from cutting.

26. In taking water from a tank 20 feet 6 inches in diameter, an engine lowered the water level 10 inches.

Find the weight of the water taken.

27. Compute the weight of the rim of the fly-wheel on some engine in the shop or laboratory.

28. What is the weight of the largest sphere which can be turned from a cube of brass weighing 32.5 pounds?

29. A piece of steel is tapered to a center at one end at the other is $3\frac{3}{16}''$ in diameter, the taper being uniform throughout the entire length of $33\frac{5}{8}''$ measured along the taper, as shown.

Determine the taper per foot and the weight of the piece.

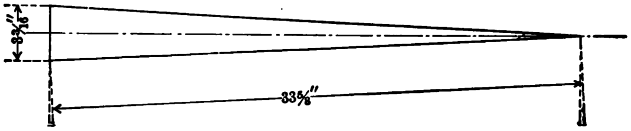


FIG. 129.

30. Measure such of the models at the desk, as may be assigned, and formulate and compute the weight.

Enter measurements and results in a table which includes number of the piece, kind of material, and shape.

31. Measure at least three pieces of your own selection in the shop, and formulate and compute the weight.

Show dimensioned sketch and tabulation as in Problem 30.

32. A freight car 36′ long and $8\frac{1}{2}'$ wide, is filled with rye to an average depth of 5 feet.

Formulate and compute the weight and the value of the shipment at 97 cents a bushel.

33. What weight of shelled corn can be shipped in a car measuring 36′×8′ 6″×8′?

Compute the value of the corn at current prices.

34. The piece shown is an iron casting whose shape is the frustum of a regular pyramid of the following dimensions:

Height $h=11$ inches;
Base edge $S=21$ inches;
Top edge $s=15$ inches;
Diam. hole $D=2.5$ inches.

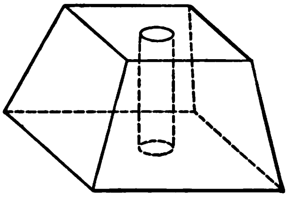


Fig. 130.

Determine the weight of the casting.

35. The base of the grooved piece shown measures $15\frac{3}{8}''\times 11\frac{1}{16}''$. The grooves are $1\frac{1}{2}''$ in width and $2\frac{1}{4}''$ in depth.

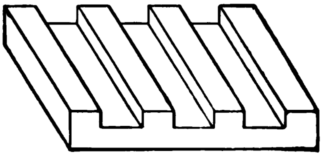


Fig. 131.

The thickness from the bottom of the groove is $2\frac{1}{8}''$. Compute the weight.

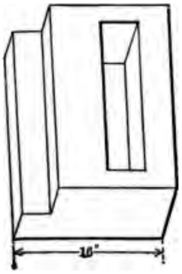


Fig. 132.

36. The thickness of the bronze piece in the sketch is $4\frac{5}{8}$ inches, the width of the slot is $4\frac{7}{16}''$.

The width of the cut in the edge is the same as its depth which is 2.5″.

The length of the piece is 31″ and the length of the slot is 27″.

Determine the weight of the piece.

37. Derive a formula for the weight of the anchor plate here shown and use it in completing the entries in the following table in which

V = volume,
W = weight.

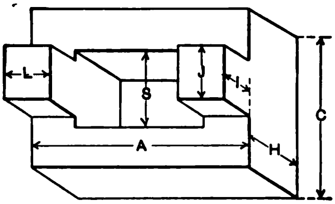


FIG. 133.

Entries in the table are in inches.
Enter volume in cubic inches and weight in pounds.

TABLE 119

ANCHOR PLATES

No.	A	C	H	S	I	J	L	V	W
1	$3\frac{1}{8}$	$2\frac{5}{8}$	$\frac{15}{16}$	$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{3}{8}$		
2	4	$3\frac{1}{4}$		$2\frac{1}{4}$		$1\frac{5}{8}$	$1\frac{1}{8}$		
3	$3\frac{3}{8}$	$3\frac{1}{16}$		$1\frac{7}{8}$		$1\frac{1}{16}$	$1\frac{1}{16}$		
4	$2\frac{7}{8}$	$2\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{32}$	$\frac{5}{16}$	$\frac{5}{8}$	$\frac{11}{16}$		
5	$2\frac{9}{16}$	$2\frac{5}{16}$	$\frac{7}{8}$	$1\frac{3}{16}$	$\frac{7}{16}$	$\frac{11}{16}$	$\frac{15}{16}$		
6	$2\frac{1}{8}$	$1\frac{7}{8}$	$\frac{1}{2}$	1	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{9}{16}$		
7	$4\frac{3}{16}$	$3\frac{7}{16}$	$1\frac{3}{8}$	2	$\frac{11}{16}$	$1\frac{1}{4}$	$1\frac{3}{4}$		
8	$5\frac{1}{16}$	$3\frac{7}{8}$	$2\frac{1}{16}$	$2\frac{3}{8}$	$1\frac{7}{8}$	$1\frac{15}{16}$	$1\frac{5}{8}$		
9	$3\frac{1}{2}$	$2\frac{3}{16}$	$\frac{3}{4}$		$\frac{3}{8}$	$\frac{21}{32}$	$1\frac{1}{8}$		
10	$2\frac{3}{32}$	$1\frac{7}{8}$		$\frac{11}{16}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{7}{16}$		

38. Measure the table on two different planers in the shop and compute the weight of each.

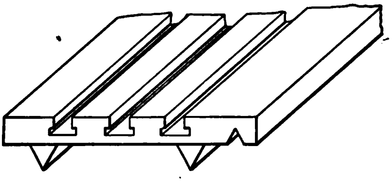


FIG. 134.

39. Measure and compute the weight of the circular table on some machine in the shop.

40. Determine the weight of the block shown, when dimensioned as follows:

$A = 4\frac{1}{8}''$ $B = 29''$

$C = 3\frac{1}{16}''$ $D = 3''$

$E = 1\frac{1}{8}''$ $F = 2\frac{1}{4}''$

$G = 3\frac{3}{8}''$ $I = \frac{3}{16}''$

$J = \frac{7}{8}''$ $K = 2\frac{5}{16}''$

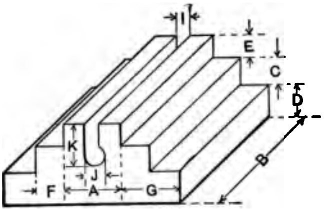


FIG. 135.

41. Measure a chuck in the shop similar to the one here shown and compute its approximate weight.

Check by weighing it.

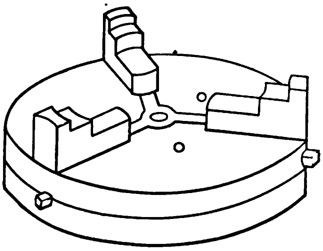


FIG. 136.

42. The flanged coupling shown has the following dimensions:

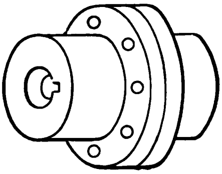


FIG. 137.

Diameter of shaft $= 12''$

Diameter of core $= 5''$

Width flange face $= 3''$

Ext. diam. flange rim $= 25''$

Thickness flange $= \frac{5}{8}''$

Compute the weight of the coupling in steel, allowing for eight bolts weighing 83.5 pounds per 100 bolts.

TABLE 120

SHRINKAGE

Metal.	Allowance per Foot.
Aluminum	Seven thirty-seconds of an inch
Lead	" " "
Steel	" " "
Zinc	" " "
Brass	Three-sixteenths of an inch
Tin	" " "
Bronze	Five thirty-seconds of an inch
Iron	One-eighth of an inch

43. The cradle pattern shown has the following dimensions:

Length $=31.75''$
Width base $=17''$
Thickness base $=2\frac{1}{8}''$
Thickness piece $A=1\frac{7}{8}''$
Width piece $A=8\frac{1}{2}''$

Compute the weight of the dry-pine pattern and the weight of the iron casting, making allowance for shrinkage.

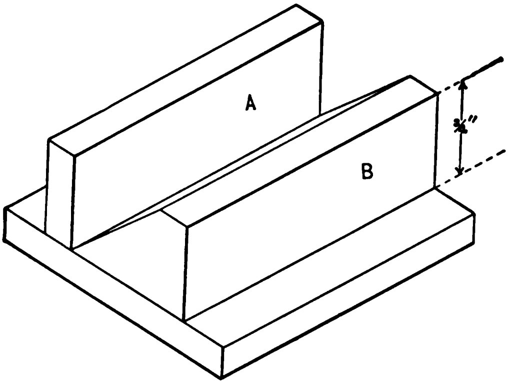


FIG. 138.

CHAPTER IX

SCREW CUTTING

SECTION 1. SHAPES OF THREADS. SECTION 2. SIMPLE-GEARED LATHE. SECTION 3. COMPOUND-GEARED LATHE.

§ 1. SHAPES OF THREADS

186. Definitions. There are two ways of forming a screw; either by spirally winding a thread around a cylinder as in Fig. 139, or by spirally grooving a cylinder, as in Fig. 140.

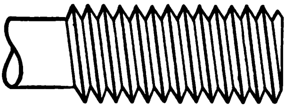


FIG. 139.

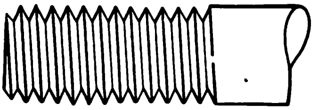


FIG. 140.

A cylinder threaded in either way is called a screw.

Screws are right-handed or left-handed. A **right-handed** screw is one which will enter a nut or other threaded hole when turned in a clockwise direction.

A **left-handed** screw is one which will enter a nut or other threaded hole when turned to the left or counter-clockwise.

A screw thread is cut by a die operated by hand or machine, or by a threading tool in a screw-cutting lathe.

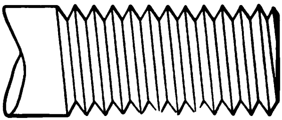
In this chapter only the latter method will be considered.

Following are the names applied to the different parts of the thread:

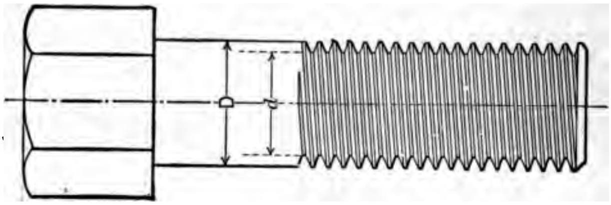
The **root** is the line or surface in which the threads meet at the bottom of the cut.

The **point** is the line or surface in which the threads meet at the top of the cut.

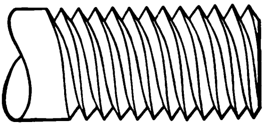
The **height** is the vertical depth of the cut. If a straight-edge were laid on the points of the thread, the depth of the thread would be the perpendicular distance from the bottom of the groove to the straight-edge. The double depth is twice this distance.



Single Thread.



Double Thread.



Triple Thread.

Fig. 111.

The **diameter** of a thread is the distance across the points of the thread, perpendicular to the axis. It is the diameter of the bolt on which the thread is cut.

The **root diameter** is the thread diameter minus the double depth.

A screw is either single-threaded or multiple-threaded.

A screw has a single thread when at each turn it advances a distance equal to the distance between the threads.

When at each turn it advances twice the distance a screw is double-threaded; when three times the distance, triple-threaded.

In order to determine whether a screw is single-threaded or double-threaded, pass the finger along the groove for one complete turn. If the finger is in the first groove from the starting point the screw is single-threaded; if the finger is in the second groove the screw is double-threaded, etc.

187. Pitch and Lead. The distance that a screw advances in one turn depends on two things:

(1) The number of threads to the inch.

(2) Whether it is single-, double-, triple-, or quadruple-threaded.

The **pitch** of a screw is the distance between the threads measured across the points, on a line parallel to the axis of the screw.

Pitch therefore equals the reciprocal of the number of threads per inch.

$$\text{Pitch} = \frac{1}{\text{number of threads per inch}}$$

The **lead** is the distance the screw advances in one revolution.

Lead therefore equals the reciprocal of the number of turns for one inch advance.

$$\text{Lead} = \frac{1}{\text{number turns for 1 inch advance}}.$$

The lead of a single-thread screw is therefore equal to the pitch.

If such a screw has 7 threads to the inch the pitch is $\frac{1}{7}$ inch and the lead is $\frac{1}{7}$ inch.

A double-thread screw has twice as many threads per inch as the turns required to advance it one inch. The lead of a double-thread screw is therefore twice the distance between the threads.

The lead of a triple-thread screw is three times the pitch. The lead therefore equals three times the distance between the threads.

188. Problems.

1. State how you would determine whether a screw is right-handed or left-handed.

2. How is the diameter of a thread measured?

3. Write a formula for root diameter.

4. State how you would measure the depth of a thread.

5. Take three screws at random from the desk. Are they double, single, or otherwise threaded?

Are they right- or left-handed?

6. What is the pitch of each of the screws in Problem 5? Why? Show formula and computation.

7. Select two other screws at random from the desk. What is the lead of each?

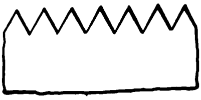
State exactly how it was determined?

8. A quadruple-threaded screw has 12 threads to the inch.

What is its pitch? Why?

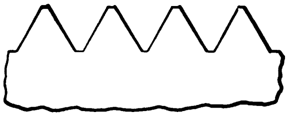
What is its lead? Why?

189. Thread Shapes. The threads in common use in the United States are as follows:



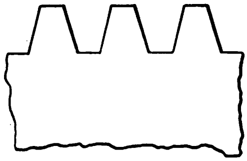
Sharp V Thread

FIG. 142.



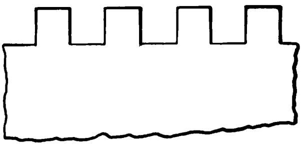
U.S. Standard Thread

FIG. 143.



29°Acme Thread

FIG. 144.



Square Thread

FIG. 145.

British Association, Whitworth, and metric screw threads are also used, but rarely, because it is difficult and expensive to keep the thread tools ground to the proper curves for cutting the Whitworth and the British Association, and the metric is avoided by manufacturers.

190. Sharp V Thread. This thread is frequently called the 60° V thread from the fact that its sides form an angle of 60° with each other as shown in the figure.

Since the sum of the angles of a triangle equals 180°, the sides of the thread not only make a 60° angle at the point, but form a 60° angle with the axis of the screw, so that the cross-section of the thread from point to root is an equilateral triangle.

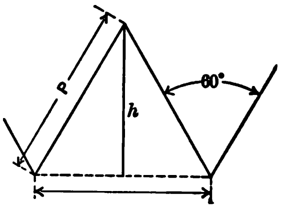


Fig. 146.

If therefore a perpendicular be drawn from the point of the thread it will bisect both the base line and the angle at the point.

Observe that *h* is the altitude of a right triangle and that the base of this triangle is ½ the pitch or $\frac{P}{2}$.

In a right triangle

(1) $\frac{\text{Side opposite an acute angle}}{\text{side adjacent}} = \text{the tangent of the angle.}$

(2) $\frac{\text{Side opposite an acute angle}}{\text{Hypotenuse}} = \text{the sine of the angle.}$

(3) $\text{Side opposite} = \sqrt{\text{hypotenuse}^2 - \text{adjacent side}^2}$.

In the figure,

P = hypotenuse;
h = side opposite 60° angle;
$\frac{P}{2}$ = side adjacent 60° angle.

Therefore the following *formulas for the depth of a* 60° *thread:*

$$\frac{h}{\frac{P}{2}}=\tan\ 60°,$$

(1) $$h=\frac{P}{2}\tan 60°,$$

$$\frac{h}{P}=\sin 60°,$$

(2) $$h=P\sin 60°,$$

(3) $$h=\frac{\sin 60°}{n},$$

(4) $$h=\sqrt{P^2-\frac{P^2}{4}}$$

(5) $$h=P\sqrt{.75}$$

Observe that (3) is the same as (2) except that $\frac{1}{n}$ is substituted for P, since the pitch of a screw equals the reciprocal of the number of threads to the inch.

In order to compute the depth of a thread by any of the five formulas, therefore, the number of threads per inch must be determined.

If a thread or pitch gage is not available for this purpose, a scale may be applied as shown in the cut.

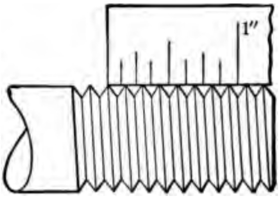


Fig. 147.

When the number of threads is not integral, but ends in a fraction, say $3\frac{1}{2}$, it is best to take the measurement for two inches instead of one, provided the length of the thread permits.

In case the thread is less than an inch long, count the number of threads or grooves in a half inch; twice this number is the number per inch.

191. Specification of Threads. When a blue-print calls for "6 threads per inch," single-thread is understood. If multiple thread is wanted, it must be specified by "6 threads per inch double," or "6 threads per inch triple," as may be required. If this were still more definitely stated as $\frac{1}{6}$ pitch, $\frac{1}{3}$ lead; or $\frac{1}{6}$ pitch, $\frac{1}{2}$ lead, there would never be a question whether "6 threads per inch double" might not mean 12 threads per inch, which it should not mean in any shop or drafting-room.

192. Size of Tap-Drill for a V Thread. When a V thread is cut in a nut or hole, the hole is said to be "tapped with a V thread." If you will think about it for a moment, you will understand why the size of drill for threading or tapping a nut equals the root diameter of the thread on the bolt on which the nut is to be used.

Now,

$$\text{root diam.} = \text{thread diam.} - \text{double depth};$$

if

$$d = \text{root diameter};$$
$$D = \text{thread diameter};$$
$$h = \text{depth},$$

then

$$d = D - 2h.$$

By this formula, the size of drill required is computed to the decimal of an inch.

The drill which should be used for the job must be selected to the nearest 16th or 32d or 64th, as the case may be, by reference to the table of decimal equivalents in the back of the book or as computed in Problem 2, paragraph 65.

193. Problems.

1. A bolt threaded with a 60° V thread has 8 threads to the inch.

Formulate and compute its pitch both in inches and millimeters.

2. Copy formulas (1), (2), and (3) from paragraph 190. Read the sine and the tangent in the table, and substitute the reading in the formulas.

The depth of a 60° V thread may be obtained by multiplying half the pitch by what number; or by multiplying the pitch by what number; or by dividing what number by the number of threads to the inch?

3. Compute the depth of the thread in Problem 1 by formula (4) of paragraph 190 and by each of the three formulas obtained in Problem 2.

4. A blue-print reads 9 threads per inch double. This means what pitch and lead? Why?

5. Copy the formula of paragraph 192, and in it substitute each value of h from the formulas which were used in Problem 3.

6. Formulate and compute the size of drill required for drilling a nut for a bolt having 5 threads per inch, taking the thread diameter from a table of V threads in a handbook.

7. By the simplest formulas compute to 3 decimal places, and fill in all omitted entries in the table below, in which

d = diameter of drill;
D = thread diameter;
n = number of threads per inch;
P = pitch.

TABLE 121

V-THREAD MACHINE SCREWS

No.	D	n	P	d	No.	D	n	P	d
1	.085	72			6		28		.157
2	.125	44			7		24		.166
3	.140	40			8	.270	22		
4	.165	36			9	.320	20		
5	.190	32			10	.375	16		

8. Copy formula (4) of paragraph 190. Perform the subtraction under the radical.

Extract the square root of $\frac{P^2}{4}$, leaving the other factor under radical.

Therefore $h = \frac{P}{2}\sqrt{\text{what number?}}$

Write the formula with the square root extracted to three decimal places. Is the formula the same or different from the tangent formula for depth?

Therefore tan 60° = $\sqrt{\text{what number?}}$

194. U. S. Standard Thread. There are three objections to the V thread:

(1) The weakening of the bolt because of the deep cut.
(2) The reduction of size by use.
(3) The easy breaking or other injury of the sharp edges.

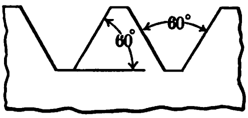


FIG. 148.

All these objections are overcome to a large extent in the U. S. standard thread here shown. This thread has the same angle as the V thread, but is flat at the point and at the root. The width of the flat both at the point and root in all U. S. standard threads is exactly one-eighth of the pitch.

To get a correct idea of this thread, imagine a V thread cut off parallel to the axis of the screw at a distance from the point equal to $\frac{1}{8}$ of the pitch measured directly down the side of the thread, and the cut-off piece fitted in at the root of the thread.

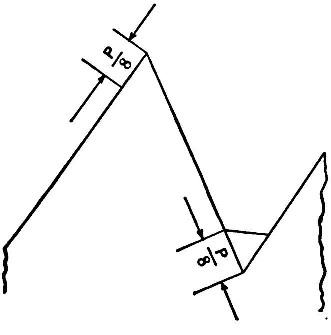


FIG. 149.

The side of the thread would then measure $\frac{1}{8}P$ less at the point plus $\frac{1}{8}P$ less at the root, or exactly $\frac{3}{4}P$ as illustrated P being the pitch.

195. Formulas for U. S. Standard Thread.

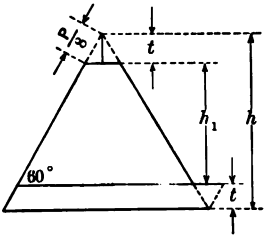


FIG. 150.

If

h_1 = depth U. S. st'd thread,
h = depth V thread,
t = distance cut off point and left uncut at root,
P = pitch,
n = number of threads per inch,
f = (width of) flat,

the formulas are as follows:

(1) $$P=\frac{1}{n},$$

(2) $$f=\frac{P}{8},$$

(3) $$f=\frac{1}{8n},$$

(4) $$t=\frac{P}{8}\sin 60°,$$

(5) $$t=\frac{P}{16}\tan 60°,$$

(6) $$h_1=\frac{.6495}{n},$$

(7) $$h_1=.6495P,$$

(8) $$h_1=\tfrac{3}{4}P\sin 60°,$$

(9) $$h_1=\tfrac{3}{8}P\tan 60.$$

196. Problems in U. S. St'd Thread.

1. Copy formulas (1) and (2), and in (2) substitute the value of P from (1).

Therefore how is formula (3) obtained from (2)?

2. Compute the flat on a U.S. standard thread having 12 threads to the inch.

3. Formulate in two ways and compute the distance the V point has been cut off to form a U. S. st'd thread with 30 threads to the inch.

4. Copy formula (2) of paragraph 190 and formula (8) of paragraph 195.

As shown by these two formulas, the depth of a U. S. st'd thread is exactly what part of the depth of a V thread of the same pitch?

5. A V thread has 24 threads per inch. Formulate and compute its depth.

Compute the depth of the same thread cut U. S. standard.

6. Study the figure. Observe that the depth of the U. S. standard thread equals h the depth of the V thread, minus $2t$.

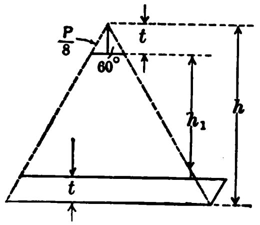


Fig. 151.

Draw the figure and formulate h_1 in terms of h and t.

But t is opposite a 60° angle, and therefore equals the hypotenuse $\frac{P}{8}$ times the sine of 60°.

Therefore $t = \frac{P}{8} \sin 60°$.

In the formula for h_1 substitute this value of t.

In paragraph 190, h equals what times sin 60°?

In the formula substitute this value of h.

7. If no mistake is made in the solution of Problem 6, the final formula reads:

$$h_1 = P \sin 60° - \frac{P}{4} \sin 60°$$

This may be written

$$h_1 = P(\sin 60° - \tfrac{1}{4} \sin 60°) \tag{10}$$

Copy both the formulas.

The expression within the parenthesis means that $\frac{1}{4}$ sin 60° is to be subtracted from sin 60°.

If the subtraction is performed, the difference is how many times sin 60°?

In (10) substitute this difference.

This problem shows how formula (8) in paragraph 195 is obtained.

8. Look at the figure in Problem 6. The base of one of the small right triangles at the point of the thread equals what part of the pitch?

Therefore by the law for hypotenuse

$$t=\sqrt{\frac{P^2}{64}-\text{what?}}$$

Show that this reduces to

$$t=\frac{P}{16}\sqrt{3}$$

9. Copy formula (8) in paragraph 195. Read sin 60° in the table, substitute the reading in the formula, and multiply it by $\frac{3}{4}$ as indicated.

Substitute the result in the formula.

Therefore in formula (7) .6495 equals what part of the sine of 60°?

10. A U. S. standard screw with a pitch of $\frac{1}{5}''$, makes 5 turns to the inch.

How is it threaded?

What is the depth of the thread?

11. Examine and measure not less than two of the U.S. standard screws at the desk.

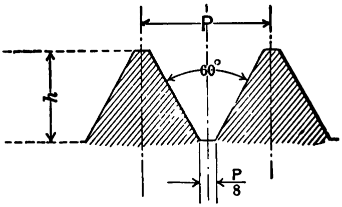


Fig. 152.

U. S. Standard.

Formulate and compute the pitch, lead, flat, and depth.

12. By reference to paragraph 192, formulate and compute the diameter of the tap drill which must be used in threading the hole for a 2¾ inch bolt with 4 threads per inch, U. S. standard.

13. By the use of the formulas complete the entries in the following table in which

n = number of threads per inch;
D = thread diameter;
h_1 = depth;
d = diameter of drill.

TABLE 122

U. S. STANDARD SCREWS

No.	D	n	h_1	d	No.	D	n	h_1	d
1	¼	20			11	2½	4		
2	⅜	16			12		3½		2.879
3	½	13			13		3		3.317
4	⅝	11			14		2⅞		3.798
5	⅞			.731	15	4½	2¾		
6	1⅛	7			16	4¾			4.256
7	1⅜			1.1585	17	5			4.480
8	1⅝			1.389	18		2½		4.730
9	1⅞	5			19	5¾	2⅜		
10	2¼			1.962	20	6			5.423

197. The Square Thread. This thread is correctly named for it has a square section, and the point, the width of the groove at top and bottom, the thickness at the root, and the depth of the thread, are supposed to be exactly the same, and therefore equal to half the pitch. Sometimes the depth is made .01 of an inch greater than the flat, but there are no standards for these or the other dimensions.

In the absence of standards, thepractice of manufacturers seems to be to make the depth and the width of the groove, or the space between the teeth, slightly greater than the other dimensions.

198. The 29° Thread. There are three 29° threads:

Brown & Sharpe 29° thread,
Acme 29° tap thread,
Acme 29° screw thread.

In its proportions none of these is exactly like the others of the same size and pitch, nor is the angle of the thread as cut with standard dimensions always exactly 29°.

The thread dimensions are figured to the 10-thousandth of an inch, and if manufacturers uniformly increased the 10-thousandths figure by 1 when the 100-thousandths figure is 5 or greater, the variation of angle from exact 29° in some instances would be lessened.

It is evident that a dimension which is actually, say .39218″, should not be cut .3921″; yet such is the practice in some cases.

199. The Acme 29° Screw Thread. A coarse pitch thread which is largely used in place of the square thread and the V thread, is the Acme or 29° thread, so called because its sides are at an angle of 29° with each other.

The height of the thread in the Acme and in the square thread screw is the same for the same pitch.

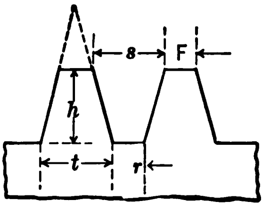


Fig. 153.

In the formulas for the different parts the following notation is used:

W_t = width of point of tool for cutting Acme thread;
F = width of point of screw thread;
h = depth of thread;
P = pitch of screw;
D = diameter of screw (over points of thread);
t = thickness of thread at root;
d = root diameter of screw;
s = width of space between threads at point;
n = number of threads per inch.

$$W_t=\frac{.3707}{n}-.0052, \tag{1}$$

$$h=\frac{1}{2n}+.01, \tag{2}$$

$$d=D-(P+.02), \tag{3}$$

$$F=\frac{.3707}{n}, \tag{4}$$

$$t=.6293P+.0052, \tag{5}$$

$$s=.6293P, \tag{6}$$

$$r=.3707P-.0052. \tag{7}$$

200. Problems in Acme Screw Thread.

1. The depth of an Acme screw thread is .1350.

Formulate and compute the number of threads per inch.

2. A bolt is to have an Acme thread with 8 threads to the inch.

Formulate and compute the width of the point of the tool for cutting it.

3. Copy formula (1) and formulate W_t in terms of P instead of n.

4. An Acme screw thread has a depth of .0655″.

Formulate and compute the pitch.

5. A $\frac{1}{2}$-inch bolt has an Acme thread of $\frac{1}{9}$ pitch.

Formulate and compute the root diameter.

6. A 2″-bolt is threaded with an Acme thread of $\frac{1}{5}$″ pitch.

Compute everything formulated in paragraph 199.

7. Complete the entries in the Table below.

TABLE 123

29° OR ACME SCREW THREAD

No.	n	W_t	t	h	No.	n	W_t	t	h
1	3				6			.2150	
2				.1767	7		.0689		
3			.0951		8				.0725
4		.0875			9	7			
5	9				10				.0600

201. Acme 29° Tap Thread. The Acme tap thread has all dimensions exactly the same as the Acme screw thread except the width of the space between threads at the point, which is exactly the same as the thickness of the thread at the root.

Write the seven formulas for the Acme tap thread, writing as formula (6)

$$d_t = D + .020'',$$

in which d_t = diameter of tap,

and D = diameter of screw.

202. Brown & Sharpe 29° Worm Thread. The formulas for this thread are as follows, the notation being shown on the figure:

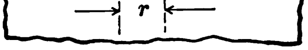


FIG. 154.

$$h = .6866P,$$

$$F = .335P,$$

$$r = .31P,$$

$$A = .3183P,$$

$$T = .5P,$$

$$c = \frac{T}{10},$$

$$s = .665P,$$

$$t = .69P.$$

203. Problems in B. & S. 29° Worm Thread.

1. Compute and tabulate the omitted entries in the following table in which the notation is the same as in the formulas of the preceding paragraph, n denoting the number of threads to the inch:

TABLE 124

B. & S. 29° WORM THREAD

n	P	h	F	r	A	T	c	s	t
2									
3½									
4									
7									
9									
16									
20									
8									
5									
1½									

204. Whitworth Standard Thread. The British standard thread is the Whitworth. The angle of the sides with each other is 5° less than in the V and U. S. standard threads, so that the angle is 55° instead of 60°.

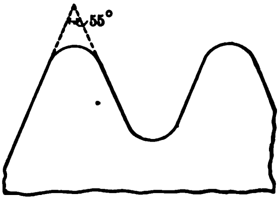


FIG. 155.

The point of this thread is rounded off for a distance exactly equal to one-sixth the total depth the thread would have if cut a 55° V thread instead of a Whitworth.

The root is rounded in a similar manner, the radius of both point and root arcs being the same.

Because of its rounded point and root and greater root diameter, this thread is obviously preferable to both the V thread and the U. S. standard. It is, however, more expensive to make because of the shape of the tool required for cutting it, and the difficulty of keeping the tool properly ground.

205. Formulas for Whitworth Thread. In the formulas below, the notation is as follows:

t = thickness of 55° V thread (at root);
d = depth 55° thread if *not* rounded off at point and root;
v = distance from point of 55° V to point of Whitworth;
v = distance from root of 55° V to root of Whitworth;
D = diameter bolt over thread;
D_r = root diameter of thread;
h = depth of Whitworth standard thread;
r = radius of point and root;
P = pitch;
n = number of threads per inch.

$$t=P=\frac{1}{n}, \tag{1}$$

$$d=\frac{P}{2\tan 27\tfrac{1}{2}^\circ}, \tag{2}$$

$$d=\frac{P\cot 27\tfrac{1}{2}^\circ}{2}, \tag{3}$$

$$d=.96049P, \tag{4}$$

$$r=.1373P, \tag{5}$$

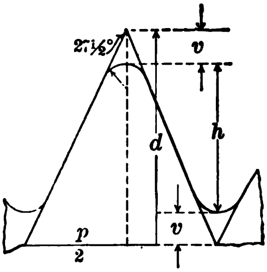


FIG. 156.

$$v=\frac{d}{6}, \tag{6}$$

$$h=P\left(.96049-\frac{1}{6}\cot 27\tfrac{1}{2}^\circ\right); \tag{7}$$

$$h=.96049P-\frac{d}{3}, \tag{8}$$

$$h=.64033P, \tag{9}$$

$$D_r=D-2h. \tag{10}$$

206. Problems in Whitworth Thread.

1. A screw has 8 threads to the inch, Whitworth standard. What is the lead and the pitch?

2. State how a Whitworth thread differs from a U. S. standard thread.

3. The total distance rounded off the point and left uncut at the root of a Whitworth thread, equals exactly what part of the depth if cut 55° V thread?

4. By each of the three formulas compute the depth from the V point to the V root, of the thread in Problem 1.

5. Compute the radius of the point of a Whitworth thread having 16 threads to the inch.

6. Copy formula (10) and substitute for h from (7), (8), and (9).

7. A $\frac{3}{16}''$-bolt has a Whitworth thread with 24 threads to the inch.

Compute the depth of the thread and the root diameter.

8. Copy formula (2), read the tangent in the 6-place table, substitute and simplify.

9. Copy formula (3), read the cotangent in the 6-place table, and substitute and simplify.

10. Simplify formula (7) in the same way as specified in Problem 8. Is the result the same as formula (9), or different? Why?

11. Formulate, compute, and tabulate the omitted entries as assigned in the following table in which the notation is the same as in paragraph 205. Make each computation by all the formulas which apply.

TABLE 125

WHITWORTH SCREW THREADS

D	n	P	r	v	d	h	D_r
$\frac{5}{16}$	18						
$\frac{3}{8}$	16						
$\frac{1}{2}$	12						
$\frac{5}{8}$	11						
$\frac{13}{16}$	10						
$1\frac{1}{8}$	7						
$1\frac{3}{8}$	6						
$1\frac{7}{8}$	$4\frac{1}{2}$						
$2\frac{1}{2}$	4						
$2\frac{3}{4}$	$3\frac{1}{2}$						

207. British Association Screw Thread. This thread has the same form as the Whitworth but the angle is 7½° less and the dimensions are different.

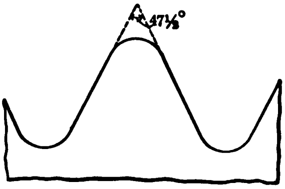


FIG. 157.

The sides of the thread are at an angle of 47½° and the point is rounded off for a distance below the 47½° sharp V point equal to approximately .268 of the pitch.

The root is rounded off for the same distance above the 47½° sharp V root, and the depth equals exactly three-fifths of the pitch.

This thread, which was first designed and used in the watch factories of Switzerland, is the English standard for small-sized screws, but the difficulty of forming the cutting tools for it and the increased difficulty of grinding, have practically prevented its use in the United States where watch screws are threaded 60° or 45° sharp V, depending on the material.

208. Formulas for British Association Thread.

The formulas below are written in the following notation:

d = depth of thread if *not* rounded off at the point and root, but cut 47½° sharp V;
v = distance rounded off at top and bottom;
P = pitch;
n = number of threads to the inch;
h = depth of thread;
r = radius point and root;
$\theta = 47\frac{1}{2}°$ = thread angle.

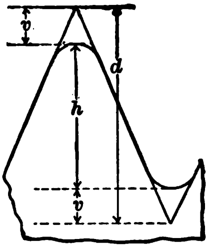


FIG. 158.

(1) $$d = \frac{P}{2 \tan \frac{\theta}{2}},$$

(2) $$h = .6P,$$

(3) $$r=\frac{2}{11}P,$$

(4) $$v=\frac{P}{2}\left(.5\cot\frac{\theta}{2}-.6\right).$$

209. Problems in British Association Thread.

1. Compute the number of threads per inch and the depth of a British Association thread having a pitch of .0028″.

2. Using the same pitch as in Problem 1 compute the distance that the point and root are rounded off.

3. Compute *d* for the thread in Problem 1, from the results in Problems 1 and 2.

4. Copy formula (1) and compute *d* for the thread in Problem 1.

5. The proportions of the British Association thread as designed and used by the Swiss, were expressed in metric units.

Compute and fill in such omitted entries in the following table, as may be assigned, carrying the results to the nearest 10-thousandth of an inch.

TABLE 126

BRITISH ASSOCIATION STANDARD THREAD

Thread No.	Diameter.		Pitch.		Thread No.	Diameter.		Pitch.	
	M/m.	Inches.	M/m.	Inches.		M/m.	Inches.	M/m.	Inches.
0	6.0		1.0		11	1.5		0.31	
1	5.3		0.90		12	1.3		0.28	
2	4.7		0.81		13	1.2		0.25	
3	4.1		0.73		14	1.0		0.23	
4	3.6		0.66		15	0.90		0.21	
5	3.2		0.59		16	0.79		0.19	
6	2.8		0.53		17	0.70		0.17	
7	2.5		0.48		18	0.62		0.15	
8	2.2		0.43		19	0.54		0.14	
9	1.9		0.39		20	0.48		0.12	
10	1.7		0.35		21	0.42		0.11	

210. International Standard Thread. There are two standard metric threads, the French and the international. Both have the sides at an angle of 60° with each other and therefore inclined 60° to the axis of the screw.

They are flattened at the point for a distance equal to one-eighth of the height of a V thread and are supposed to be flatted at the root for a distance equal to one-eighth of the height if cut a V thread.

The thread is therefore of the same form as the U. S. standard except that by recommendation of the International Congress the root is rounded instead of flatted, at the discretion of the manufacturers.

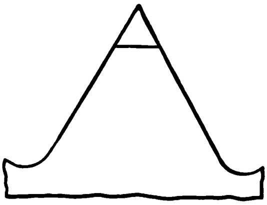


Fig. 159.

With the exception of the root when rounded, the formulas for international standard thread are the same as for the U. S. standard for which see paragraph 195.

When the root is left flat, the flat equals one-eighth of the pitch.

211. Problems in International Standard Thread.

1. Copy the formulas of paragraph 195.

2. A bolt 60 millimeters in diameter has a thread with a pitch of 5.5 millimeters.

Compute the depth of the thread and its root diameter, assuming the thread to be flatted and rounded in standard proportions.

3. Compute and tabulate the omitted entries below, as assigned, carrying results to the nearest 10-thousandth.

Table 127

INTERNATIONAL STANDARD THREADS

Diameter.		Pitch.		Diameter.		Pitch.	
M/m.	Inches.	M/m.	Inches.	M/m.	Inches.	M/m.	Inches.
6		1		36		4	
9		1.25		45		4.5	
12		1.75		52		5.	
22		2.5		56		5.5	
30		3.5		72		6.5	

4. By substitution in the formulas of Problem 1 complete the entries in the following table in which D, P, and d are in millimeters: (For notation, see paragraph 199.)

TABLE 128

INTERNATIONAL STANDARD THREADS

No.	D	P	d	No.	D	P	d
1	6	1		6	42	4.5	
2	9	1.25		7	39	4	
3	10	1.5		8	60	5.5	
4	11	1.5		9	68	6	
5	14	2		10	27	3	

212. Distribution of Stretch. The point of greatest weakness in a bolt is where the thread merges into the body. This is due to the fact that the body, being of greater diameter, will not stretch uniformly with the thread part of the bolt. In order to overcome this, a hole is sometimes drilled from the center of the head to the beginning of the thread, of such size as to make the cross-sectional area of the body the same as that at the root of the thread, thereby equalizing the stretch throughout the entire length of the bolt.

Compute the diameter of the hole for distributing the stretch in

(1) U .S. Standard thread bolt,
(2) Whitworth Standard thread bolt,
(3) Metric screw thread bolt,
(4) Acme thread bolt.

Determine diameters from the thread tables in any handbook.

§ 2. SCREW CUTTING ON A SIMPLE-GEARED LATHE.

213. Revolution of Spindle and Leadscrew. The figure shows a screw-cutting lathe.

When a screw thread is to be cut the split nut attached to the carriage is closed so as to engage the leadscrew.

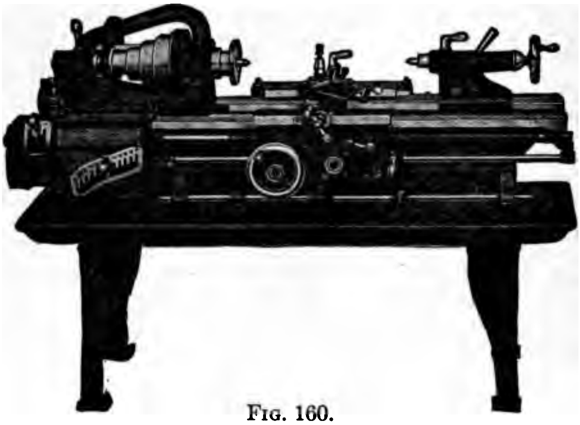


FIG. 160.
(Courtesy of The Hendey Machine Company.)

At each turn of the leadscrew the carriage accordingly moves a distance equal to the pitch of the leadscrew.

By the definition of pitch given at the beginning of the chapter, a leadscrew having 8 threads to the inch has a pitch of $\frac{1}{8}''$. A carriage operated by such a screw moves $\frac{1}{8}$ of an inch at one revolution of the screw.

If a bolt on which a thread is to be cut is placed in a lathe so as to revolve with the spindle, and the lathe is so geared that the leadscrew revolves the same number of times as the spindle, the number of threads to the inch which are cut on the bolt by the tool attached to the carriage will exactly equal the number of threads to the inch on the leadscrew. Hence in a lathe so geared, 8 threads will be cut to the inch when the leadscrew has 8 threads to the inch, and 6 threads will be cut to the inch when the leadscrew has 6 threads per inch.

It will be evident that the number of threads per inch which may be cut, can differ from the number of threads per inch on the leadscrew only when spindle and leadscrew revolve at different rates.

Therefore if 16 threads per inch are to be cut with a $\frac{1}{8}$ pitch leadscrew, the spindle must make two turns while the leadscrew makes one; if 28 threads per inch are wanted, the spindle must turn three and one-half times as fast as the $\frac{1}{8}$ pitch leadscrew.

214. Change-gears on a Simple-geared Lathe. A lathe is simple-geared when there is only one change of speed between stud and leadscrew gears, that is between *C* and *L*.

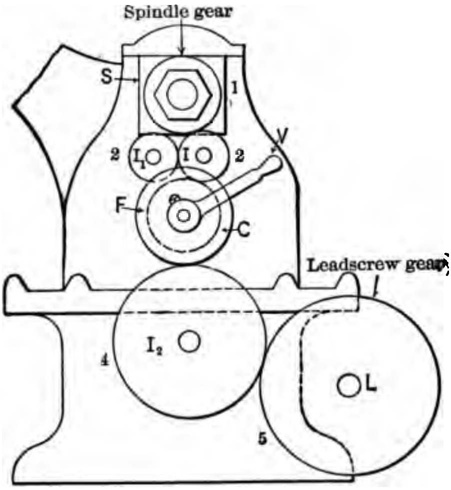


Fig. 161.

A study of the figure will show how this change is accomplished.

S is the spindle gear which revolves with the spindle.

I and I_1 are two idler gears employed for turning the leadscrew in either direction.

C and F are the gears on the stud, both being keyed to a hollow sleeve and revolving as one gear. It is a convenience to regard F as the fixed gear on the stud sleeve and C as the change gear.

I_2 is another idler used only for transmitting the revolution of C to L, the leadscrew gear.

215. Effect of Idlers. When the lever V is in the position shown, both spindle and leadscrew will turn in the same direction. For example, suppose S to revolve clockwise. I will turn counter-clockwise and F and C clockwise. I_2 will therefore turn counter-clockwise and L clockwise, that is, in the same direction as the spindle.

Considering the gears as numbered on the figure, the odd-numbered gears will turn clockwise; the even-numbered, counter-clockwise.

If the lever V is pressed down so as to release I and to engage I_1 with S, the motion of S will no longer be transmitted to I direct, but will be transmitted through I_1. The motion of I and therefore of the leadscrew will consequently be reversed, and spindle and leadscrew will turn in opposite directions.

Hence, with V up or down, the idlers I and I_1 do not affect the speed of the leadscrew relatively to the spindle, but determine whether spindle and leadscrew shall turn together or oppositely.

216. Effect of the Change-gears. If a lathe has gears S and F of the *same size,* they will evidently revolve at the same speed.

C is keyed to the same sleeve as F and will also revolve at the same speed as F and the spindle.

Suppose now, that C has 24 teeth and that L has 72 teeth. C and therefore the spindle will make three revolutions while L and the leadscrew make one. If C has 48 teeth and L has 96, the leadscrew will make one revolution in two revolutions of the spindle. In other words, the relation between the rotations of leadscrew and spindle depends on, and is the same as, the relation between the number of teeth on spindle and leadscrew gears.

If L has a greater number of teeth than C, it will turn slower; if it has a less number, it will turn faster than C.

The number of turns of spindle and leadscrew and accordingly the number of threads per inch which may be cut, are therefore dependent on the number of teeth in stud and leadscrew gears; hence the name *change gears.*

217. Notation. In the problems on the simple-geared lathe the following notation is used:

L = the number of teeth in the leadscrew gear;
C = the number of teeth in the change gear on the stud;
F = the number of teeth on the fixed gear on the stud;
S = the number of teeth on the spindle gear;
T_L = the number of threads per inch on the leadscrew;
T_S = the number of threads per inch which are to be cut.

Observe that the notation is significant, L and C being the initial letters in leadscrew and change, respectively, T_L and T_S signifying leadscrew threads and threads to be cut on screw, and S and F signifying spindle gear threads and fixed stud gear threads.

218. Standard Gears. There are two series of standard gears: one increasing by a common difference of 7, the range being from a gear of 21 teeth to a gear of 105 teeth; the other increasing by a common difference of 4, the range being from a gear of 24 teeth to a gear of 120 teeth. In other words, one series begins with a 21 gear and increases by 7 to a 105 gear; the other series begins with a 24 gear and increases by 4 to a 120 gear.

219. Problems.

1. A lathe having a leadscrew with 6 threads per inch is equipped as follows:

$$L = 96, \quad C = 48, \quad S \text{ and } F \text{ are equal.}$$

Compute the number of threads which will be cut on the screw when the carriage has moved two inches.

2. Determine the number of threads per inch which will be cut by the lathe in Problem 1 when $L=72$, and $C=24$; also when $L=40$ and $C=40$.

3. A leadscrew has 4 threads per inch. What gears must be placed on stud and leadscrew in order to cut a U. S. standard thread on a bolt $1\frac{7}{8}$ inches in diameter?

4. Under the heading, Standard Gears, write out all the gears in the two series described in paragraph 218, denoting gears by their number of teeth, as 21, 28, 35, etc.

220. Simple Method for Determination of Gears. Suppose a lathe has a leadscrew with 4 threads to the inch and that 12 threads to the inch are wanted on a bolt. The threads on bolt and leadscrew are in the ratio of 12 to 4, and the spindle must therefore turn three times as fast as the leadscrew.

If it were possible, this ratio in speeds might be produced by putting a gear with 12 teeth on the leadscrew and a gear with 4 teeth on the stud. But a gear of 4 teeth is impossible; it is therefore necessary to select from the gears furnished with the lathe, two gears which are the same multiple of 4 and 12 and are therefore in the same ratio as 4 and 12.

By reference to the standard gears tabulated in Problem 4 you will observe that the first gear in the first series, into which 4 will divide without a remainder, is 28 which is 7 times 4. Multiplying the denominator also by 7 we have 84. Therefore if a 28 gear is placed on the stud and an 84 gear on the leadscrew the speeds of the spindle and leadscrew will be in the ratio of 28 to 84, or 4 to 12, or 1 to 3.

$$\frac{4\times7}{12\times7}=\frac{28}{84}.$$

Stud gear $=S=28$
Leadscrew gear $=L=84$

By the method just illustrated determine the gears required to cut 24 threads per inch with a lathe whose leadscrew has 10 threads to the inch.

221. Formula for Change-gears when S and F are Equal. The preceding work fully explains the following formula in which

T_S = number of threads to be cut;
T_L = number of threads on the leadscrew;
L = number of teeth in leadscrew gear;
C = number of teeth in stud change-gear.

$$\frac{T_S}{T_L}=\frac{L}{C}.$$

This formula can be used for determining the change gears on any single-geared lathe on which *spindle gear and stud fixed gear have the same number of teeth.*

222. Problems.

1. By substitution in the formula determine the gears for stud and leadscrew when 14 threads per inch are to be cut with a leadscrew whose pitch is $\frac{1}{8}''$.

2. The leadscrew is $\frac{1}{6}$ pitch, the screw is to be cut $\frac{1}{16}$ pitch, the change gear on the stud has 36 teeth.

What gear must be placed on the leadscrew?

3. The leadscrew has a 105 gear and 6 threads per inch. What gear on the spindle will cut 32 threads per inch?

4. A lathe having an 88 gear on the leadscrew and a 32 gear on the stud, cuts 22 threads per inch.

What is the pitch of the leadscrew?

5. A lathe with a leadscrew having 4 threads per inch, has gears on leadscrew and stud of the same number of teeth. If it cuts 8 threads per inch, what is the ratio of the number of teeth on the fixed stud gear to the number of teeth on the spindle gear?

223. Formula for Change Gears when Spindle Gear and Fixed Stud Gear are of Different Sizes. In order to select the change gears for stud and leadscrew it is necessary to know the ratio of the number of teeth on the spindle gear to the number of teeth on the fixed stud gear. On some lathes this may be determined by counting, but on many the stud gear is concealed in the headstock. When this is the case, some other means than counting must be resorted to.

Write a statement specifying how you would determine the ratio of the spindle gear to the fixed stud gear when the construction of the lathe makes counting impossible.

The following formula can be used for determining the change gears on *any single-geared* lathe:

$$\frac{T_S S}{T_L F}=\frac{L}{C}.$$

It will be observed that when $S=F$, these two quantities cancel each other and the formula becomes the same as that in paragraph 221.

224. Problems. Solve the following problems by substitution in the formula of the preceding paragraph.

1. Determine the change gears for cutting 24 threads to the inch with a leadscrew having 6 threads to the inch, when spindle gear and fixed stud gear have 16 and 32 teeth respectively.

2. The spindle gear has 20 teeth, the fixed stud gear 60 teeth, the leadscrew has 10 threads per inch.

What change gears are required to cut 8 threads per inch?

3. Under the conditions named in the preceding problem could a 24 gear be used on the leadscrew?

Show all work upon which your answer is based.

§ 3. SCREW CUTTING ON A COMPOUND-GEARED LATHE

225. Change Gears on a Compound-geared Lathe. The drawing shows the arrangement of the gears on a compound-geared lathe.

A lathe is compound-geared when there are *two changes of speed* between stud and leadscrew gears, that is, between C and L.

A study of the figure will show how these two changes in speed are produced.

It will be noticed that the motion of *C* is not transmitted to *L* through an idler gear as in the simple-geared lathe, but that *C* engages with a gear *D*, and *L* with a gear *E*, both *D* and *E* being keyed to the same sleeve and therefore revolving together.

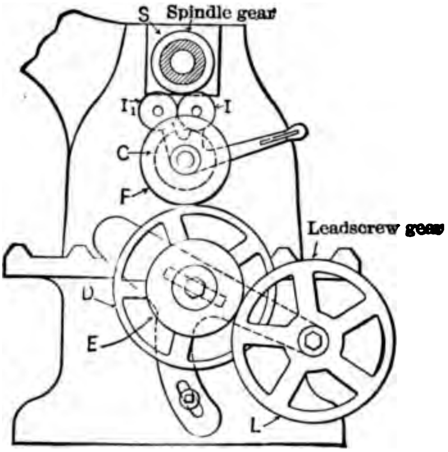


FIG. 162.

Owing to their difference in size, *C* and *D* revolve at different linear velocities, as do also *E* and *L*, thereby giving *two* changes of speed between *C* and *L*.

The range of screw cutting is much greater on a compound-geared lathe than on one having only one change of speed between stud and leadscrew gears. This is due to the fact that on a compound-geared lathe there are four change gears.

These gears may be determined by two different methods:

(1) Compound ratio.

(2) Formula.

226. Change Gears by Compound Ratio. Suppose it is required to cut 4 threads per inch with a compound-geared lathe whose leadscrew has 6 threads per inch, and on which the spindle gear and the fixed stud gear are of the same size.

First form the following fraction, cancelling S and F because they are equal to each other.

$$\frac{{}^{*}T_S S}{T_L F}.$$

Substituting the numbers given for T_S and T_L, we have the fraction $\frac{4}{6}$.

Regarding this as a compound ratio, we resolve or separate it into two ratios whose product equals $\frac{4}{6}$, as follows:

$$\frac{4}{6}=\frac{4}{3}\times\frac{1}{2}=\frac{D}{C}\times\frac{L}{E}.$$

The two fractions obtained by resolving the compound ratio $\frac{4}{6}$ show that in order to cut the number of threads required, we place on the lathe

two gears D and C in the ratio of 4 to 3,

and

two gears L and E in the ratio of 1 to 2.

We now inspect the change gears in the set furnished with the lathe and find the following gears:

18, 24, 36, 42, 48, 54, 60, 72, 96.

From these, the four gears may be selected as shown in paragraph 220.

227. Problems.

1. Copy the literal fraction in the preceding paragraph; show data, cancellation, substitution, and resolution.

Compute the different change gears which may be used from the set, in order to cut the required number of threads.

2. Make a different resolution of the compound ratio $\frac{4}{6}$, and compute the change gears which may be used from the set in paragraph 226.

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* For notation see paragraph 217.

3. Determine whether it is possible to resolve $\frac{4}{6}$ so that the required number of threads cannot be cut with the lathe set of gears.

228. Drivers and Driven. In a train of gears as illustrated in the cut in paragraph 225, some gears are drivers, others are driven.

Thus *S* drives *F*, *C* drives *D*, and *E* drives *L*. In the determination of drivers and driven, idlers are disregarded.

The gear train of a compound-geared lathe therefore has the following gears:

Drivers, *S*, *C*, *E*,
Driven, *F*, *D*, *L*.

229. Problems.

1. Tabulate drivers and driven in Problems 1 and 2 in paragraph 227.

By trial on the lathe, or by reading and inquiry, or both, determine if it makes any difference whether the smaller or the larger of the two drivers is placed on the stud to engage with *D*.

2. With the set of gears in paragraph 226, compute and tabulate as drivers and driven, all the gears which may be used to cut 12 threads per inch when the leadscrew has 2 threads per inch and *S* equals *F*.

3. Under the same conditions as in the preceding problem determine the gears which are necessary for $3\frac{1}{2}$ threads per inch.

230. Formulas for Change Gears in a Compound-geared Lathe. The immediately preceding paragraphs explain the following formulas for the change gears on any compound-geared lathe.

$$\frac{T_S S}{T_L F}=\frac{DL}{CE}, \tag{1}$$

$$\frac{T_S}{T_L}=\frac{FDL}{SCE}. \tag{2}$$

Formula (2) which is only formula (1) multiplied by $\frac{F}{S}$, is the expression in symbols, of the following

LAW FOR COMPOUND GEARING. The number of threads to be cut to the inch, divided by the number of threads to the inch on the leadscrew, equals the product of all the driven gears divided by the product of all the drivers.

231. Problems in Compound Gearing.

1. Determine the change gears when

$$T_S = 1\tfrac{1}{2},$$
$$T_L = 4,$$
$$S = 24,$$

and

$$F = 72,$$

assuming the same set of gears as in paragraph 226.

2. By inspection or inquiry, determine the gears in the set with which any compound-geared lathe in the shop is equipped, and compute all the change gears which will make it possible to cut 24 threads per inch, if the leadscrew has 6 threads per inch and spindle and fixed stud gears are in the ratio of 1 to 2.

3. Determine the actual number of threads to the inch and the actual ratio of spindle and fixed stud gears on the lathe used in Problem 2, and compute the change gears which may be used to cut 24 threads per inch; also the gears for 3½ threads per inch.

4. Determine the gears which may be used on one of the compound-geared lathes in the shop, to cut the following:

1. Any two Sellers' threads.
2. Any two Whitworth threads.
3. Any two metric threads.
4. Any two Acme threads.
5. Any two 60° V threads.

Make your selection of threads per inch from the screw tables in some standard handbook.

Specify number of lathe and the manufacturer.

5. Compute and tabulate the omitted entries below:

TABLE 129

COMPOUND-GEARED LATHE

No.	T_S	T_L	S	F	D	L	C	E
1	18	6	1	1				
2	14	6	1	1				
3	7	10	1	1				
4	5	6	1	1				
5	3½	6	1	1				

6. Make the entries in the following table, for the same lathe and gear set as in Problem 2, showing all computations.

TABLE 130

A COMPOUND-GEARED LATHE

No.	T_S	T_L	S	F	D	L	C	E
1								
2								
3								
4								
5								
6								

232. Gear Plates. On most screw-cutting lathes the gears which must be used in cutting different threads per inch are tabulated on a plate attached to the headstock or gear-box.

On modern lathes the gears are concealed in a gear-box and are thrown into mesh in their various combinations by knobs or pins whose manipulation is indicated on the lathe plate. The only objection to this method of shifting the change-gears is the expense of the repairs necessitated by careless workmen who attempt to shift the gears *while the lathe is in motion!*

In a shop equipped with modern screw-cutting lathes, the first thing to learn is to *change gears only when the lathe is not running.*

233. Problems.

1. Following are gear plates from different lathes, with some entries omitted.

Rule each plate in your work-book and complete the entries by computation, showing all work in full.

Determine by inspection whether any of the lathes in the shops have plates corresponding to those here given. In a sentence or two state the result of your inspection, specifying the manufacturer and the number of lathe.

Determine also whether the lathes mentioned have the full set of gears called for on the gear plate.

Under each of the plates in the work-book specify the number of threads per inch on leadscrew and show how determined.

1

Cut	Stud	Scr.
2	60	24
3	60	36
4		48
	40	40
6		48
	40	56
8	40	
9		72
	20	40
11		
	20	48
	20	52
14		
16		
	20	68
18		

2

Cut	Stud	Scr.
2	72	24
	48	24
4		24
5	48	40
	24	24
7		28
8	24	
	24	36
	24	40
11		
	24	46
12		
	24	52
14		
	24	60
16		
	24	68
18		

3

Cut	Stud	Scr.
3	48	24
4		
	48	40
6		
7		
	48	64
9		
10		
11		
	24	48
13		
	24	56
	24	60
16		
18		
Compound		
20	24	40
22		44
24	24	
	24	52
	24	56
30		
	24	64
36		

2. Make a drawing of the plate on lathe No. 21 or No. 52, or some other lathe if none in the shop has these numbers, and explain the mathematics of its operation, and its use, together with such other drawings as may be necessary to show the manipulation of the knobs or levers indicated by the plates.

3. Make a drawing of the plate on lathe No. 20, or some other if no lathe in the shop has this number, and explain its use and the mathematics of its operation, together with such other drawings as may be necessary to show the manipulation of the knobs or levers indicated by the plates.

4. By the formula of paragraph 230 compute the compound gears for use in connection with plate 3. The entries may be determined by observing on the plates the ratio of the threads cut compound for any set of stud and leadscrew gears, to the threads cut simple for the same combination of gears.

If you do not understand the preceding statement, try to think it out by observing the relation of the entries from 20 to 36 in the first column of plate 3, *under* the word compound, and the entries from 10 to 18, in the first column *above* the word compound.

5. Make a drawing of the plate on lathe No. 11 or some other and explain the mathematics of its operation, and its use, together with such other drawings as may be necessary to show the manipulation of the knobs or levers indicated by the plates.

6. Make a drawing of the plate on lathe No. 19 or some other in the shops and show the mathematical computation for the entries.

7. Make a drawing of the plate on lathe No. 17 or some other in the shops and show the mathematical computation for the entries.

8. Make a drawing of the plate on lathe No. 18 in the shops and show the mathematical computation for the entries.

9. Make a drawing of the plate on lathe No. 15 or 16 in the shops and show the mathematical computation for the entries.

10. Make a drawing of the plate on lathe No. 51 or some other in the shops and show the mathematical computation for the entries.

11. State exactly how to determine from a gear plate the pitch of the leadscrew, and the ratio of the compound gears from a plate like plate 3 in Problem 1.

234. Multiple Threads. A bolt is multiple threaded when the number of threads to the inch is a multiple of the number of turns required to advance the bolt or nut through one inch. When this multiple is 2, the bolt is double-threaded; when 3, triple-threaded; when 4, quadruple-threaded.

The advantage of a multiple thread is its decreased depth for a given pitch, and the consequently greater strength of the bolt.

The figure below shows how a double thread is cut.

A groove of the required pitch is first cut around the screw to such a depth that the width of the cut is exactly equal to the distance between cuts.

The work is then given a half turn, either by turning the dog over, or by disconnecting the stud gear and giving the spindle a half revolution, the half being determined by a mark placed on the gear by the operator. At the end of the half turn, the point of the tool will be opposite the center of the distance between the cuts. The gear is then connected and the double thread finished.

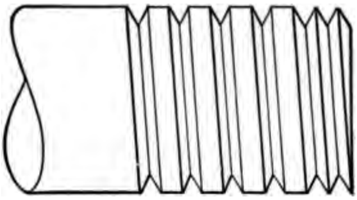

FIG. 163.

A triple thread is cut in a similar manner except that the first cut is made so that the distance between the cuts is exactly twice the width of the cut.

After the gear has been disconnected, and the spindle given one-third of a revolution, a second cut is taken and a third in like manner.

It will be observed that a half revolution and a third of a revolution are possible as described only when the number of teeth in the stud gear is divisible evenly by 2 and 3 respectively.

When the lathe set does not include such a gear, either the required gear is cut, or a specially prepared dog and a face plate accurately slotted for receiving the tail of the dog at the half, third, quarter, or such other multiple as may be required, are used.

235. Problems.

1. Write detailed, numbered directions for cutting a double-thread screw of $\frac{1}{7}$ pitch, on a lathe equipped with plate 2 as given in paragraph 233.

Could this thread be cut with an ordinary face plate by turning the dog over?

Give reason for your answer.

2. State the difference in procedure from that in Problem 1 in order to cut a triple-threaded screw of the same pitch on the same lathe.

3. State exactly how you would direct one not knowing the business to cut a quadruple-thread screw of $\frac{1}{3}$ pitch on a lathe equipped with plate 3.

CHAPTER X

PULLEYS

SECTION 1. RIM VELOCITY. SECTION 2. WEIGHT OF A PULLEY. SECTION 3. VELOCITY RATIO OF PULLEYS. SECTION 4. CENTRIFUGAL TENSION. SECTION 5. BACK GEARS.

§ 1. RIM VELOCITY

236. Velocity in Terms of Radius. In the work-book draw a pulley whose circumference measured to the outside of the rim is C and whose radius is R.

When the pulley has made one complete revolution, any point on the rim has moved through a distance equal to how many times the length of the radius?

When it has made 5 revolutions, the point has moved through a distance equal to how many times the radius?

The circumferential or rim velocity of a pulley is the distance through which a point on the rim moves per time unit.

If V denotes rim velocity, and R is in inches, how many revolutions per second does the pulley make when

$$V=2\pi R \text{ inches per second?}$$

How many when

$$V=5\times 2\pi R \text{ inches per second?}$$

How many when

$$V=N\times 2\pi R=2\pi NR \text{ inches per second?}$$

If the velocity is expressed in feet per second and R is in inches,

$$V = \frac{2\pi NR}{?} \text{ feet per second?}$$

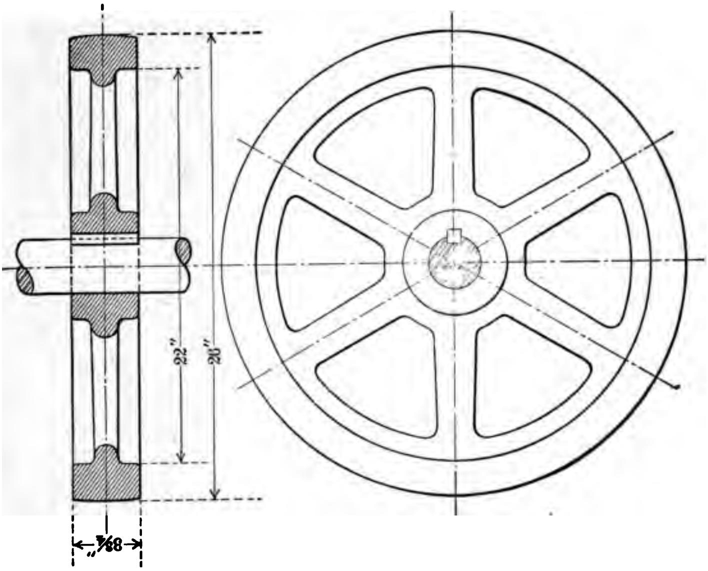

Fig. 164.

If the velocity is expressed in feet per minute and N denotes the number of revolutions per second and R is in inches,

$$\frac{V = ? \times 2\pi NR}{?}.$$

237. Formula for Rim Velocity. The circumferential or rim velocity of a pulley is also known as linear velocity or the distance a point on the rim travels per time unit, if the distance through which it has revolved were laid off on a straight line.

If

D = diameter of pulley in inches,
R_f = radius in feet,
R_i = radius in inches,
N = number of revolutions per minute,
V = rim velocity in feet per minute.

Write a formula for V in terms of R_f.
Write a formula for V in terms of R_i.
Write a formula for V in terms of D.

238. Problems. Complete the entries in the following table:

TABLE 131

RIM VELOCITY OF PULLEYS

No.	D	N	V	No.	D	N	V
1	$4\frac{3}{4}$	300		6	14	75	
2	$6\frac{1}{2}$	270		7	20	86	
3	$9\frac{1}{4}$	240		8	$8\frac{1}{2}$	125	
4	12	100		9	21	65	
5	$16\frac{3}{4}$	90		10	$5\frac{3}{4}$	150	

§ 2. WEIGHT OF A PULLEY

239. Formulas for Weight of a Pulley. The variation in the proportions of the different parts of a pulley makes any formula for weight an approximation. The following from Cromwell's Treatise on Belts and Pulleys is claimed to be sufficiently accurate for ordinary purposes.

(1) Cromwell's formula,

$$W = w^3\left[.163\frac{R}{w} + .015\left(\frac{R}{w}\right)^2 + .00309\left(\frac{R}{w}\right)^3\right],$$

in which

W = approximate weight of pulley in pounds;
R = radius of pulley in *inches*;
w = width of belt in inches.

The formula may be used to calculate the weight of an unbelted pulley by taking

$$w = \frac{4}{5}F$$

in which

F = width of face of pulley in inches.

(2) Clarke's Formula. Unwin gives the following as Mr. D. K. Clarke's formulas:

For rough castings

$$W = 7.6D - 1.5 \quad \text{to} \quad 12D - 9.5.$$

For finished pulleys

$$W = 7D - 1.75 \quad \text{to} \quad 11.6D - 9.25,$$

in which

W = approximate weight in pounds per inch of width;

D = diameter in feet.

240. Problems.

1. By Cromwell's formula compute the weight of a 10-inch pulley operated by an 8-inch belt.

2. Compute the weight of the pulley in Problem 1 by Clarke's formula.

3. Compute the weight of some *detached* pulley in the shops, and test the degree of accuracy of the formula by weighing the pulley.

4. If possible make a second test of the formula with another pulley whose dimensions are considerably different from those of the pulley used in the first test.

5. Compute the weight of some pulley in *use* in the shops.

6. By Clarke's formula determine the weights of the pulleys in Problems 3, 4, and 5.

7. Determine the per cent of error in the weight of the pulley in Problems 1 and 2 as computed by Cromwell's and by Clarke's formula, entering your work in the following form:

Weight by Cromwell's formula =
Weight by Clarke's formula =
Actual weight =
Per cent of error by Cromwell's =
Per cent of error by Clarke's =

8. By both formulas compute the weight of the largest and the smallest pulley in use in the shops.

§ 3. VELOCITY RATIO OF PULLEYS

241. The Belting of Pulleys. The figure shows pulleys connected both by an open belt and a crossed belt.

It will be evident that when connected by an open belt, both pulleys will move in the same direction; if the belt is crossed, one will move in an opposite direction to the other.

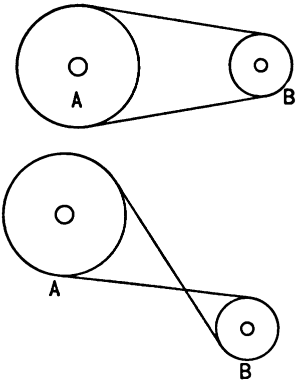

FIG. 165.

Observe that the arc of contact, and hence the friction between belt and pulley, is greater with a crossed belt. Therefore for a given combination of pulleys greater power may be transmitted by a crossed belt. The objection to the use of a crossed belt is the excessive wear caused by the edges rubbing against each other.

242. Problems in Pulleys.

1. Suppose in the figure that A has a circumference of 36 inches and B a circumference of 12 inches. When the belt has moved through a distance of 12 inches, A has made what part of a revolution?

B has made what part of a revolution?

A has revolved through how many degrees?

B has revolved through how many degrees?

If α denotes the angle of revolution of A and θ denotes the angle of revolution of B, α equals how many degrees?

And θ equals how many degrees?

Which pulley has moved with the greater angular velocity?

One pulley has how many times the angular velocity of the other pulley?

When the belt has moved through a distance of 24 inches, A has made what part of a revolution?

B has made what part?

Then α equals how many degrees?

And θ equals how many degrees?

Which has moved with the greater velocity, the larger or the smaller pulley?

Therefore, when two pulleys are connected by a belt, which pulley has the greater velocity?

Which makes the greater number of revolutions?

Therefore the greater the diameter of a pulley, is its velocity greater or less?

Therefore are the revolutions of pulleys directly or inversely proportional to their diameters?

2. If

$V_L =$ the velocity of A,

$V_S =$ the velocity of B,

$D_L =$ the diameter of A,

and

$D_S =$ the diameter of B,

does

$$\frac{V_L}{V_S}=\frac{D_L}{D_S} \text{ or does } \frac{V_L}{V_S}=\frac{D_S}{D_L}?$$

State the reason for your answer.

3. When two pulleys are connected by a belt,

If

$N_L =$ the number of revolutions of the larger,

and

$N_S =$ the number of revolutions of the smaller, is

$$\frac{N_L}{N_S}=\frac{D_S}{D_L}=\frac{C_S}{C_L}=\frac{\frac{1}{2}D_S}{\frac{1}{2}D_L}=\frac{R_S}{R_L}?$$

State the complete reason for your answer.

4. If A and B in the figure of paragraph 241 have the circumferences specified in Problem 1, what is the numerical value of $\frac{N_L}{N_S}$?

5. If the diameter of A is $5\frac{3}{4}$ inches and the diameter of B is $2\frac{3}{4}$ inches, what is the value of $\frac{N_L}{N_S}$?

Therefore the velocity ratio of two pulleys may be determined from what measurements of the pulleys?

6. Under a lettered heading in your work-book, write a law and a formula for the velocity ratio of pulleys.

7. In the combination of pulleys shown in the figure, the driven pulleys B, C, and D are in different planes, and the width of the face of the driver A prevents interference of the belts with each other.

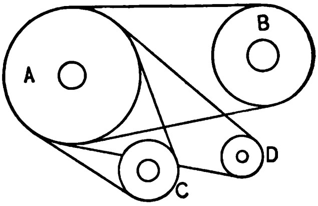

Fig. 166.

In the table, D represents diameter in inches, N the number of revolutions, and the subscripts the several pulleys.

Complete the entries as determined by the formula of Problem 6.

TABLE 132

VELOCITY RATIO OF PULLEYS

No.	D_A	D_B	D_C	D_D	N_A	N_B	N_C	N_D
1	22	7	5	4	150			
2	24				75	200	175	120
3	26		12	8		190	250	
4	28	16	9		125			300
5	30		6	5		225		800

8. The drawing shows an arrangement of pulleys for a factory, pulleys 1 to 4 being in the third story; 5, 6, and 7 in the second; and 8 on the first. 9 is the fly-wheel of the engine.

The respective diameters are as follows:

Pulleys 1 and 2,	16 inches;
Pulleys 3 and 4,	13 inches;
Pulleys 5, 6, and 7,	21 inches;
Pulley 8,	27 inches;
Fly-wheel,	142 inches.

Compute and tabulate the velocity ratio.

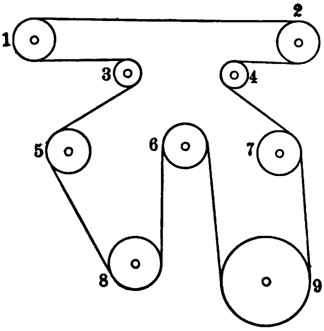

Fig. 167.

9. Make a drawing of the pulley arrangement on the different floors in the machine shop.

10. In a drawing, exhibit the pulley and belt connections between the driving shaft, counter shaft, and the various machines in the shops.

243. Step Cones. For work of various character and materials, it is necessary that it shall be possible to operate at different speeds the machines on which the work is being manipulated. This is accomplished by the use of speed-cones or stepped pulleys as shown in the figure, one being on the driving shaft and one on the mandrel of the machine.

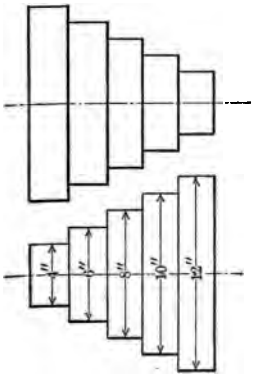

Fig. 168.

As shown, the diameters of the respective steps are the same in both cones, but one points in the opposite direction from the other.

244. Problems.

1. The steps of a pair of speed cones are 4, 6, 8, 10, and 12 inches respectively, as shown in Fig. 168.

Compute and tabulate the velocity ratios.

2. Measure the diameters of the steps of each pair of speed cones for all the machines in the shop and compute and enter in separate tables the velocity ratios for each machine, specifying machine by name, number, and manufacturer.

245. Pulleys in Trains. The drawing shows a train of pulleys, 1 being the pulley on motor.

2 and 3 are on the same shaft and revolve together, as do 4 and 5.

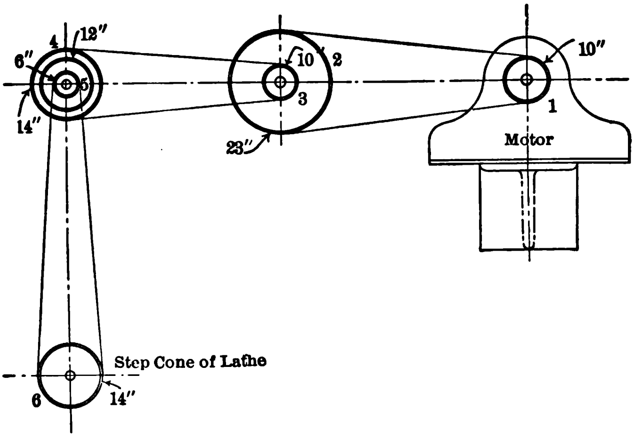

FIG. 169.

It is evident that 1 drives 2, 3 drives 4, and 5 drives 6. In other words, the *odd*-numbered pulleys 1, 3, and 5 are drivers, the *even*-numbered, 2, 4, and 6 are the *driven.*

In all trains of pulleys similar to that in the figure, the velocity ratio of the first and the last pulley may be computed by the following law:

The ratio of the number of revolutions of the *last driven* to *first driver* equals the ratio of the product of the diameters of all the *drivers* to the product of the diameters of all the *driven.*

246. Problems.

1. Under the heading, Velocity Ratio of a Train of Pulleys, enter in the work-book a formula for velocity ratio as determined from the preceding law, using the following notation:

N_L = number of revolutions of last driven;
N_F = number of revolutions of first driver;
D_1, D_2, D_3, etc. = diameter of drivers;
d_1, d_2, d_3, etc. = diameter of driven.

2. In the figure of paragraph 245, suppose the drivers beginning with 1 to have diameters of 7, 12, and 10 inches respectively, and that the driven have the diameters dimensioned.

By the formula of Problem 1, compute the velocity ratio of the train.

3. In the figure the pulleys have the following diameters:

$D_1 = 18$ inches;
$D_2 = 5$ inches;
$D_3 = 16$ inches,
$D_4 = 4\frac{1}{2}$ inches;
$D_5 = 14\frac{1}{2}$ inches;
$D_6 = 4$ inches;
$D_7 = 12$ inches;
$D_8 = 3\frac{1}{2}$ inches.

Compute the velocity of pulley 8 when pulley 1 makes 100 revolutions per minute.

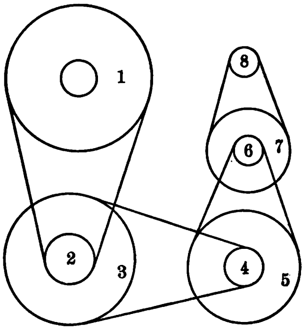

Fig. 170.

4. Compute the velocity ratio of the train of pulleys through which each machine in the shops is operated, taking your data from the drawings made in Problems 9 and 10 in paragraph 242.

Tabulate the results for each machine, with specifications of number, name, and manufacturer.

§ 4. CENTRIFUGAL TENSION IN A PULLEY RIM

247. Cause of Centrifugal Tension. The word centrifugal means flying from the center. Centrifugal tension is a tension due to the revolution of the pulley and is caused by the tendency of each particle in the rim to move in a straight line tangent to the rim. In other words, each particle of the rim tends to fly from the center in the same way water does from a grindstone.

With increase in velocity the centrifugal tension and consequently the tendency of the rim to burst, increases greatly.

Because of this and of the additional stresses to which a pulley is subjected from the pull of the belt, the contraction in casting, and in some cases, the non-coincidence of the center of gravity of the pulley with the center of revolution, the speed of a cast-iron pulley should not exceed 100, or even less, feet per second.

248. Law and Formula for Centrifugal Tension. The centrifugal tension in pounds on each cubic inch in the rim of a revolving pulley equals the weight of the rim per cubic inch times the square of the velocity of the rim in inches per second divided by 32.2, the force of gravity.

We therefore have the following formula:

$$T=\frac{WV^2}{32.2},$$

in which

T = centrifugal tension per square inch, in pounds;
W = weight of rim per cubic inch;
V = velocity of rim in *inches* per second.

249. Problems. Work the problems in the table below, as assigned.

Rule only so much of the table in the work-book as may be required for tabulation of the problems worked.

TABLE 133

CENTRIFUGAL TENSION

No.	V	T	No.	V	T
1	20		6	100	
2	40		7	125	
3	60		8	150	
4	80		9	200	
5	90		10	250	

As shown by entries in the completed table, doubling the velocity makes the tension about how many times as great?

§ 5. BACK GEARS

250. Velocity Reducing Mechanism. The drawing shows a cone pulley riding loose on the spindle of a lathe, and the reducing mechanism common to large lathes, consisting of train of toothed wheels *A*, *B*, *C*, and *D*, called back gears.

Pinion *A* is attached to the cone pulley and revolves with it.

B and *C* are both keyed to the same piece and revolve together.

D is keyed to the spindle.

Observe that the revolution of the cone pulley is not transmitted directly to the spindle, but indirectly through the train of gears.

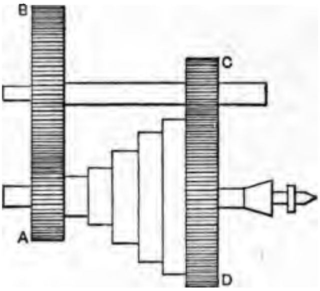

FIG. 171.

The velocity ratio of the mechanism therefore equals the velocity of the cone pulley (and therefore *A*) divided by the velocity of *D* (and therefore the spindle).

251. Problems.

1. If *A* has 12 teeth and *B* has 36, *B* makes what part of a revolution for each revolution of *A*?

Therefore *B*'s velocity is what part of *A*'s?

If *C* has 10 teeth and *D* has 60, *D* makes what part of a revolution for each revolution of *C*?

Therefore *D*'s velocity is what part of *C*'s?

Therefore the velocity of the spindle is what part of the velocity of the cone pulley?

2. Under the heading, Formula for Velocity Ratio of Back Gears, enter in the work-book the formula in the following notation:

V_s = velocity of spindle;
V_c = velocity of cone pulley;
A, B, C, D = number of teeth on the respective wheels of the reducing mechanism.

The arrangement of the formula can be determined by reference to the law of paragraph 245 and by observance of the fact that the teeth of gears are proportional to their diameters.

3. Determine V_c when $V_s=30$, $A=10$, $C=60$, $B=10$, and $D=56$.

4. Complete the entries in the following table, as assigned:

TABLE 134

VELOCITY RATIO WITH BACK GEARS

No.	V_c	A	B	C	D	V_s
1	1200	12	10	48	60	
2	1000	8	12	24	72	
3		8	12	24	72	75
4	800	12	14		70	40
5	500	12	10	24		25

5. Compute and tabulate the velocity ratio of cone pulley and spindle on each machine in the shops, when running back-geared.

6. Under a lettered heading, Advantages of Back Gears, state the advantages which result from the use of back gears, numbering each in succession.

7. What caution would you give a beginner who has learned the operation of a lathe in single gear, and is to run it back-geared for the first time?

8. Under proper lettered heading make a sectional drawing to scale, of the headstock of some lathe in the shops, showing back-gear train and change-gear train.

Specify lathe by name, manufacturer, and number.

9. A lathe is running back-geared with the following train:

A and C each $=20$;
B and D each $=60$.

The belt runs on a $11\frac{1}{2}$-inch step on the cone on the counter-shaft, and on a $4\frac{1}{2}$-inch step on the cone on the spindle.

How many revolutions does the counter-shaft make to one revolution of the spindle?

CHAPTER XI

BELTS

SECTION 1. FUNDAMENTAL PRINCIPLES. SECTION 2. LENGTH OF AN OPEN BELT. SECTION 3. LENGTH OF A CROSSED BELT. SECTION 4. EFFECTIVE PULL.

§ 1. FUNDAMENTAL PRINCIPLES

252. Transmission of Motion. Motion is transmitted from one piece of machinery to another in three different ways:

(1) Direct contact, as in gearing.
(2) Link connectors, as in the crank and the connecting rod of an engine.
(3) Belts, or other wrapping connectors.

In all cases where an exact velocity ratio is not required, belts are the most convenient connector, and in machinery subject to sudden shock in its operation, are the best means of transmission, since their slipping lessens the shock.

In order to avoid heating, unnecessary wear and power loss, it is important that belts be of proper length.

The problems immediately following are therefore preparatory to the determination of formulas for the length of both open and crossed belts.

These formulas require a knowledge of the following fundamental principles, as developed in the next paragraph, Problems 1 to 6 inclusive:

1. The relation of acute angles whose sides are perpendicular to each other.

2. The length relation of the opposite sides of a parallelogram.

3. The sine and the cosine of an acute angle.

4. The length of any part of the circumference of a circle in terms of the radius.

253. Problems.

1. Under the lettered heading, Relation of Acute Angles Whose Sides are Respectively Perpendicular, draw an acute angle A and measure it with the protractor.

A = how many degrees?

Draw an acute angle B, whose sides are respectively perpendicular to the sides of A. (Use extreme care regarding the perpendicularity).

Measure B with the protractor.

B = how many degrees?

What therefore is the relation of A to B?

Draw an acute angle C considerably smaller than A.

Draw an acute angle D whose sides are respectively perpendicular to the sides of C.

Measure both C and D with the protractor.

C = how many degrees?

D = how many degrees?

What therefore is the relation of C to D?

Draw an acute angle E larger than A.

Draw an acute angle F whose sides are respectively perpendicular to the sides of E.

Measure as before and state the relation of E to F.

Therefore acute angles whose sides are perpendicular each to each are always in what relation to each other?

(Write or letter plainly your answer and double underline it.)

2. By definition, a parallelogram is a four-sided flat figure whose opposite sides are parallel.

Under the lettered heading, Length Relation of Opposite Sides of a Parallelogram, draw a large parallelogram having side a opposite side b, and side c opposite side d.

By the scale determine accurately the following:

a = how many inches?

b = how many inches?

Therefore what is the relation of a to b?

c = how many inches?

d = how many inches?

Therefore what is the relation of c to d?

Therefore the opposite sides of a parallelogram are always in what length relation to each other?

(Write or letter, and underline your answer as in Problem 1.)

3. Under the lettered heading, Sine and Cosine of an Angle, draw a right triangle.

Denote the right angle by C and the acute angles by A and B.

Denote the side opposite A by Opp and the side adjacent to A by Adj.

Denote the hypotenuse by $Hypot$.

In a right triangle, the side opposite an acute angle, divided by the hypotenuse, is called the sine of the angle.

Sine is abbreviated sin, written with a small letter without punctuation. Therefore

$$\text{(1)} \qquad \sin A = \frac{Opp}{Hypot}.$$

The side adjacent to an acute angle of a right triangle, divided by the hypotenuse, is called the cosine of the angle.

Cosine is abbreviated cos, written with a small initial letter, without punctuation. Therefore

$$\text{(2)} \qquad \cos A = \frac{Adj}{Hypot}.$$

Enter equations (1) and (2) under the figure you have drawn.

Which is the longer line in the figure, the hypotenuse or the side opposite A?

Therefore in the sine ratio, which is greater, the denominator or the numerator?

Therefore is the numerical value of the sine of an acute angle of a right triangle, integral or decimal?

Which is longer, the hypotenuse or the side *adjacent* to A?

Therefore in the cosine ratio, which is greater, the denominator or the numerator?

Therefore is the numerical value of the cosine of an acute angle of a right triangle, integral or decimal?

4. The following is copied from a table of natural sines and cosines:

TABLE 135

SINE AND COSINE

Deg.	Sin.	Cos.	Deg.	Sin.	Cos.
30°	500	866		940	342
40°	643	766	75°		
	766		80°	988	174
		500	85°	996	

According to Problem 3, are these sines and cosines integral or decimal?

Why are no decimal points used in the table?

In the solution of a problem in which sines and cosines are used, how should the numbers from the table be written?

Draw the table in the work-book and complete the entries by reference to the 3-place table.

5. Enter in the work-book the formula for circumference of a circle in terms of its radius. If you do not remember the formula, refer to paragraph 136.

C = how many degrees?

Therefore 1° of circumference = what part of $2\pi R$?

And 5° of circumference = how many 360ths of $2\pi R$?

And 8° of circumference = how many 360ths of $2\pi R$?

Therefore $\theta°$ of circumference = how many 360ths of $2\pi R$?

$$\frac{\theta}{360} \times 2\pi R = \text{how many 180ths of } \pi R? \quad \text{Why?}$$

6. If the arc of contact of a belt with a pulley is 125°, the length of the belt which touches the rim of the pulley equals how many 360ths of $2\pi R$?

If $R = 12$ inches, formulate and compute the arc of contact.

§ 2. LENGTH OF AN OPEN BELT

254. Exact Formula. An open belt as shown in the cut, is one which does not cross the center line of the pulleys. When two pulleys of unequal size are connected by an open belt, the arc of belt contact with the larger pulley is just as much *more* than 180°. as the arc of contact with the smaller pulley is *less* than 180°.

Therefore if the arc of contact with the larger pulley is 191°, the arc of contact with the smaller pulley is 169°.

Fig. 172 shows an open belt connecting two pulleys of unequal size.

AB and CD are diameters, each perpendicular to OO', the line of centers.

$O'E$ and OF are radii to the points of tangency of the belt and the pulleys, and are therefore perpendicular to *EF*.

L = total length of belt in feet;
d = distance between centers in feet;
R = radius of pulley II in inches;
r = radius of pulley I in inches;
θ = number of degrees in angle $AO'J$;

$$L = .2618\left[R + r + \frac{\theta}{90}(R - r)\right] + 2d\cos\theta.$$

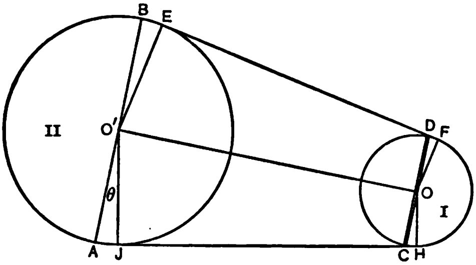

FIG. 172.

Observe on the figure, that the length of the belt equals 2*FE*, plus the arc of contact with the large pulley, plus the arc of contact with the small pulley.

255. Problems.

1. Draw a large figure similar to that in paragraph 254, making one pulley considerably smaller than the other.

From *O*, draw a line parallel to *FE*, terminating in *O'E* at *V*.
Then *OVEF* is what kind of a figure?
Therefore *FE* = what other line?
And what is the length relation of *r* and *VE*?
Therefore what is the relation between *O'V* and $R - r$?
Enter $R - r$ on your figure, using a dimension line.
Dimension *d* on the figure.
What kind to triangle is *OVO'*?
Therefore *VOO'* is what kind of an angle of what kind of a triangle?

Therefore

$$\sin VOO' = \frac{R-r}{?}?$$

2. Length of Arcs of Contact. By the construction of the figure, the sides of the angle VOO' are in what relation to the sides of the angle $BO'E$?

Therefore what is the relation of $BO'E$ to VOO'?

If $VOO' = \theta°$, $BO'E =$ how many degrees?

And $DOF =$ how many degrees?

And $COH =$ how many degrees?

And $AO'J =$ how many degrees?

Therefore $BE = \frac{?}{360} \times 2\pi R$?

Therefore $BE = \frac{?}{180} \times \pi R$?

Therefore $2BE = \frac{?}{90} \times \pi R$?

Now $DF = \frac{?}{360} \times 2\pi r$?

Therefore $DF = \frac{?}{180} \times \pi r$?

Therefore $2DF = \frac{?}{90} \times \pi r$?

Therefore total arc of contact = what?

3. Length of Belt not in Contact with Pulleys.

$$\cos \theta = \frac{OV}{?}?$$

But $OV =$ what other line?

Therefore $\cos \theta = \frac{FE}{?}$?

Therefore $FE = ? \quad \times \cos \theta$?

Therefore $2\,FE = 2\ ? \quad \cos \theta$?

4. Total Length of an Open Belt. By reference to the results obtained in the three preceding problems, show that—

$$L=\frac{\pi}{12}\left[R+r+\frac{\theta}{90}(R-r)\right]+2d\cos\theta$$

in which L = length of belt in feet;
R = radius large pulley in inches;
r = radius small pulley in inches;
d = center distance in feet;
θ = half the arc of contact greater or less than a semicircumference.

How does the formula compare with that in paragraph 254?

256. How to Use the Formula. In order to compute the length of an open belt by the formula of Problem 4, observe that it is necessary to determine the following:

R and r, the radii of the pulleys;
d, the distance between centers;
θ, half the arc of contact in excess of 180° on the large pulley, and half the arc of contact less than 180° on the small pulley.

θ is determined from its sine which in all cases equals the difference of the two radii, divided by the distance from the center of one pulley to the center of the other.

Thus

$$\text{sine }\theta=\frac{\text{Difference between radii in inches}}{\text{Distance between centers in inches}},$$

or

$$\sin\theta=\frac{R-r}{d}.\quad\text{(See paragraph 255, Problem 1.)}$$

257. Problems in Open Belts.

1. Two pulleys with radii of 6″ and 8″ and with their centers 10 feet apart, are connected by an open belt.

Formulate and compute the arc of contact with each pulley.

2. In the table below, the notation is the same as in paragraph 255. Compute and fill in the omitted entries, as assigned.

TABLE 136

ARC OF BELT CONTACT

No.	R	r	d	$\sin \theta$	θ
1	7	5	11		
2	9	7½	12		
3	10	8½	9		
4	11	9¼	10		
5	9¼	6¼	8½		
6	8¾	5	9½		
7	7½	5¼	7		
8	12	9	8¼		

3. Two pulleys have their centers 20 feet apart; their diameters measure 23″ and 16″.

Compute the length of belt required.

4. As assigned, compute and fill in the omitted entries except in the last column, in the table below, in which D_1 and D_2 denote the pulley diameters in inches; R and r the radii in inches; d, the center distance in feet; and L, the length of the belt in feet.

TABLE 137

LENGTH OF OPEN BELTS

No.	D_1	D_2	d	R	r	$\frac{R-r}{d}$	θ	$\cos \theta$	L	
									Exact	Approx.
1	12	9	11							
2	13	9	9½							
3	14	10	10							
4	15	8	12							
5	16	7	8							
6	17	11	7½							
7	18	10	13							
8	19	12	6¾							
9	20	13	14							
10	28	18	15							

258. Approximate Formula for Length of an Open Belt. The following approximate formula is commonly used for the length of an open belt connecting pulleys of *unequal* size:

$$L=3\tfrac{1}{4}(R+r)+2d.$$

Copy the formula into the work-book.

Remove the parenthesis by multiplying the $3\frac{1}{4}$ into it.

In the multiplier $3\frac{1}{4}$ greater or less than π? How much?

Are the products obtained by removal of the parenthesis, therefore, greater or less than the semi-circumferences of the pulleys?

Look at the cut in paragraph 254 and explain why this should be so.

259. Problems in Open Belts.

1. With the same data as in Problem 1, paragraph 257, compute the length of the belt by the approximate formula.

2. With data unchanged, compute the length of the belt in Problem 1 by the exact formula.

3. Using the data in the table of Problem 2 in paragraph 257, compute and tabulate the length of the belts as may be assigned, both by the exact and approximate formulas.

4. As may be assigned, compute and fill in the entries in the last column of the table in Problem 4, paragraph 257.

5. With a tape-line, measure the length of belt required for connecting any two pulleys in the shops.

Take such other measurements as may be necessary for the computation of the length by both formulas.

Make the computations, and tabulate all measurements and results.

How closely do the results agree?

§ 3. LENGTH OF A CROSSED BELT

260. Exact Formula. As shown in the cut, a crossed belt is so named because it crosses the center line of the pulleys.

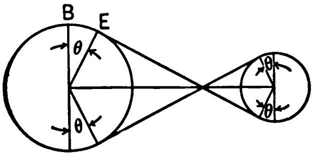

FIG. 173.

If d = distance between pulley centers in *feet*,
R = radius large pulley in inches,
r = radius small pulley in inches,
L = length of belt in feet,

the exact formula for the length of a crossed belt is as follows:

$$L=\frac{\pi}{12}\left[R+r+\frac{\theta}{90}(R+r)\right]+2d\cos\theta.$$

261. Problems.

1. The figure shows a crossed belt, with the radius of the large pulley produced through E the point of tangency, a distance equal to the radius of the small pulley.

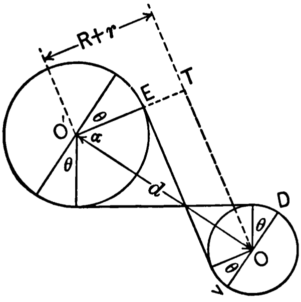

FIG. 174.

Observe that the arc of contact in excess of 180° or πR, on each pulley, is $2\theta°$.

Draw the figure in the workbook.

2. In the right triangle $O'TO$, cos x = what in terms of sides as dimensioned?

And $\theta = 90° -$ what?

From the two equations and the table of cosines, compute θ

when $d = 8\frac{1}{2}$ feet,
$R = 9$ inches,
and $r = 4$ inches.

3. Arc of Contact on Large Pulley.

If C = circumference large pulley,
R = radius large pulley,
and C = how many times R?

Therefore arc θ = how many times R?

Therefore arc 2θ = what in terms of R?

Therefore total arc of contact on the large pulley equals what?

4. Arc of Contact on Small Pulley.

If c = circumference small pulley,
and r = radius small pulley,
c = how many times r?

Therefore arc θ = how many times r?

Therefore arc 2θ = what in terms of r?

Therefore total arc of contact on the small pulley equals what?

5. Length of Belt not in Contact. On Fig. 174 place V at the lower point of tangency of the belt with the small pulley.

Show that the angle $O'OT = \theta$, and mark θ within the angle.

In the right triangle $O'TO$, d is what side of the triangle?

OT is what side with respect to θ?

Does $OT = VE$? Why?

Does $\cos \theta = \frac{OT}{d}$? Why?

In the equation substitute the equal of OT and show that

$$VE = d \cos \theta.$$

Therefore $2VE$ = what?

6. Exact Formula for Length of Crossed Belt. By reference to the figure in Problem 1, and the preceding problems, show that

$$L = .2618\left[R + r + \frac{\theta}{90}(R+r)\right] + 2d \cos \theta,$$

when L and d are in feet and R and r are in inches.

Is the formula the same as that in paragraph 260? Why?

7. By the exact formula compute the length of a crossed belt when

$R = 15$ inches,
$r = 10\frac{1}{2}$ inches,
$d = 18$ feet.

262. Approximate Formula for Length of a Crossed Belt.

The following formula is commonly used for the length of a crossed belt:

$$L = 3\tfrac{3}{8}(R+r) + 2d,$$

in which all dimensions are in the same unit.

263. Problems in Length of Belts.

1. Under headings, letter the *approximate* formulas for both open and crossed belts, and state exactly how they differ.

2. By the approximate formula compute the length of a crossed belt required for two pulleys measuring 18″ and 12″ in diameter and having their centers 21 feet apart.

3. Compute the length of a crossed belt by the approximate formula using the same data as in Problem 7, paragraph 261.

4. Make the same computations for a crossed belt, as those made for an open belt in Problem 3, paragraph 257.

5. By the approximate formulas, compute and tabulate the length of the belts in the shops, as may be assigned, in the following form. Show dimensioned drawings with belt numbers corresponding to those in the table.

TABLE 138

LENGTH CROSSED BELTS IN SHOPS

No.	R	r	$3\frac{3}{8}(R+r)$	$2d$	L
1					
2					

6. By the approximate formulas compute and fill in the omitted entries below:

TABLE 139

LENGTH OF BELTS

No.	D_1	D_2	R	r	$3\frac{1}{4}(R+r)$	$3\frac{3}{8}(R+r)$	d	$2d$	L_o	L_c
1	$15\frac{1}{2}$	$5\frac{3}{8}$					10′			
2	$13\frac{5}{16}$	$9\frac{1}{4}$					11′			
3	$21\frac{13}{64}$	$11\frac{1}{2}$					13′			
4	$17\frac{3}{16}$	$7\frac{11}{32}$					9′			
5	$19\frac{3}{8}$	$9\frac{1}{4}$					12′			
6	$23\frac{1}{2}$	$13\frac{1}{4}$					9′			
7	29	$15\frac{5}{8}$					11′			

Open

$$L_o = 3\tfrac{1}{4}(R+r) + 2d.$$

Crossed

$$L_c = 3\tfrac{3}{8}\,(R+r) + 2d.$$

264. Length of Belting in a Coil. The length of a belt in a closely rolled coil may be computed by the following formula, in which

L = length of belt in *feet;*

D = diameter of roll in inches;

d = diameter of eye in inches;

n = number of turns in the coil;

$$L = .1309n(D+d).$$

265. Problems. Solve the following by the formula of the preceding paragraph.

1. The diameter of a coil of closely rolled belting is $16\frac{1}{2}$ inches, the diameter of the eye is 5 inches, and the number of turns is 15. Compute the length of belting in the coil.

2. Determine the length of the belting in some coil in the shops both by the formula and by actual measurement.

Compute the per cent of error in the length as determined by the formula.

3. At 25 cents per running foot, what is the cost of a roll of belting having 18 turns, the diameter being 25 inches, and the eye measuring 7 inches.

266. Weight of Belts.

If L = the length of the belt in feet,

W = the width in inches,

P_s = the approximate weight of a single belt in pounds,

and

P_d = the approximate weight of a double belt in pounds,

then

$$P_s=\frac{LW}{13},$$

and

$$P_d=\frac{LW}{8}.$$

267. Problems. Solve the following by the formulas of paragraph 266.

1. Write a law for the approximate weight of a single belt, and a law for the approximate weight of a double belt.

2. Compute the approximate weight of a $4\frac{1}{2}$-inch belt, 44 feet $6\frac{3}{4}$ inches long, both when single and when double.

3. Determine the approximate and the actual weight of some belt in use in the shops, and the per cent of error in the weight as determined by the formula.

§ 4. EFFECTIVE PULL OF A BELT

268. Definition. The effective pull of a belt is the difference between the tension on the tight side and the tension on the slack side.

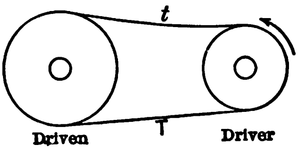

FIG. 175.

The figure shows two pulleys joined by a belt, the tension in the belt due to its tightness being indicated by T and t.

When the pulleys are at rest, the tension is equalized throughout the belt, so that

$$T=t.$$

If the driver revolves in the direction indicated by the arrow, T is increased and t diminished until the difference of the two tensions becomes large enough to cause revolution in the follower.

Therefore if

E = effective pull in pounds,

T = tension on tight side in pounds,

t = tension of slack side in pounds,

$$E=T-t.$$

269. Formula for Effective Pull.

E = the effective pull in pounds,

V = the velocity of the belt in feet per minute,

H.P. = the horse-power transmitted by the belt,

then

$$E=\frac{33000\text{ H.P.}}{V}$$

In the formula, friction is disregarded.

270. Problems.

1. By the formula compute the effective pull of a belt running at the rate of 2000 feet per minute and transmitting 10 horsepower.

2. A driving pulley $24\frac{1}{2}$ inches in diameter and carrying a belt transmitting 13 horse-power makes 300 revolutions per minute. Determine the effective pull.

3. Compute and tabulate the omitted entries in the following table, in which

D = the diameter of driver in inches,

r = the number of revolutions per minute.

and the other symbols are the same as in paragraph 269.

TABLE 140

EFFECTIVE PULL

No.	D	r	V	H.P.	E	No.	D	r	V	H.P.	E
1	$10\frac{3}{4}$	300		25		9	$12\frac{3}{8}$	425		80	
2	$8\frac{1}{2}$	650		40		10	16	300		72	
3	$21\frac{1}{4}$	375		32		11	24	280		64	
4	18	330		50		12	72	200		29	
5	24	250		60		13	48	225		48	
6	9	410		35		14	50	345		100	
7	32	290		21		15	26	400		125	
8	40	160		70		16	$12\frac{1}{2}$	300		200	

CHAPTER XII

HORSE-POWER

SECTION 1. WORK. SECTION 2. HORSE-POWER TRANSMITTED BY A BELT. SECTION 3. HORSE-POWER OF A STEAM ENGINE. SECTION 4. HORSE-POWER OF A GAS ENGINE. SECTION 5. ELECTRIC HORSE-POWER. SECTION 6. HORSE-POWER OF A PUMP.

§ 1. WORK

271. Definition and Illustration. Work is the effort of a force in overcoming resistance.

In a locomotive drawing a train, the motive power is the steam which presses against the pistons with a force sufficient to overcome the combined resistance of friction of bearings, inertia and weight.

When a weight is raised, or falls under gravity; when velocity is imparted to a piece of material, or a velocity increased, or a constant velocity maintained under a varying resistance, work is done.

272. Law and Formula of Work. The law of work should be committed to memory:

Work equals force times the distance through which it acts.

In order to formulate this law for use in computation, the following symbols may be employed:

W = work in foot-pounds;

F = force in pounds;

S = distance in feet.

Then

$$W = F \times S.$$

This formula tells us that the work done by any force can be determined only when we know two things:

(1) The number of pounds of force;
(2) The distance in feet through which the force acts.

The formula also tells us that when these two things are known, the *work done may be computed by multiplying force in pounds by distance in feet.* Observe that the product is not called pounds-feet or feet-pounds, but *foot-pounds.*

It should now be clear that the work done by any force is determined by multiplying the force in pounds by the distance in feet, and that the product is the work in foot-pounds, or is the number of foot-pounds of work. Therefore,

$$\text{Work (in foot-pounds)} = \text{Force (in lbs.)} \times \text{Distance (in ft.)};$$
$$\text{Work} = \text{Force} \times \text{Distance};$$
$$W = F \times S;$$
$$W = FS.$$

273. Problems in Work. Under the usual headings solve the following problems, arranging your work so that

(1) Formula, substitution, and solution are near the left margin;
(2) Data are near the right margin.

In all computations on work and horse-power, π may be taken as 3.14.

1. A weight of 525 pounds is lifted vertically through a distance of 51 feet.

Compute the work done.

2. A baseball, dropped from one of the windows at the top of the Washington monument, was caught by a professional catcher at the foot of the monument.

Assuming the window to be 545 feet from the ground, the catcher's hands 4 feet from the ground, and the ball to weigh 5¼ ozs., compute the work done by the ball.

3. The piston of the Corliss engine in the applied mechanics laboratory at Pratt Institute moves through a distance of 30 inches at each stroke.

Compute the work done by the piston in one stroke when the steam presses against the piston with an average force of 348 pounds.

4. The average pressure of the steam against the piston of an engine was 60 pounds per square inch. If the area of the piston was 78.5 square inches, what was the work done by the engine in 1 hour when making 50, 25-inch strokes per minute?

5. Compute the work done by the piston of Problem 3 in 8 strokes.

6. Compute the work done by the piston of Problem 3 in 3 minutes, when the engine is making 100 strokes per minute.

A single-acting engine is one in which the steam is admitted to only one side of the piston.

A double-acting* engine is one in which the steam is admitted alternately to each side of the piston.

The end of the cylinder through which the piston rod works is called the crank end; the other is called the head end.

In computing the steam pressure on the piston of a double-acting engine, deduction must be made for the area of the piston rod.

7. The piston of the Corliss engine in the mechanical laboratory is $12\frac{27}{64}$ inches in diameter, and the piston rod is 2½ inches.

Compute the area of the piston against which the steam presses both for the head end and the crank end.

8. Relation of Revolutions to Stroke. At each revolution of an engine, the piston travels through a distance equal to how many times the length of the stroke?

Therefore the number of strokes per minute equals how many times the number of revolutions?

R.P.M. or r.p.m. is the abbreviation for number of revolutions per minute.

---

* Unless otherwise specified, the engines in the problems which follow are double-acting.

9. Compute the work of the engine in Problem 7 when running 55 strokes per minute with a mean effective pressure of 50 pounds to the square inch.

10. If the number of strokes in Problem 9 were only half as many per minute, what part as much work would be done by the piston per minute?

What part as much in 2 minutes?

What part as much in 5 minutes?

Prove your answers by computation.

11. If the number of strokes were the same as in Problem 9, and the pressure were only half as much, what part as much work would be done per minute?

Prove your answer by computation.

12. Formulate and compute the approximate diameter of the piston in Problem 4.

Size of an Engine. The size of an engine is indicated by the product of two numbers, commonly called the bore and the stroke:

(1) The diameter of the piston in inches.

(2) The length of the stroke in inches.

Thus an engine 10×24 is an engine having a piston 10 inches in diameter and a stroke of 24 inches.

13. Interpret the following engine sizes:

$$10\times30, \quad 12\times30, \quad 24\times48, \quad 32\times72.$$

14. The Ball engine in the applied mechanics laboratory of Pratt Institute is $8\frac{1}{2}\times8$.

Formulate and compute the piston area and the work done when the engine is making 40 revolutions per minute with an average pressure of 50, no allowance being made for size of rod.

15. A Hamilton Corliss engine 26×42, with a piston rod $4\frac{3}{8}''$, is making 75 r.p.m., with the gage at 125 lbs.

Formulate and compute the work done in 8 minutes, if 50% of the gage pressure is effective.

16. A belt is driven by a $9\frac{3}{4}$-inch pulley making 150 r.p.m.

Formulate and compute the work done by the belt when the effective pull is 184 lbs.

17. If the belt in Problem 16 is 8 inches wide formulate and compute the effective pull per inch of width and the work per inch of width.

18. A $15\frac{5}{8}$-inch pulley running 114 r.p.m., drives a 12-inch belt under an effective pull of 38 lbs. per inch of width.

Formulate and compute the work of the belt per hour.

§ 2. HORSE-POWER TRANSMITTED BY A BELT

274. Illustrations of a Rate. If a train runs 150 miles in 3 hours, its rate is 50 miles an hour. A motor-boat which covers 45 miles in 2 hours has made the distance at $22\frac{1}{2}$ miles per hour. The rate of a belt which moves at a velocity of 50 feet in a second, is 3000 feet per minute. The rim of a 9-inch pulley making 140 r.p.m. has a velocity or rate of 330 feet per minute. A force of 20 pounds, acting through a distance of 300 feet in 5 minutes, does 6000 foot-pounds of work at a rate of 1200 foot-pounds per minute. A hoisting engine which lifts 550 pounds to a height of 60 feet in 1 minute, works at a rate of 33,000 foot-pounds per minute.

All of the preceding illustrations show what a rate is, and should make clear the following definition.

275. Definition of Horse-Power. Horse-power is the unit of rate at which work is done. This unit is 550 foot-pounds per second or **33,000 foot-pounds per minute.**

We therefore have the following law and formula:

$$\text{Horse-power} = \frac{\text{Work in foot-pounds}}{33000 \times \text{time in minutes}}$$

If H.P. = horse-power,

W = work in foot-pounds,

T = time in minutes,

F = force in pounds,

S = distance in feet through which the force acts,

then

$$\text{H.P.} = \frac{W}{33000 \times T}. \tag{1}$$

But

$$W = F \times S.$$

Therefore

$$\text{(2)} \qquad \text{H.P.} = \frac{F \times S}{33000 \times T}.$$

276. Problems. Under the usual headings solve the following problems. Friction is disregarded.

1. By reference to formula (1) and the notation, write the law for horse-power in terms of work.

2. By reference to formula (2) and the notation, write the law for horse-power in terms of force and distance.

3. A force of 300 pounds acts through 22,000 feet in 100 minutes. Compute the horse-power by substitution in one of the formulas of paragraph 275.

4. An engine does 85,000 foot-pounds of work in 5 minutes. Compute the horse-power.

5. An elevator weighing 3150 pounds ascends through 250 feet in 2 minutes. Compute the horse-power required.

6. The maximum load on an elevator is limited to 2000 pounds in excess of the weight of the elevator, which is 1400 pounds.

If the elevator must move through a height of 180 feet in 2½ minutes, compute the horse-power required.

277. Horse-power of a Belt. In paragraph 269, the following formula was used for computing the effective pull of a belt:

$$\text{(3)} \qquad E = \frac{33000 \text{ H.P.}}{V}$$

To solve this formula for H.P., we first multiply it by V, which cancels V in the denominator.

Thus

$$V \times E = \frac{33000 \text{ H.P.} \times V}{V},$$

or

$$33000 \text{ H.P.} = VE.$$

Dividing by 33,000 we obtain the following:

Formula for a Horse-power of a Belt:

$$\text{H.P.} = \frac{V \times E}{33000}. \qquad (4)$$

Now

E = the effective pull of the belt in pounds,

and

V = the velocity of the belt in feet per minute.

Therefore

E = the force in pounds,

and

V = the distance through which the force acts in 1 minute.

Therefore

$V \times E = F \times S = W.$

Therefore

$V \times E$ = the work in foot-pounds per minute.

Now horse-power is a rate of 33,000 foot-pounds of work per minute, and it should therefore be evident why *the horse-power transmitted by a belt equals its velocity in feet per minute times the effective pull in pounds, divided by 33,000.*

278. Problems in Horse-power of Belts. Under the usual headings solve the following:

1. A belt under an effective pull of 300 pounds is running at the rate of 1500 feet per minute. By substitution in formula (4) compute the horse-power of the belt.

2. A belt under an effective pull of 420 pounds is driven by a pulley whose rim velocity is 1250 feet per minute.

Compute the horse-power.

3. In computations on pulleys the rim velocity in feet per minute (when not known) must be computed from the diameter and r.p.m.

If V = rim velocity in feet per minute,
N = number of revolutions per minute,
D = diameter of pulley *in feet,*

write a formula for V in terms of D and N.

4. Under this problem number, enter formula (4) from paragraph 277 and substitute for V from the preceding problem.

Interpret the formula thus obtained, which should be numbered (5), by specifying under what conditions it would be used for computing horse-power.

5. A 14-inch pulley making 125 r.p.m., drives a belt with an effective pull of 248 pounds. What is the horse-power delivered?

6. A 4-inch belt on a $9\frac{1}{2}$-inch pulley making 160 r.p.m., has an effective pull of 36 pounds per inch of width.

Compute the horse-power.

7. Write a formula for the effective pull of a belt per inch of width, using the following notation:

e = effective pull per inch of width;
E = total effective pull;
w = width of belt in inches.

8. Multiply the formula of Problem 7 by w, and interpret the formula then obtained, by stating how E would be computed if w and e were known.

9. In formula (5) as obtained in Problem 4, substitute for E from the formula of Problem 8, numbering the new formula (6).

10. By substitution in formula (6), compute the horse-power delivered by a $6\frac{1}{2}$-inch belt driven by a 26-inch pulley making 120 r.p.m., under an effective pull of 30 pounds per inch of width.

11. A belt is to be driven by a $12\frac{3}{8}$-inch pulley with an effective pull of 32 pounds per inch of width. How much more horse-power will be transmitted by a $7\frac{1}{2}$-inch belt than by a 5-inch belt under these conditions, at 140 r.p.m.?

12. A pulley whose diameter is $15\frac{5}{16}$ inches, making 135 r.p.m. drives a 7-inch belt with an effective pull of 34 pounds per inch of width. Compute the horse-power transmitted.

13. An electric light and power company uses a 72-inch belt which transmits 1500 horse-power at a speed of 5657 feet per minute. Compute the effective pull per inch of width.

14. Compute the horse-power transmitted by a 78-inch belt having the same speed and effective pull per inch of width as the belt in the preceding problem.

15. A belt is driven by an 18-inch pulley. How many r.p.m. will give a velocity of 4050 feet per minute?

279. Width of Belt for a Given Horse-power. In the trade catalogue of the New York Leather Belting Company, the following rules are given for determining the width of belt required to transmit a given horse-power:

(1) For Single Belt. "Multiply the horse-power to be transmitted by 33,000, and divide the product by the product of 50 multiplied by the speed of the belt in feet per minute."

(2) For Double Belt. The same rule as for a single belt except that 90 instead of 50 is the multiplier of the speed in feet per minute.

280. Problems. Under proper headings solve the following problems:

1. By reference to rule (1) write a formula for the width of a single belt required to transmit a given horse-power.

2. By substitution in the formula of Problem 1 compute the width of single belt required to transmit 120 horse-power with a speed of 3600 feet per minute.

3. By reference to the rules preceding, formulate and compute the width of double belt required under the following conditions: Speed 4200 feet per minute, 280 horse-power required.

4. A double belt driven by an 18-inch pulley at 4050 feet per minute, transmits 400 horse-power. Compute

(1) The width of the belt;
(2) The effective pull per inch of width;
(3) The number of revolutions per minute.

5. At the mills of the Merrimack Paper Company in Lawrence, Mass., a 10-inch 5-ply Victor-Balata belt is driven by a 5-foot pulley making 100 r.p.m. Compute the effective pull per inch of width when the belt is delivering 40 horse-power.

6. The belt in Problem 5 drives a 14-inch pulley. How many r.p.m. does the pulley make under the conditions specified?

7. In a tannery at Newark, N. J., a 6-inch belt is driven by a 24-inch pulley at 750 r.p.m. Compute the velocity of the belt in feet per minute.

8. What width of belt will deliver 900 horse-power over a 4½-foot driving pulley making 2000 r.p.m.?

9. A belt driven by a 12½-foot pulley at 85 r.p.m., under an effective pull of 850 pounds, is to transmit 45 horse-power. Compute the width of belt required.

10. A belt on a 20-foot driver at 70 r.p.m. delivers 500 horse-power. How wide is it?

11. A belt driven by a 70-inch pulley at 275 r.p.m., is to transmit 300 horse-power. Compute the width of belt required.

12. The data in the following table are from Barth's Studies in Belting, as reported in the Transactions of the A.S.M.E., '09.

Rule the table in the work-book and formulate, compute, and fill in the omitted entries:

V = velocity of belt in feet per minute;
T_1 = tension in lbs. on tight side;
T_2 = tension in lbs. on slack side;
E = effective pull in lbs.;
H.P. = horse-power delivered to machine.

TABLE 141

HORSE-POWER DELIVERED TO BELT-DRIVEN MACHINES

ARC OF CONTACT, 180°

V	T_1	T_2	E	H.P.
500	210	60		
1000	212	54		
2000	211	57		
3000	207	68		
4000	198	84		
5000	187	107		
6000	173	134		

13. Following is another tabulation from Barth's Studies as referred to in Problem 12.

Compute and fill in omitted entries in the work-book.

TABLE 142

HORSE-POWER DELIVERED TO BELT-DRIVEN COUNTER-SHAFTS

ARC OF CONTACT, 180°

V	T_1	T_2	E	H.P.
500	140	40		
1000	141	38		
2000	140	41		
3000	134	53		
4000	125	60		
5000	114	92		

14. The following formula is sometimes used for computing the approximate width of a *single* leather belt, to transmit any required horse-power:

With of Single Belt.

$$w=\frac{50000\times \text{H.P.}}{D\times \text{r.p.m.}\times \text{oz.}},$$

in which w = width of belt in inches;

H.P. = required horse-power;

D = diameter of driving pulley in inches;

oz. = weight of belt in ounces per sq.ft.

A single leather belt $\frac{3}{16}''$ thick, under an effective pull of 40 pounds per inch of width, is required to deliver 120 horse-power at 480 r.p.m. The belt weighs 16 oz. per square foot, and the driving pulley is 28 inches in diameter. Compute the width of belt required.

15. The following formula is sometimes used for computing the approximate width of a *double* leather belt, to transmit any required horse-power:

Width of Double Belt.

$$w = \frac{67500 \times \text{H.P.}}{D \times \text{r.p.m.} \times \text{oz.}}.$$

The notation in this formula is the same as in Problem 14.

Compute the width of double belt required to transmit 30 horse-power over a 16-inch pulley at 180 r.p.m., the belt weighing 32 ozs. per square foot.

16. A single leather belt weighing 18 ounces per square foot is to run on a 28-inch driving pulley.

How wide must it be to deliver 120 horse-power when the pulley makes 150 r.p.m.?

17. It is required to transmit 210 horse-power by a double leather belt weighing 36 ounces per square foot, driven by a 42-inch pulley at 500 r.p.m. What width of belt must be used?

18. In Woodcraft for October, 1908, the following formulas are given by Mr. L. R. Pomeroy for the horse-power delivered by electrically driven belts operating heavy woodworking machinery: For Single Belt:

$$\text{H.P.} = (D \times F \times R) \div (12 \times 400).$$

For Double Belt:

$$\text{H.P.} = (D \times F \times R) \div (12 \times 400 \times .7),$$

in which D = diameter smaller pulley in inches;

F = face of pulley in inches;

R = r.p.m.

Under the proper headings write each of the formulas with a fraction line instead of the division symbol, and with the parentheses and multiplication symbols omitted.

19. Compute the horse-power transmitted by a $7\frac{1}{2}$-inch pulley with a $3\frac{1}{4}$-inch face at 1350 r.p.m., by both a single and a double belt.

20. In the article referred to in Problem 18, the following formula from Reuleaux is also given:

$$\text{H.P.} = T \times w \times F \times C,$$

$$\text{H.P.} = (T \times w \times D \times R \times C) \div 4,$$

in which T = thickness of belt in inches;
w = width of belt in inches;
D = diameter of pulley in inches;
F = velocity of belt in feet per minute;
R = r.p.m.;
$C = .0062$ to $.0098$ (for leather belts);
$C = .0036$ to $.0068$ (for cotton belts);
$C = .0050$ to $.0082$ (for rubber belts).

Write both formulas for each kind of belting, using a fraction line in place of the parenthesis and division symbol, and omitting the multiplication symbols.

21. With a working stress (effective pull) of 60 lbs. per inch of width, what horse-power will a belt transmit for each inch of width, when traveling 550 feet per minute? Why?

If the working stress is reduced one-half, the belt must travel how many times as fast to transmit the same horse-power as before? Why?

22. In Power and Engineer for Nov. 1, 1910, the following law is given for the horse-power transmitted by a belt:

$$\text{H.P.} = \frac{\text{velocity in ft. per min.} \times \text{width} \times \text{stress}}{550 \times 60},$$

in which the width is in inches and the stress is the effective pull per inch of width.

Under an effective pull of 40 lbs per inch of width, compute the horse-power which a 12-inch double belt is transmitting when running over a 6-inch pulley making 200 r.p.m.

281. Size of Shaft Required to Transmit a Given Horse-power. When a shaft is so placed as to be subject to torsion or twisting only, bending being eliminated, the diameter required for transmitting any specified horse-power with a given number of revolutions, may be computed by the following approximate formula from Larard and Golding's Practical Calculations for Engineers.

Formula for Diameter of Mild Steel Shaft:

$$d = \sqrt[3]{\frac{40\ \text{H.P.}}{n}}$$

in which

d = diameter of shaft in inches;

H.P. = horse-power to be transmitted by shaft;

n = number of revolutions of shaft per minute.

This formula is based on an assumed safe shearing stress for mild steel, of 8000 lbs. per square inch.

282. Problems.

1. The original form of the formula in paragraph 281 is

$$\frac{f_s\frac{\pi}{16}d^3}{63024}=\frac{\text{H.P.}}{n}$$

which may be written

$$\frac{f_s\frac{\pi}{16}}{63024}\times d^3=\frac{\text{H.P.}}{n}\quad\text{Why?}$$

Determine the value of the fraction which multiplies d^3, when

$$f_s=8000,\text{ and }\pi=3.14.$$

Substitute the value in the formula.

2. Copy the formula obtained by substitution in Problem 1, and divide it by the multiplier of d^3.

Therefore

$$d^3=\frac{\text{H.P.}}{\text{How many}\times n}?$$

Therefore

$$d^3=\frac{\text{What (to 2 decimal places)}\times\text{H.P.}}{n}$$

Therefore

$$d=\sqrt[3]{\frac{?\times\text{H.P.}}{n}}$$

How does this compare with the formula of paragraph 281?

3. By the approximate formula compute the size of shaft required to transmit 1200 H.P. at 375 r.p.m.

4. Formulate, compute, and fill in the diameters below, as assigned:

TABLE 143

DIAMETERS MILD STEEL SHAFTS

No.	H.P.	n	d^3	d
1	50	150		
2	175	300		
3	200	200		
4	275	340		
5	300	500		
6	450	225		
7	500	200		
8	850	100		
9	900	150		
10	1000	400		

§ 3. HORSE-POWER OF A STEAM ENGINE

283. How Horse-power is Determined. In computations on the horse-power of an engine three different kinds of horse-power may be considered:

Calculated horse-power;
Brake horse-power;
Indicated horse-power.

Calculated Horse-power is determined from the formula in paragraph 284 by the use of a table giving the average effective pressure on the piston in pounds to the square inch for various initial pressures and points of cut-off. As this takes no account of heat and friction losses the calculated horse-power is always greater than the horse-power actually developed.

Brake Horse-power is the power delivered at the driving pulley of the engine and is therefore the actual horse-power of the engine. It is determined by clamping a Prony brake or some similar device to the driving pulley.

Indicated Horse-power is the power actually developed in the cylinders of an engine. It is determined by the use of an indicator and the formula in the next paragraph, and is the horse-power commonly meant when one speaks of the horse-power of an engine.

284. Formula for Indicated Horse-power. Since horse-power is 33,000 foot-pounds of work per minute, the horse-power of an engine can be determined only when the number of foot-pounds of work is known. The two factors required, pounds and feet, are obtained as follows:

(1) Pounds, from the mean-effective pressure per square inch and the area of the piston in square inches;

(2) Feet, from the length of the stroke in feet and the number of revolutions per minute; that is, from the piston travel in feet per minute.

We therefore have the following standard
Formula for the Horse-power of an Engine:

$$\text{H.P.} = \frac{PLAN}{33000},$$

in which H.P. = the indicated horse-power;
P = the mean-effective pressure on the piston in pounds per square inch;
A = the area of the piston in square inches;
N = the number of strokes per minute
= 2 times r.p.m.;
L = the length of the stroke in feet
= 2 times the length of the crank in feet.

The mean-effective pressure, or M.E.P. as it is called, can be determined only by attaching an indicator and taking an indicator card. Without an indicator, the **M.E.P.** may be roughly taken as *half the boiler gage pressure of a stationary engine,* and 40 *per cent of the boiler gage pressure of a locomotive.*

285. Problems. Under the usual headings solve the following:

1. A 22×30 Corliss engine under 90 M.E.P., makes 80 r.p.m. Formulate and compute the indicated horse-power.

2. Compute the horse-power of an engine with a piston 32 inches in diameter, a crank 24 inches in length, making 70 r.p.m., under 125 pounds M.E.P.

3. What is the horse-power of a 32×72 engine at 60 r.p.m., and 105 M.E.P.?

4. A 14×42 engine is working under 80 pounds M.E.P. How many strokes per minute will give 120 horse-power?

5. By solution of the formula for horse-power, write formulas for N, L, A, and P.

6. Formulate number of strokes per minute in terms of number of revolutions.

7. Formulate length of stroke in terms of length of crank, L_c.

8. Formulate piston travel in one revolution in terms of length of crank.

9. A compound engine is one having a high-pressure cylinder and a low-pressure cylinder, arranged either tandem or one above the other.

The horse-power of a compound engine equals the horse-power of both high- and low-pressure cylinders.

If one cylinder is 18 inches in diameter with 85 lbs. M.E.P., and the other cylinder 30 inches in diameter with 25 M.E.P., compute the horse-power of the engine when the stroke is 18 inches and there are 260 r.p.m.

10. A compound engine has a 28-inch high-pressure cylinder with 55 lbs. M.E.P., and a 52-inch low-pressure cylinder with 20 lbs. M.E.P.

Compute the horse-power when the 28-inch crank makes 100 r.p.m.

11. Formulate and compute the horse-power of the engine in Problem 15, paragraph 273, under the conditions there specified.

12. What horse-power will be developed in a 22×28 engine at 350 r.p.m., with the gage at 70?

13. Compute the horse-power at 400 r.p.m., of an engine with two cylinders each 18×32, under 110 gage pressure.

14. The cylinders of a locomotive are each 16×28, and the drive wheels are 5 feet 8 inches in diameter.

Compute the horse-power developed when the speed is 45 miles per hour, and the gage is at 160.

15. Taking the effective power at the draw-bar as 45% of the horse-power developed in the cylinders, formulate and compute the power at the draw-bar of the locomotive in Problem 14.

16. If possible, determine the data necessary to the determination of the effective power at the draw-bar, of one of the best locomotives in the passenger service of any road entering New York City.

17. By solution of the formula for H.P. in paragraph 281, write a formula for N.

18. By solution of either the N formula or H.P. formula write a formula for P.

19. By solution of any of the three preceding formulas write a formula for L.

20. Formulate A in terms of H.P., L, N, and P.

21. Formulate r.p.m., in terms of H.P. L, N, A, and P.

22. Formulate length of crank in terms of H.P., N, A, P, and r.p.m.

23. A pair of 36″×60″ engines hoist a tank containing 2600 gallons of water, to the top of a 1550 foot shaft in 45 seconds.

What horse-power is delivered by each engine? (Eng. Mag., Sept., 1905.)

286. Allowance for Area of Piston Rod. The horse-power developed in the cylinder of an engine can be computed only when three things are definitely determined:

(1) The piston travel in feet per minute;
(2) The piston area in square inches;
(3) The M.E.P. or mean effective pressure.

These factors are determined as follows:

To determine the *piston travel*, it is necessary to count the number of revolutions of the crank shaft per minute, for several minutes, and to divide the total count by the number of minutes during which the count was made.

The piston travel in feet per minute equals the average number of revolutions thus determined, multiplied by twice the length of the stroke of the piston in feet.

Formulate piston travel in feet per minute.

The *piston area* against which the steam presses is not the same on both sides of the piston.

At the head end of the cylinder the piston area is equal to the area of the bore.

Formulate head-end piston area in square inches in terms of radius.

At the crank end of the cylinder the piston area is equal to the area of the bore minus the area of the piston-rod.

Formulate crank-end piston area in square inches in terms of radius of piston and radius of piston-rod.

The **M.E.P.** can only be determined by the use of an indicator, as described in Chapter 18 of the Practical Elementary Algebra of this course.

287. Formula for Actual Indicated H.P. in Double-acting Engines. The notation below is used in the formulas which follow:

A = area of head end of piston in square inches;

a = area of piston-rod in square inches;

$A-a$ = what?

L = length of stroke in feet;

r.p.m. = number of revolutions of crank-shaft per minute;

P_h = mean-effective pressure on head end in lbs. per square inch;

P_c = mean-effective pressure on crank end of piston in lbs. per square inch;

H.P. = indicated horse-power developed in the cylinder.

$$\text{H.P.}=\frac{P_hLA\text{ r.p.m.}}{33000}+\frac{P_cL(A-a)\text{ r.p.m.}}{33000}$$

$$\text{H.P.}=\frac{L\text{ r.p.m.}}{33000}[P_hA+P_c(A-a)].$$

288. Problems. By substitution in both formulas, compute the horse-power developed in the cylinder of a

Corliss engine under the following conditions. The pressures given were determined by the indicator.

1. Engine 12×36, diameter piston-rod $2\frac{1}{8}''$, 90 r.p.m., $P_h=50.5$, $P_c=50.8$.

2. Engine 17×35, diameter piston-rod $2\frac{9}{16}''$, 85 r.p.m.,, $P_h=50.5$, $P_c=50.8$.

3. Engine 28×48, diameter piston-rod $4\frac{5}{8}''$, 70 r.p.m., $P_h=50.5$, $P_c=50.8$.

4. Engine 30×60, diameter piston-rod 5″, 65 r.p.m., $P_h=85.5$, $P_c=85.75$.

5. Engine 22×48, diameter piston-rod $3\frac{1}{2}''$, 75 r.p.m., $P_h=65.5$, $P_c=65.75$.

§ 4. HORSE-POWER OF A GAS ENGINE

289. Kinds of Gas Engines. A gas engine is a machine for producing power by the explosion of gas in a closed cylinder.

Four operations are necessary for the production of power:

(1) Admission;
(2) Compression;
(3) Ignition and Expansion;
(4) Exhaustion.

A cycle is a completed series of these four operations.

There are two kinds of gas engines: 2-cycle and 4-cycle.

A two-cycle engine is one in which the cycle is completed in two strokes and therefore at each revolution.

A four-cycle engine is one in which the cycle is completed in four strokes and therefore in two revolutions.

Two-cycle therefore means a two-stroke cycle and four-cycle means a four-stroke cycle.

It will be evident that in a 2-cycle engine the piston receives an explosive impulse at each revolution, while in a 4-cycle engine explosion occurs at every other revolution. It would seem that a 2-cycle engine would develop twice the horse-power of a 4-cycle engine of the same size under the same conditions, but this is not the case owing to fundamental differences in design and operation which tend to diminish the power of the 2-cycle engine. Under ordinary and similar working conditions, the horse-power of a 2-cycle engine may be assumed to be 1.65 times as much as that of a 4-cycle engine of the same size.

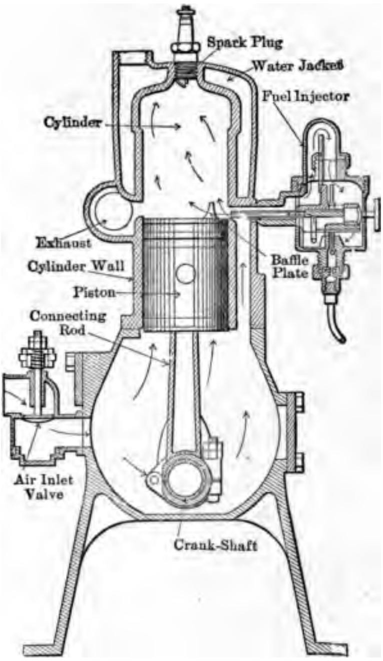

FIG. 176.
2-cycle "Columbia" gas engine.

290. Horse-power of a Gas Engine. The horse-power actually developed by an explosive motor can be determined only under test by a Prony brake or other similar device. From the nature of the fuels used, the pressure against the piston is difficult to measure and therefore the horse-power, which by definition is 33,000 foot-pounds per minute, is not easily computed in terms of work. For this reason quite a number of different formulas are employed which give what is called a *rated horse-power.*

The standard formula in the United States for rated horse-power of 4-cycle gasoline motors is that of the American Licensed Automobile Manufacturers, now the Society of Automotive Engineers. The rating of a gasoline motor by this A.L.A.M. formula is called its S.A.E. rating. The rating is for a piston speed of 1000 feet per minute, although the rated horse-power of the modern, long-stroke, multi-cylinder motor is only about half its actual horse-power.

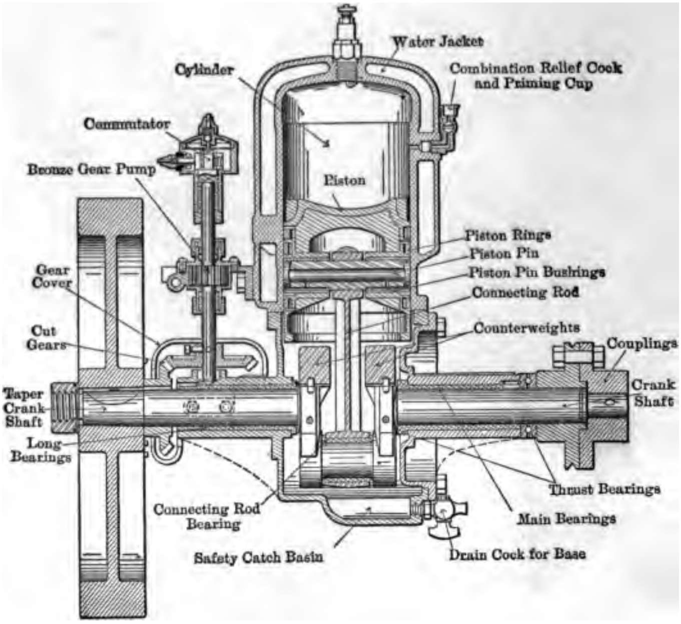

FIG. 177.

2-cycle " Gray " gas engine.

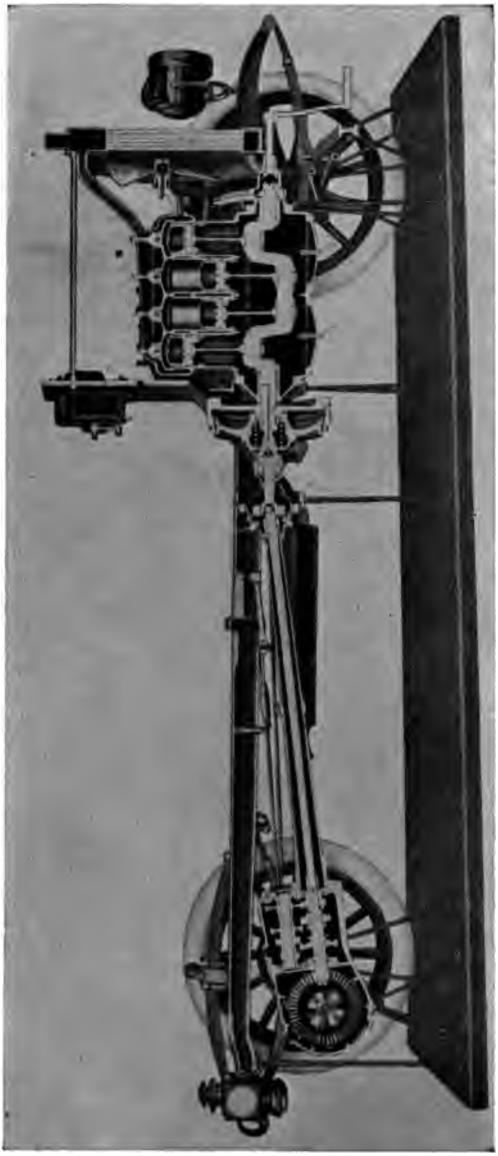

FIG. 178.

Sectional view of "Flanders-20" chassis, showing water cooling system, cylinders, clutch, universal joint, transmission gears, and differential.

291. Formulas for Rated Horse-power of Explosive Motors. The following notation is used in the formulas in this paragraph:

D = diameter of cylinder in inches;
N = number of cylinders;
L = length of stroke in inches;
r.p.m. = number of revolutions per minute;
P = mean-effective pressure on piston, considered about 90 lbs. per square inch;
A = area piston in square inches.

FOUR-CYCLE MOTORS:

A.L.A.M. Formula.

$$\text{H.P.}=\frac{D^2N}{2.5}=.4D^2N. \tag{1}$$

British Royal Auto Club Formula.

$$\text{H.P.}=0.405D^2N. \tag{2}$$

French Auto Club Formula.

$$\text{H.P.}=0.45D^2N. \tag{3}$$

American Power Boat Association Formulas.

$$\text{H.P.}=\frac{D^2N}{2.5338}, \tag{4}$$

and for motors under 6 inches stroke,

$$\text{H.P.}=\frac{D^2LN}{15.200}. \tag{5}$$

Formulas (4) and (5) are for high-speed racing-boat engines. The horse-power of ordinary engines may be taken as $\frac{2}{3}$ of that obtained by these formulas.

Royal Auto Club Formula.

(6) $$\text{H.P.} = \frac{(D+L)^2N}{9.92}.$$

Swedish Royal Auto Club Formula.

(7) $$\text{H.P.} = \frac{D^2LN \text{ r.p.m.}}{15240}.$$

A Manufacturer's Formulas

(8) $$\text{H.P.} = \frac{PLD^2N}{1000}.$$

(9) $$\text{H.P.} = .36LD^2.$$

Roberts' Formula.

(10) $$\text{H.P.} = \frac{D^2LN \text{ r.p.m.}}{c},$$

in which $c = 18{,}000$ for gasoline engines,

and $c = 19{,}000$ for coal-gas engines.

N. Y. Motor Boat Association Formula.

(11) $$\text{H.P.} = \frac{ALN \text{ r.p.m.}}{850},$$

in which L is in *feet* instead of inches.

TWO-CYCLE MOTORS:

Roberts' Formula.

(12) $$\text{H.P.} = \frac{D^2LN \text{ r.p.m.}}{13500}$$

American Power Boat Association Formulas.

(13) $$\text{H.P.} = \frac{D^2LN}{12.987}.$$

(14) $$\text{H.P.} = \frac{D^2N}{2.10008}.$$

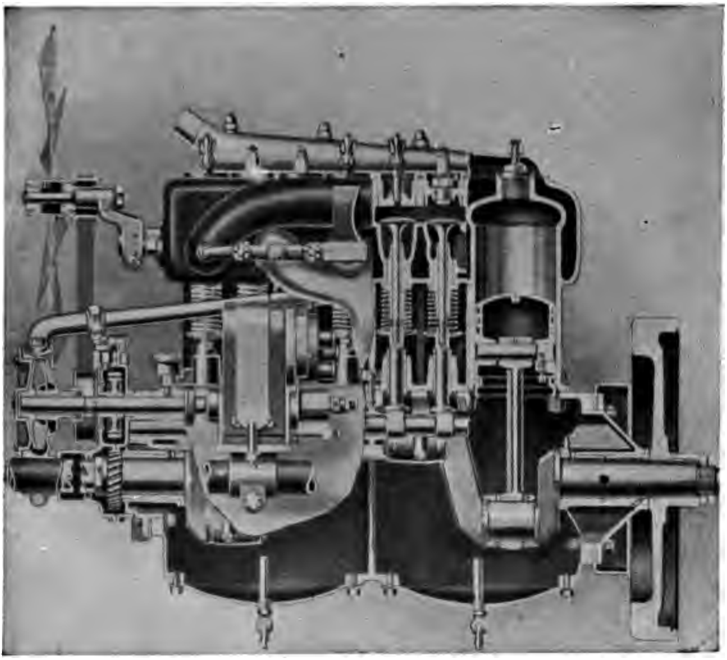

FIG. 179.
Sectional view of "Flanders-20" motor.

Formulas (12), (13), and (14) are for high-speed racing-boat engines; the horse-power of ordinary engines is about $\frac{2}{3}$ as much.

Formula.

(15) $$\text{H.P.} = \frac{D^2LN \text{ r.p.m.}}{21000}.$$

292. Problems.

1. A 2-cylinder 4-cycle, $5\frac{1}{2}\times 5$ aero-motor is rated under the A.L.A.M. formula by the manufacturers, at 25 H.P.

Does the formula give this rating?

2. Compute and tabulate the omitted entries in the following table as assigned. The numbers in the H.P. columns refer to the formulas in the preceding paragraph.

TABLE 144

RATED HORSE-POWER 4-CYCLE MOTORS

No.	D	N	L	r.p.m.	Horse-power.										
					1	2	3	4	5	6	7	8	9	10	11
1	$3\frac{1}{2}$	2	4	650											
2	$3\frac{3}{8}$	4	4	650											
3	$4\frac{9}{16}$	6	5	500											
4	$4\frac{1}{2}$	4	$4\frac{1}{2}$	600											
5	$5\frac{1}{8}$	6	$5\frac{1}{2}$	450											
6	$4\frac{1}{4}$	4	$4\frac{1}{4}$	625											

3. Compute the horse-power of a 4-cycle, 2-cylinder coal gas, $3\frac{5}{8}\times 3\frac{5}{8}$ engine, at 350 r.p.m.

4. A $4\frac{3}{8}\times 6$, 4-cycle, 4-cylinder gasoline engine, makes 450 r.p.m. What is the horse-power?

5. A $4\frac{1}{2}\times 9$, 4-cycle, 1-cylinder gasoline engine makes 400 r.p.m. Compute the horse-power by formulas (6) and (10).

6. The 4-cycle, $3\frac{5}{8}\times 3\frac{3}{4}$ motor in Figs. 178 and 179 is rated at 20 H.P. Which formula gives this rating? What is the H.P. by formula (15) at 5000 r.p.m.?

7. Compute the horse-power of a 2-cycle, 1-cylinder, $3\times 3\frac{1}{2}$ motor at 600 r.p.m. by the four formulas for 2-cycle engines.

8. The motor in Fig. 180 is $4\times 4\frac{1}{2}$. Compute its horse-power by the A.L.A.M. formula.

9. A $4\frac{1}{2}\times 6\frac{1}{2}$ motor has 6 cylinders. What is the horse-power by formulas (1) and (2) and N. Y. Motor Boat Association formula at 500 r.p.m.

10. What is the horse-power developed in a 4-cycle, 4-cylinder motor, $5\times 7\frac{1}{2}$, at 475 r.p.m. under 90 M.E.P.

11. The following tabulation is arranged from a trade catalogue of 4-cycle, 1-cylinder, gasoline engines.

Determine by computation whether any of the formulas of the preceding paragraph give approximately the horse-power here tabulated. Compute also the speed of the belt in feet per minute.

TABLE 145

HORSE-POWER OF GASOLINE ENGINES

No.	Horse-power.	Bore, Inches.	Stroke, Inches.	Speed, r.p.m.	Pulley Diam. in Inches.	Belt Travel.
1	2	4	6	450	8	
2	4	4½	9	400	12	
3	6	5½	10	375	16	
4	8	6½	12	350	16	
5	10	7½	14	300	18	

12. The tabulation below is arranged from a trade catalogue of motor-boat engines.

Determine whether the tabulated horse-power is approximately the same as that computed by the N. Y. Motor Boat Association formula.

TABLE 146

HORSE-POWER OF MOTOR-BOAT ENGINES

No.	H.P.	Cylinders.	r.p.m.	Bore.	Stroke.	H.P. N.Y.M.B.
1	6	2	800	3½	3½	
2	15	3	800	4	4	
3	20	4	800	4	4	
4	18	2	700	5	5	
5	27	3	700	5	5	

293. Maximum Horse-power. A series of tests on twenty-eight American motors were made at Cornell University in order to determine the value of a constant by means of which the maximum horse-power which a motor should develop, could be determined.

The result of these tests was the following:

Maximum H.P. Formula.

$$\text{H.P.}_{\max}=\frac{D^2NL\ \text{r.p.m.}}{1120}$$

in which L = stroke in *feet.*

294. Problems. Compute the maximum horse-power of the following.

1. 6-cylinder motor, $4\frac{3}{8}\times5$, 950 r.p.m.

2. 1-cylinder motor, $3\frac{1}{2}\times3\frac{3}{4}$, 900 r.p.m.

3. The same motor and conditions as in Problem 1, paragraph 292.

4. The same motor and conditions as in Problem 4, paragraph 292.

5. The same motor and condition as in Problem 5, paragraph 292.

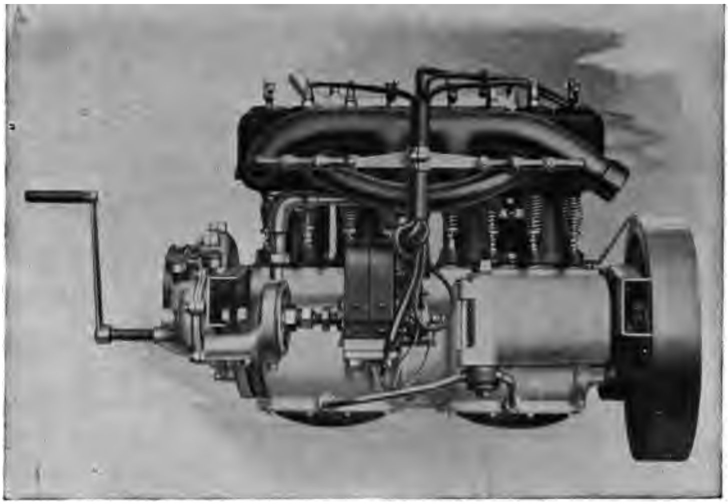

Fig. 180.

E.M.F. "30" motor. Courtesy of Mr. Herbert L. Carpenter.

295. Indicated Horse-power of a Gas Engine. The following formula may be used for computing the horse-power developed in the cylinders of a gas-engine:

$$\text{I.H.P.}=\frac{PLA\times \text{e.p.m.}}{33000},$$

in which P = mean effective pressure;
L = stroke in feet;
A = area piston in square inches;
e.p.m. = number of explosions per minute.

In a gas engine the mean effective pressure is hard to determine. One of the simplest means is by the use of the Frederic Grover Formula for M.E.P.

$$P=2C-.01C^2$$

in which P = mean effective pressure;
C = compression pressure in pounds per square inch above atmospheric pressure.

As the fuels used in gas engines have different characteristics, the use of the Grover formula without a change of constants for different fuels can give only roughly approximate values.

296. Problems. By the formulas of paragraph 295, compute the horse-power of the following:

1. Engine $5\frac{1}{4}\times 8\frac{1}{4}$, with 137 explosions per minute, and 45 pounds per square inch compression pressure above atmosphere.

2. $6\frac{1}{2}\times 8$ engine, 100 explosions per minute, 60 compression pressure above atmosphere.

§ 5. HORSE-POWER OF AN ELECTRIC MOTOR

297. Power. The power developed in an electric circuit or by an electric machine is expressed in terms of watts or kilowatts.

One kilowatt = 1000 watts, kilo meaning thousand. The kilowatt is the unit of electric power transmission.

Power is determined from the following law:

Power in watts = current in amperes × e.m.f. in volts, or

watts = amperes × volts.

(1) Formulate this law, using the following notation:

W = power in watts;
I = current in amperes;
E = electromotive force in volts.

(2) Formulate kilowatts in terms of current and voltage, denoting kilowatts by Kw.

298. Problems in Power. Solve the following:

1. An incandescent light takes .5 ampere at 100 volts. Formulate and compute the power consumed.

2. A dynamo supplies 250 lamps at 120 volts, each taking .4 of an ampere. Compute the power.

3. A generator with a voltage of 325 supplies 48 amperes. What is the power?

4. In an electric light station the voltmeter reads 120 and the ammeter 350 at the same instant.

Compute the load on the generator in kilowatts.

5. The station in Problem 4 operates a 2-wire direct-current system having 55-watt, 110-volt incandescent lamps.

How many lamps were burning at the instant the reading was taken?

6. A dynamo furnishes power for the following equipment at 115 volts:

25 arc lights of 10 amperes each.
525 incandescent of .55 ampere each.
10 electric heaters taking a total of 20 amperes.
Compute the load on the dynamo in kilowatts.

7. What power will be required for the following at 125 volts?
6 electroplating baths of 2.5 amperes each.
18 motors of 25 amperes each.
50 storage batteries taking a total of 25 amperes.

299. Ohm's Law. If one were to select the most fundamental and important law in electrical computation, Ohm's law would undoubtedly be given first place. This law states the relation of current to electromotive force and resistance, and therefore makes possible the computation of current when e.m.f. and resistance are known.

The law briefly stated is as follows:

The current in amperes equals the electromotive force in volts divided by the resistance in ohms.

This law is often shortened to

$$\textbf{amperes} = \frac{\textbf{volts}}{\textbf{ohms}}.$$

(1) Write a formula for this law, using the following notation:

I = current in amperes;
E = voltage, or e.m.f. in volts;
R = resistance in ohms.

If, therefore, voltage and resistance are known, the current can be determined by what operation?

Formerly current in amperes was denoted by C, this symbol being still used in some books. The standard symbol, however, is I.

(2) Formulate the law again, using C instead of I to denote amperes.

300. Voltage and Resistance. Voltage is the force or pressure with which a current flows through a conductor.

(3) Write a formula for voltage in terms of current and resistance, by multiplying formula (1) in the preceding paragraph, by R.

Therefore if the amperage and resistance are known, by what operation can voltage be determined?

(4) Write a formula for resistance in terms of voltage and current, by dividing formula (3) by current.

Therefore if voltage and current are known, by what operation can resistance be determined?

301. Problems. Solve the following by substitution in the preceding formulas:

1. Compute current when the voltage is 375 and the resistance 8.

2. Compute voltage when the current is 65 and the resistance 18.

3. Compute resistance when the voltage is 1240 and the current 92.

4. A wire carries 200 volts with a resistance of $16\frac{2}{3}$ ohms. Compute the current.

5. A current of 35 amperes is flowing through a resistance of 14 ohms. Compute the voltage.

6. If 95.5 amperes flow under 425 volts, what is the resistance?

7. A lamp with a resistance of 210 ohms is supplied by a main carrying 110 volts. Compute the current flowing through the lamp.

8. What current will flow through a lamp of 150 ohms resistance under a voltage of 150?

9. What voltage will maintain a current of 12 amperes through a lamp whose resistance is 5.2 ohms?

10. A 60-watt lamp is lighted by a current at 120 volts? Compute the number of amperes through the lamp.

302. Electric Power Formulas. In paragraph 297 the following was given as a

Power Formula. $W = IE.$

(5) Write this formula and state exactly what each symbol denotes.

But I equals what in terms of E and R?

In formula (5), substitute this value of I.

E equals what in terms of I and R?

In formula (5), substitute this value of E.

What therefore are the three formulas for power?

Write three formulas for power in kilowatts.

303. Problems. The following problems should be solved by substitution in the formulas of the preceding paragraph:

1. Compute the power when the resistance is 300 and the voltage 120.

2. What is the power when the current is 18 and the resistance 50.5?

3. A Leclanché battery has 10 cells, each with 1.5 volts e.m.f. and a resistance of .25 ohm. Compute the power furnished by the battery.

4. If a Daniell cell has a voltage of 1.05 and a resistance of .6 ohm, how much power will be generated by 15 cells?

5. A generator is required to operate a 600-light installation, each lamp being standard 55-watt, at 110 volts. Compute the capacity of the generator in kilowatts.

6. How much power is required to operate 20 arc lamps, each having a resistance of 5.1 ohms and requiring 55 volts?

304. Electric Horse-power. The determination of electric horse-power was made possible in 1878 when the great English physicist, Joule, published the final result of his almost lifelong experiments. The figures of this publication that concern us at this point in the study of mathematics, are the following:

A current of 1 ampere under a pressure of 1 volt, when flowing continuously for 1 second, does .7375 foot-pound of work.

In paragraph 297 it was stated that

Ampere × volts = what?

Therefore 1 ampere × 1 volt = what?

Therefore 1 watt = how many ft.-lbs. per sec.?

Therefore 1 watt = how many ft.-lbs. per min.?

But 1 horse-power = how many ft.-lbs. per min.?

Therefore

(1) 1 horse-power = approximately how many watts?

(2) and 1 watt = what part of a horse-power?

Therefore

(3) 1 kilowatt = how many horse-power to two decimal places?

Formulate (1), (2), and (3).

If the preceding questions have been correctly answered, the following law will be easily understood:

Law for Electric Horse-power. The horse-power developed by an electric current equals the current in amperes times the electromotive force in volts, divided by 746.

(4) Formulate the preceding law.
(5) In formula (4) substitute for I and E in terms of W.
(6) In formula (4) substitute for E in terms of I and R.

305. How to Compute Electric Horse-power. By reference to the formulas of paragraph 304 state exactly how to change

(1) Watts into horse-power.
(2) Kilowatts into horse-power.
(3) Horse-power into watts.
(4) Horse-power into kilowatts.

306. Problems. Solve the following problems:

1. A generator delivers 12 amperes at 600 volts. What is the horse-power of the generator?

2. A generator supplies 250 volts through a resistance of 50 ohms. Compute the horse-power.

3. What is the horse-power of a dynamo which furnishes a current of 25 amperes through 12 ohms resistance?

4. What horse-power is being developed at the instant when the switchboard instruments show the following readings?

Ammeter 15,
Voltmeter 225.

5. The cut shows a generator which supplies 5 amperes at 150 volts.

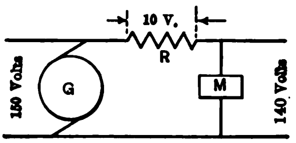

FIG. 181.

The resistance *R* takes 10 volts and the motor *M* takes 140 volts. Compute

(1) The approximate horse-power furnished by the generator.
(2) The power taken by *R*.
(3) The power taken by *M*.

6. By the size of a generator is meant its kilowatt capacity, which is the number of kilowatts it can deliver.

What size generator would be required for the following installment with a constant pressure of 125 volts with a resistance of 4.5 ohms in the circuit?

8 motors each taking 15 amperes,
50 incandescents of .6 ampere each,
40 electric heaters requiring 7 amperes each,
12 arc lamps requiring 5.5 amperes each.

7. How many amperes will be supplied by a 50 horse-power dynamo under a load of 175 volts?

8. What is the voltage of a generator which supplies 75 watts to an incandescent lamp having 240 ohms resistance, if 6 volts are lost in the line?

9. What horse-power is required for the transmission of 25 amperes under a pressure of 200 volts? How many Kw?

10. What must be the kilowatt capacity of a dynamo to supply 35 amperes through 100 ohms resistance?

11. A dynamo forces a current at a voltage of 900 through 300 ohms resistance. Compute the horse-power and the kilowatt capacity.

12. A plant is operated by power supplied by a 95 horse-power engine. If a motor is to be substituted for the engine, what size motor will be required?

13. An elevator weighing 5000 pounds when loaded to its maximum capacity, is to be installed to ascend through 400 feet in $1\frac{1}{2}$ minutes. What horse-power engine will be required?

If a motor is selected instead of an engine, what must be its kilowatt capacity?

14. An engine with a stroke of 12 inches, and a piston area of 7.35 square inches, makes 220 revolutions per minute.

An indicator card shows 75 pounds M.E.P.

What must be the kilowatt capacity of a generator to furnish the same power?

15. A 3-horse-power generator with an efficiency of 95% will maintain at full candle-power how many 16 c.p. 55-watt incandescent lamps, each taking $\frac{1}{2}$ an ampere?

16. A current of 32 amperes passes through a conductor having a resistance of 15 ohms. Compute the power expended.

If a resistance of 25 ohms is added to the conductor in series, and still another resistance of 20 ohms additional in series, compute the power in each instance.

307. An Alternating-current Circuit. When power is supplied by an alternating generator, both current and voltage are constantly changing. In other words the number of amperes and the number of volts continually alternate from one value to another.

When both reach their greatest and least and all other values at the same instant, they are said to be in phase. When the current passes through its successive values ahead of the voltage, the current is said to lead and is called a *leading* current. When the voltage passes through its successive values ahead of the current, the current is said to lag and is called a lagging current.

Therefore an A.C. current may be in the following relations with the voltage:

(1) In phase.
(2) Lead.
(3) Lag.

308. The Power of A.C. Circuits. The power of an alternating-current circuit depends not only on current and voltage, but on how much the current leads or lags.

The amount of lead or lag is called the PHASE ANGLE and is therefore expressed in degrees.

The formulas used in the computation of power in direct currents cannot therefore be used for A.C. computation unless changed so as to make allowance for a difference in phase between the current and the voltage.

If

ϕ = phase angle;

$\cos\phi$ = cosine of the phase angle, or the *power factor;*

W = power in watts;

I = current in amperes;

E = e.m.f. in volts;

we have the following

A.C. Power Formula: $W = IE\cos\phi$.

Compare this formula with formula (5) of paragraph 302 and state exactly how it differs from that formula.

Under suitable heading, write this formula in the workbook, the two other formulas for power in watts, and the three for power in kilowatts as previously given, being sure to multiply the second part of every formula by $\cos\phi$.

309. Problems. In order to solve the following problems the values of the cosine for the different phase angles must be taken from the table of cosines.

1. Compute the power of an A.C. circuit for the following instrument readings and phase angle:

Ammeter 18;
Voltmeter 120;
Phase angle 25°.

2. In the notation of paragraph 308, cos ϕ is called the *power factor*. From the table of cosines fill in the omitted entries in the following table:

TABLE 147

POWER FACTOR

No.	ϕ	cos ϕ.	No.	ϕ	cos ϕ.	No.	ϕ	cos ϕ.
1	0°		6	50°		11	85°	
2	10°		7	60°		12	90°	
3	20°		8	70°		13		.469
4	30°		9	75°		14		.588
5	45°		10	80°		15		.122

3. Look at the power formula in paragraph 308 and state exactly what seems to you to be the reason why the cosine of the phase angle is called the power factor.

4. The phase angle is 32°, the current 30.2, and the e.m.f. 300. Compute the power.

5. The nominal rated horse-power of the D.C. 4-motor locomotives, 1904 type, on the New York Central is 2200 under 625 volts. Compute the current, and capacity of the motors.

6. Electric energy is furnished to consumers at a certain price per kilowatt-hour.

If watt-hours = watts × hours, what do kilowatt-hours equal? Write formulas for watt-hours in terms of

(1) Current and voltage.
(2) Current and resistance.
(3) Voltage and resistance.

7. Write formulas for kilowatt-hours in terms of

(1) Current and voltage.
(2) Current and resistance.
(3) Voltage and resistance.

8. 20 lamps taking $\frac{1}{2}$ an ampere each, are supplied by a 100 volt circuit.

If the lamps are burned for 4 hours every evening except Sundays, what should be the consumer's bill for January of the present year, at 12 cents per kilowatt hour?

9. A room is lighted by 16 lamps each taking .5 ampere on a 110-volt circuit.

Compute the cost of lighting for one evening if the lamps are turned on at 6:50 and are burned until 9:45.

10. The efficiency of a lamp is expressed in watts per candle power.

Thirty-two incandescent 16-c.p. carbon lamps having an efficiency of 3.4 watts, are burned for 5 hours on a 110-volt circuit.

Compute the cost at 12 cents per kilowatt hour.

11. Tungsten lamps have an efficiency varying from 1 to 1.5 watts. What would be the saving in 5 hours if 16-c.p. tungsten lamps of highest efficiency were substituted for the carbon lamps in Problem 10?

What would be the saving if lowest efficiency lamps were used?

12. A 25-watt lamp has an efficiency of 1.2 watts. What is its candle power? If its efficiency were 1.6 watts, would it be of greater candle-power or less? How much?

13. The following table showing the decrease in life of standard 3.1 watt incandescent lamps, owing to increased voltage, is from Foster's Handbook:

TABLE 148

LIFE FACTOR OF LAMPS

% of Normal Voltage.	Life Factor.	Efficiency in watts per Candle
100	1.000	3.1
101	.795	2.99
102	.615	2.9
103	.49	2.8
104	.40	2.7
105	.34	2.62
106	.29	2.54

According to these figures, the life of a 3.1-watt lamp is shortened approximately one-half by what per cent increase above normal voltage?

What per cent increase above normal shortens its life about two-thirds?

If the normal voltage is 110, what voltage will increase its efficiency .4?

If the life of the lamp under normal voltage is 500 hours, what will be its life when its efficiency is increased .48?

14. A Moore vacuum tube 176 feet long and 1⅞ inches in diameter on an alternating circuit of 60 cycles in the United Engineering Societies' Building in New York, was tested at 21.5 amperes under 220.3 volts, the power factor being 73%.

The total lumens (light units) produced was 17,400.

Compute the lumens per

(1) Apparent watt.
(2) True watt.
(3) Foot.

15. The following data are from the report of the Heating Committee of the association of Edison Illuminating Companies in September, 1905:

One pound of coal produces 5 cubic feet of gas containing 3000 B.T.U. (British Thermal Units), or .25 Kw. containing 853 B.T.U.

At current prices compute to six decimal places the difference in the cost of 1 B.T.U. from gas and from electricity.

How many Kw. would be required to supply the same number of B.T.U. as are contained in 1000 cubic feet of gas.

16. To weld a 2-inch round axle by electricity, 75 H.P. for 95 seconds are required. A 2-inch square axle requires 90 H.P. for 100 seconds.

Compute the difference in cost at 12 cents per kilowatt-hour.

17. The generators in an apartment house gave an output of 82,000 kilowatt-hours in one year, with a boiler consumption of 560 tons of coal.

Compute the number of pounds of coal consumed per Kw.-hour.

The total cost of operating this plant for the year, including interest and depreciation, was $5200.

What would have been the cost from a central station at current prices per Kw.-hour?

18. To drive a certain plant employing 4100 men, a total of 2500 horse-power is required. If this is 8.20 men per *effective* horse-power, compute the loss in horse-power in the plant.

With a power factor of .85, what must be the output in kilowatts over an A.C. circuit?

19. If the true watts equals 13,400 and the voltmeter and ammeter readings are 500 and 30 respectively, compute the phase angle.

20. What is the power factor when the true watts equal 21,950 and the apparent watts equals 28,324.

310. Torque. The work done by a revolving piece such as a shaft, fly-wheel, armature, or the pulley of an electric motor is called its torque.

The fundamental law of torque is

Torque = force × leverage.

When the force is in pounds and the leverage (which equals the radius) is in feet, the product is foot-pounds or pounds-feet.

If T = torque in pounds-feet,
S = r.p.m.,
R = radius of armature in feet,

the horse-power of an electric motor is expressed by the following formula:

$$\text{H.P.} = \frac{T \times 2\pi RS}{33000},$$

from which

$$T = \frac{33000 \text{ H.P.}}{2\pi RS}.$$

311. Problems. Solve the following:

1. Show the operations by means of which the torque formula is obtained from the horse-power formula of the preceding paragraph.

2. Explain exactly why the horse-power is equal to the torque in pounds times 2π times the radius in feet times the number of revolutions per minute divided by 33,000.

3. Compute the torque of a 500-horse-power motor making 740 r.p.m., the radius of the armature being 10 inches.

4. Compute the torque when the horse-power is $28\frac{1}{2}$, the r.p.m. = 860, and the diameter = 24 inches.

5. Formulate kilowatts in terms of T, R, and S.

6. Show the computations which change two of the formulas of Problem 5 into a formula for torque.

7. Compute the kilowatts delivered by a motor having an armature 30 inches in diameter, at 500 r.p.m. with a torque of 60.

In computation on induction motors allowance must be made for slip which is the difference in the rate of rotation of the rotating field and the rotor, owing to resistance.

Slip varies from 1 to 40%, and is greatest with smallest horse-power.

8. Compute the torque of a 75 H.P. induction motor at 500 r.p.m. with an armature 18 inches in diameter, the slip being 4%, and the power factor meter reading 95½% lag.

9. The formula for torque is often given for a radius of 1 foot. Write the two formulas of paragraph 310, with this value substituted for *R*.

10. If the torque formula of Problem 9 be written in terms of watts instead of horse-power, it becomes

$$T=\frac{33000\ W}{2\pi S\ 746}.$$

Explain why, so that no one could fail to understand.

11. Simplify the formula of Problem 10 by computing $2\pi\times746$ and by dividing 33,000 by the result, to two decimal places.

12. Compute the torque at 1 foot radius of a 125-H.P. induction motor when the slip is 1½%, the r.p.m. =325, and the power factor meter reads 94 lag.

13. Determine the radius of the armature in some standard 125 H.P. induction motor, and compute the torque when the other factors are the same as in Problem 12.

14. The high-pressure fire-service pumps of the Borough of Manhattan at Gansevoort and West Streets, are operated by five 800 horse-power, 25-cycle induction motors, making 750 r.p.m. under a voltage of from 6300 to 6600. The armature has a diameter of 4 feet.

Compute the total torque of the motors, when the power factor meter on the switchboard stands at 97 lag, allowing 1½% for slip.

15. Compute the current under both voltages in Problem 14.

16. The per cent of efficiency of pumps which are operated by electric motors is determined by dividing the horse-power output, by the horse-power input received from the motors, and by multiplying the quotient by 100.

Thus $$\text{Per cent efficiency}=100\times\frac{\text{H.P. output}}{\text{H.P. input.}}.$$

Compute the per cent efficiency when the H.P. output is 708 and the H.P. input is 929.

17. Below is a tabulation of a test of the fire-service pumps operated by the induction motors of Problem 14. Compute and fill in the omitted entries.

TABLE 149
TEST OF FIRE-SERVICE PUMPS

No.	Elec. H.P. Input.	H.P. Output.	Per cent Efficiency.
1	916	711	
2	877	700	
3	920.5	703	
4	892	695	
5	952	559	
6	899	714	

18. The horse-power input of a pump, as received from the motors equals $\frac{1}{746}$ of the total watts, times the per cent efficiency of the motors.

Thus $$\text{H.P. input} = \frac{W \times \%\,\text{Eff.}}{746}.$$

Compute H.P. input when $W = 9000$ and the efficiency is 91%.

19. The horse-power output of a pump equals the number of gallons which it delivers per minute times the head times the weight per gallon, divided by 33,000.

Write a formula for this law, using the following notation:

$\text{H.P}_o.$ = horse-power output;
g = number of gallons per minute;
h = the head in feet;
w = weight per gallon in pounds.

20. State exactly why the formula of Problem 19 gives the horse-power output of a pump.

21. A pump delivers 3705 gallons of water per minute, weighing 8.34 pounds per gallon, the head being 3 feet. Compute the H.P. output.

22. Write a formula for the per cent efficiency of a pump, by substitution in the equation of Problem 16, from the formulas of Problems 18 and 19.

Simplify the result by dividing the number in the denominator into the number in the numerator, to four decimal places.

23. Compute the per cent efficiency of the following pumping service:

Horse-power of motors................	4995
Per cent efficiency of motors............	93.2
Gallons of water pumped per minute.....	18,189
Head in feet..........................	2.31×320

§ 6. HORSE-POWER OF A PUMP

312. Kinds of Pumps. The simplest classification of pumps is that which divides them into lift pumps and force pumps.

A **lift pump** in its simplest form consists of a piston or plunger working in a cylinder from which the air is exhausted by the movement of the piston.

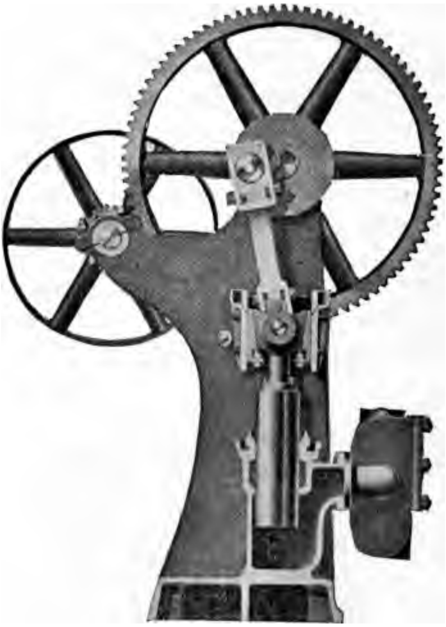

FIG. 182.

Single-acting pump.

This withdrawal of the air in the cylinder is made possible by a valve in the center of the piston, which opens when the piston is pressed into the cylinder and permits the air in the cylinder to pass through the piston to the other side. At the instant the return stroke begins, the valve closes, leaving no opening in the piston, and the air is drawn ahead of the piston from the cylinder.

A place from which the air has been exhausted is called a **vacuum.** The movement of the piston therefore creates a vacuum in the cylinder of the pump.

If a pump is connected to a well or cistern by a pipe or other air-tight connector, the pressure of the air on the water will force the water up the pipe from which the air has been drawn, and into the cylinder.

When the plunger moves to the end of the cylinder of a lift-pump the valves open and a cylinderful of water passes through the opening in the plunger to the other side.

On the return stroke, if valves and piston are water-tight, the pump delivers a cylinderful of water.

A **force pump** creates a vacuum in an enclosed cylinder from which the water is ejected under pressure of the piston or plunger.

Fig. 182 shows a single-acting force-pump; Fig. 183 shows a double-acting pump. In the work which follows pumps will be regarded as single-acting unless the contrary is specified.

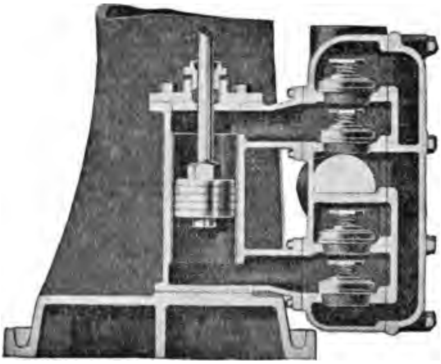

FIG. 183.

Double-acting force-pump. Courtesy of Henry R. Worthington Pump Company

313. Work Done by a Pump. Work has been defined as force times the distance through which it acts. *The work of a pump is the weight of the water discharged multiplied by the height the water is lifted.*

The work of a pump at each stroke, therefore, equals the weight in pounds of the water delivered, times the height in feet through which the water is lifted.

The work per minute in foot-pounds or pounds-feet equals the work per stroke times the number of strokes per minute.

If a pump delivers water to mains against pressure, the pressure must be changed into equivalent head by the use of a conversion factor. What this is and how it is used is explained in the next paragraph.

314. Conversion of Pressure into Head. *The height through which water is lifted by a pump is called the head.* Where water is not lifted, but is forced into mains, the pressure per square inch may be converted into head or lift by multiplying by 2.31.

Any number whose use as a multiplier converts one quantity into something else is called a **conversion factor.**

The conversion factor which converts pressure in pounds per square inch, into head in feet, is 2.31. Therefore the following law:

Head in feet = 2.31 × pressure in pounds per square inch.

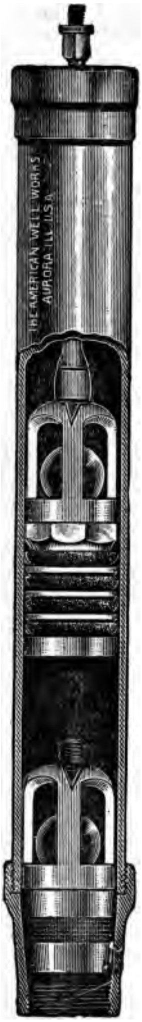

FIG. 184.
Pump cylinder showing valves.

In case water is first lifted and then forced against pressure, the

$$\left.\begin{matrix}\text{Total head}\\ \text{in feet}\end{matrix}\right\}=\left\{\begin{matrix}\text{lift in feet}+\text{equivalent}\\ \text{pressure head.}\end{matrix}\right.$$

315. Pump Laws. The following laws will be of service in solving the problems of this section:

(1) Gallons delivered per stroke.

$$\text{Gals. per stroke}=\frac{\text{cyl. area}\times\text{length stroke}}{231},$$

in which 231 is the number of cu.in. in a gallon.

(2) Pounds delivered per stroke.

$$\text{Lbs. per stroke}=\frac{8.337\times\text{cyl. area}\times\text{length stroke}}{231},$$

in which 8.337 is the weight in lbs. of one gallon of water.

(3) Gallons delivered per minute.

Gals per minute = gals per stroke × what?

(4) Pounds delivered per minute.

Lbs. per minute = lbs. per stroke × what?

(5) Horse-power of a pump.

$$\text{H.P.}=\frac{\text{lbs. per minute}\times\text{head in feet}}{33000}.$$

316. Problems. In the following, where the problem requires the writing of a formula be sure to give the notation and to letter the formula neatly under suitable heading.

1. Formulate the cylinder area called for in law (1),

(*a*) In terms of radius.

(*b*) In terms of diameter.

In (*b*) reduce $\frac{3.1416}{4}$ to a decimal.

2. Formulate the capacity of a pump per stroke in gallons,

(*a*) In terms of area of cylinder.
(*b*) In terms of radius of cylinder.
(*c*) In terms of diameter of cylinder.

3. Fomulate the capacity of a pump per stroke in pounds, in terms of the three different things of Problem 2.

Reduce $\frac{8.337}{231}$ to a 5-place decimal and substitute in each formula.

4. Formulate the capacity of a pump in gallons per minute.

5. Formulate the capacity of a pump in pounds per minute.

6. Formulate the horse-power of a pump in terms of head.

7. Formulate the horse-power of a pump in terms of pressure.

8. Formulate the horse-power of a pump in terms of head and pressure.

9. In computing the horse-power required for pumping, allowance is made for friction, which in some cases may be as much as 25%. In all cases therefore where a problem calls for the horse-power required to pump a given weight of water under a given head, the formula of Problem 6, for the horse-power actually delivered by a pump, can be changed into a formula for the horse-power required, by multiplying by

1 + per cent allowance for friction.

Write the formula with the following notation for the conversion factor: $(1+\% f)$.

10. A pump having a 5½-inch cylinder and a 14-inch stroke makes 130 strokes per minute. Compute

(1) Area cylinder.
(2) Gallons per stroke.
(3) Pounds per stroke.
(4) Gallons per minute.
(5) Pounds per minute.
(6) Horse-power delivered, with 30 feet suction lift against 200-lbs. pressure.
(7) Horse-power of pump, with 20 per cent friction.

11. A triplex (3-plunger) single-acting pump has three times the capacity of a single cylinder or plunger.

Make the seven computations required in Problem 10, for the following Deane vertical triplex single-acting plunger pumps:

TABLE 150

DEANE TRIPLEX PUMPS

Diam. Plunger.	Stroke.	Strokes Per Min.	Head.
8	7	45	50
9	10	42	100
10	12	40	150
4	6	50	250

12. In determining the size of pump required to deliver a given number of gallons through pipes, allowance must be made for friction in the pipes. This places additional work on the pump and is figured as so much additional head, which is called friction head.

If

H_f = loss of head due to friction, or friction head in feet,
L = length of straight pipe in feet,
V = velocity of flow in feet per *second*,
D = diameter of pipe in feet,

we have the following

Formula for Friction Head:

$$H_f = \frac{.02LV^2}{64.4D}.$$

Compute the friction head through a pipe 1000 feet long and 2 inches in diameter, when $V = 400$ F.P.M.

13. If allowance is made for the friction of the flow in the pipe, we have the following

Pump Horse-power Formula:

$$\text{H.P.} = \frac{\text{lbs. water per minute } (H + 2.31\ P + H_f)}{33000}$$

in which H.P. = pump horse-power required;
H = suction lift in feet;
P = pressure developed in mains in lbs. per square inch
which = pressure delivered by pump *minus* pressure at which water is delivered to pump;
H_f = friction head allowance in feet.

A pump which receives water directly from the mains at 20 pounds pressure, delivers 800 pounds of water per minute against a pressure of 300 pounds per square inch, through 1500 feet of straight pipe, the friction head being 358 per 1000 feet of pipe.

Compute the horse-power required.

14. Compute the horse-power of a pump having four single-acting plungers each 4 inches in diameter, the length of stroke being 6 inches, the suction lift 15 feet, the pressure developed 200 lbs., the friction head 400, with 150 strokes per minute.

15. A pump with water plungers $6\frac{1}{2}$ inches in diameter delivers 247 gallons per minute, the stroke being 10 inches, and the r.p.m. 43.

Compute the number of plungers in the pump with no allowances.

16. A pump delivers 3000 gallons per minute through 2000 feet of 10-inch pipe against 300 pounds pressure, the suction lift being 15 feet.

Determine the friction head from the table, and compute the horse-power.

TABLE 151

FRICTION HEAD

Size Pipe.	Friction Head in Feet per 1000 Feet of Straight Pipe.								
10″	G.P.M.	700	750	800	900	1000	1200	1400	1600
	H_f	5.1	5.9	6.6	8.3	10.0	13.8	18.4	23.6
	G.P.M.	1800	2000	2200	2400	2600	2800	3000	
	H_f	29.9	36.0	43.5	51.0	58.5	66	77.2	

From H. R. Worthington's " Turbine Pumps."

CHAPTER XIII

SOLUTION OF AN EQUATION

SECTION 1. THE FOUR FUNDAMENTAL OPERATIONS. SECTION 2. ONE UNKNOWN QUANTITY. SECTION 3. TWO UNKNOWN QUANTITIES. SECTION 4. FORMULAS. SECTION 5. THE QUADRATIC.

§ 1. THE FUNDAMENTAL OPERATIONS

317. What the Operations Are. It is impossible to learn much about the solution of equations without a knowledge of the four fundamental operations,

Addition,
Subtraction.
Multiplication.
Division.

These are called fundamental because they underlie all other operations and are constantly used in all parts of mathematics.

318. Kinds of Terms. The terms of an algebraic expression are of two kinds:

(1) Like, or numerical terms and those having one or more letters the same with the same exponents.

(2) Unlike, all other terms.

Thus in $3x+2x-8+5x^2-6x^2+9$

$5x^2$ and $3x$ are unlike terms because while the letters are the same, the exponents are different.

$3x$ and $2x$ are like terms and $5x^2$ and $-6x^2$ are like terms, because they have the same letters with the same exponents.

9 and -8 are also like terms because both are numerical.

319. Addition. The terms of an expression are added by adding the coefficients of the like terms.

Thus the terms of the expressions

$$5+4x^2+x \text{ and } 3x+2x-8+5x^2-6x^2+9$$

are added by adding the x terms, the x^2 terms, and the numerical terms as follows:

$$\begin{array}{r} x+4x^2+5 \\ 3x+5x^2-8 \\ 2x-6x^2+9 \\ \hline 6x+3x^2+6 \end{array}$$

We therefore have the following:

Addition Laws:

(1) Write like terms in the same vertical column.

(2) Set down under each column the excess of plus or minus.

320. Examples. Add the terms in the following examples:

1. $2x^3-x+5+2x^2,\ 3+2x+x^3-x^2,\ x^2-5+x^3-3x.$
2. $3b^2-4+2b-b^4,\ 3b-12b^4-2b^2+1,\ 3+4b-8b^4.$
3. $5ay+3cx-24, -5cx+12-3ay,\ 2cx+ay-3.$
4. $8d-2c+4a-3,\ 3c-2a+8-d,\ 4-3a+16c+5d.$
5. $50r^{-3}+16v^2+19s, -12v^2-25r^{-3}+34s.$
6. $10a^2x-8ax^2+3b^2c,\ 42ax^2-15a^2x-8b^2c.$
7. $4\sqrt{x}+3\sqrt{y}-6\sqrt{t},\ -2\sqrt{y}+4\sqrt{t}-2\sqrt{x}.$
8. $5\sqrt{x+y}-2\sqrt{x-y}+12\sqrt{x+y}, 8\sqrt{x-y}+7\sqrt{x+y}-9\sqrt{x-y}.$
9. $5b\sqrt[3]{x+1}-3d\sqrt{x-1}+15,\ 6d\sqrt{x-1}-8+3b\sqrt[3]{x+1}.$
10. $3x^2y^3-14x^3y+5,\ 17x^2y-x^2y^3-4.$

321. Subtraction. When one expression is to be subtracted from another, it may be indicated by a minus sign between the two expressions, or by the words from and take.

The names used in subtraction are as follows:

(1) The minuend, or the quantity or expression from which subtraction is to be made.
(2) The subtrahend, or the expression to be subtracted.

Subtraction Laws. Following are the laws of subtraction:

(1) Write like terms in the same column.
(2) Change signs of all terms in the subtrahend.
(3) Set down the excess of plus or minus under each column.

322. Examples. Perform the following subtractions:

1.	From	$5x^3-2x^2+3x+5$
	take	$2x^3+3x^2-6x+3$
2.	From	$8y^4-7y^3+8y^2-y+7$
	take	$3y^4+2y^3+5y^2+y+5$
3.	From	$12b^2c^2-7bc+5$
	take	$bc-4-11b^2c^2$
4.	From	$18x^3y-15x^2+20x^2y^2-9+3x$
	subtract	$12x^3y+9x^2-3x^2y^2-3x+14$
5.	From	$15b^4x^2+14bx+6b^2x-16$
	subtract	$14-12b^2x+13bx-21b^4x^2$
6.	From	$29x^5-14x^2-13x+21x^4-x^3+1$
	subtract	$42x^2+15x-28x^4+6-30x^5+2x^3$

323. Inclosure of Subtrahend in Parenthesis. A subtrahend is frequently denoted by inclosing the quantities in a parenthesis preceded by a minus sign.

Thus $\qquad 3x^3-6x^2+12-(2x^3+3x^2+20)$

means exactly the same as though it were written,

from $3x^3-6x^2+12$ take $2x^3+3x^2+20$.

The quantities in the parenthesis are therefore the subtrahend, and the parenthesis may be removed provided the signs of all the terms in the parenthesis are changed.

Thus $3x^3-6x^2+12-2x^3-3x^2-20$.

324. Examples. Remove the following parentheses and add the terms:

1. $5b^3+2y-ax^2+8-(3y-2ax^2+4-3b^3)$
2. $18x^4-5y^3+21-6x^2y^2-(13+2y^3+8x^4-3x^2y^2)$
3. $50x^3-40x^3y^3+8-12x^2-(4+21x^3y^3-3x^2+20x^3)$
4. $38b^4-25x^3b^2-17x^2b^3-20-(17b^4-14x^3b^2+8x^3b^3-1)$

325. Multiplication. One expression may be multiplied by another by the use of the following

Laws of Multiplication.

(1) Like signs give plus; unlike signs give minus.
(2) Multiply coefficients.
(3) Bring down letters.
(4) Add exponents of same letters.

Thus

$$\begin{array}{r}
3x^3+2x^2+5x-6 \qquad\qquad \\
2x+3 \qquad\qquad\qquad\qquad\quad \\
\hline
6x^4+4x^3+10x^2-12x \qquad\quad \\
9x^3+\ 6x^2+15x-18 \\
\hline
6x^4+13x^3+16x^2+3x-18
\end{array}$$

326. Examples. Perform the following multiplications:

1. $5x^3+8x^2-2x$ by $3x+4$
2. $10y^4-3y^3+5y^2-y$ by $8y^2+2y$
3. $12d^3-2d^2+16d-4$ by $3d+2$
4. $11z^3-4x^2-11a^2r^3$ by $3z-2x$

Multiplication is often indicated by one parenthesis following another with no sign between them.

Thus $(3x^2+5)(2x-3)$

denotes that $3x^2+5$ is to be multiplied by $2x-3$.

Perform the following indicated multiplications:

5. $(x+y)(x-y)$

6. $(x-y)(x-y)$

7. $(x+y)(x+y)$

8. $(x^3+2x+1)(3x^2+x)$

9. $(5y^4-4y^3-5)(2y-3)$

10. $(a+b+c)(a-b-c)$

11. $6(5x^2-3x+1)(3x^2+14x+3)$

12. $4(3x+8-x)^2$

327. Division. In order to divide one expression by another the terms of both dividend and divisor must be arranged in exactly the same order; either

(1) Ascending order, or
(2) Descending order.

The following are arranged in ascending order:

$$4-x+2x^2-3x^3+5x^4.$$

The following are arranged in descending order:

$$5x^4-3x^3+2x^2-x+4.$$

Division Laws. Following are the laws of division:

(1) Arrange terms of both dividend and divisor in the same order, either ascending or descending.
(2) Like signs give plus, unlike signs give minus.
(3) Divide numerical coefficient dividend by numerical coefficient divisor.
(4) Bring down letters.
(5) Subtract exponents of letters in divisor from exponents of the same letters in dividend.

328. Model Solution. Following are model solutions for division:

1
$$3a^2c^3x\overline{)\,-12a^4c^3x}$$
$$-4a^2x$$

2
$$-12xy^2\overline{)\,-24x^3y^4b^3}$$
$$2x^2y^2b^3$$

3
$$
\begin{array}{l}
3x-2)\,9x^4-16x^2-7x+10\,(3x^3+2x^2-4x-5\\
\qquad\quad 9x^4-\ \ 6x^3\\
\qquad\quad \underline{-\qquad+\quad}\\
\qquad\qquad 6x^3-16x^2\\
\qquad\qquad 6x^3-\ \ 4x^2\\
\qquad\quad \underline{\ -\qquad\ +\quad\ }\\
\qquad\qquad\qquad -12x^2-7x\\
\qquad\qquad\qquad -12x^2+8x\\
\qquad\qquad\qquad \underline{+\qquad\quad -\ }\\
\qquad\qquad\qquad\qquad\quad -15x+10\\
\qquad\qquad\qquad\qquad\quad \underline{-15x+10}
\end{array}
$$

329. Examples. Perform the following divisions:

1. Divide $28a^4b^3$ by $-7a^3b^2$
2. Divide $9a^2+12a+4$ by $3a+2$
3. Divide $x^2+15x+54$ by $6+x$
4. Divide $12-21x^2+10x^3-11x$ by $2x^2-3x-4$
5. Divide $2x^4+7x+1-x^3+4x^2$ by $3-x+x^2$
6. Divide $145x+72-2x^4+82x^2+7x^3$ by $9-x^2+8x$
7. Divide $30y+28y^4+9-71y^3-35y^2$ by $4y^2-13y+6$

8. $\dfrac{x^2+2xy+y^2}{x+y}$ 9. $\dfrac{x^2-2xy+y^2}{x-y}$

10. $\dfrac{x^3+3x^2y+3xy^2+y^3}{x+y}$ 11. $\dfrac{x^3+y^3}{x+y}$

12. $\dfrac{x^3-3x^2y+3xy^2-y^3}{x-y}$ 13. $\dfrac{x^3-y^3}{x-y}$

14. $\dfrac{x^4-y^4}{x^2+y^2}$

15. $\dfrac{x^2-8x+15}{x-3}$

16. $\dfrac{3x^2-(x^3-4x)}{x}$

17. $\dfrac{42x^{\frac{3}{2}}-7x^{\frac{5}{2}}}{x}$

18. $\dfrac{18x^2(x-1)^2}{3x(x-1)}$

19. $\dfrac{27y^6-8z^3}{3y^2-2z}$

20. $\dfrac{7y^4(y^2-1)^3}{y^2(y^2-1)^2}$

21. $\dfrac{L^3+L_1{}^3}{L^2-LL_1+L_1{}^2}$

§ 2. ONE UNKNOWN QUANTITY

330. What an Equation is. An equation is an expression which contains an equality sign.

Thus

$$A=\pi R^2$$

is an equation. If A in this equation denotes the area of a circle and R denotes the radius, the equation tells us exactly what relation exists between the area of a circle and its radius. It tells us that if we know the length of the radius of any circle in inches or feet or any other unit, the area may be determined by squaring the radius and multiplying the result by 3.1416.

All formulas which are given in preceding chapters are therefore equations because they are expressions of equality.

331. Problems. Write the following equations:

1. Under suitable headings write not less than two equations which express the relation between the floor surface or the capacity and the dimensions of this room or its equipment.

2. Write an equation for the total watts of the lamps by which the room is lighted.

3. Write an equation for the total kilowatts of the lamps.

4. Write a formula for the horse-power output of a generator which lights 100 rooms, each taking the same power.

5. If $x=12$, then $3x-2x+5x=$what?

6. If $a=15$, $b=8$, and $c=2$, what does $3a-3b+2c$ equal?

7. In pumping,* if

H = the total head,
h = suction lift,
P = pressure developed in mains,

write an equation expressing the relation of H to h and P.

332. An Unknown Quantity. The unknown quantity in an equation is the quantity for which no numerical value is given and whose value is therefore not known.

In a formula the value of the unknown quantity may be determined by computation after substitution of the known values.

In other equations the unknown must be determined by *solving* the equation. What solution is and how it is done is explained and illustrated in subsequent paragraphs.

333. Solution of an Equation. One can do very little in mathematics or even its simplest applications, without a knowledge of how to determine the value of the unknown quantity in various kinds of equations.

The solution of an equation is the process of determining the value of its unknown quantity or quantities. How this is done can be learned only by considerable practice.

First, it is necessary to learn what the different parts of an equation are, in order that the instructions given for solving an equation may be understood.

334. Members. The equality sign separates an equation into two parts called members. The part at the left and therefore the first one read is the first member; the part at the right is the second member.

Thus in the equation $\text{H.P.} = \frac{PLAN}{33000}$, H.P. is the first member; $\frac{PLAN}{33000}$ is the second.

335. Terms. The signs $+$, $-$, $=$, but no others, separate an equation into parts called terms, and the number of terms in every equation is the same as the number of parts into which the equation is divided by these signs.

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* See problem 13, page 352.

Thus $5x-2x+6x=12-x$

has five terms because the signs separate it into five parts.

The horse-power equation had the same number of terms as members because it had only one sign.

336. Coefficient. Any multiplier is called a coefficient. It is also called a factor. There are therefore three words in mathematics which mean exactly the same thing:

Multiplier, Coefficient, Factor.

Thus in the equation $746\ \text{H.P.}=I^2R$,

746 is the coefficient of H.P., I^2 is the coefficient of R, and R is the coefficient of I^2.

The coefficient of I^2R is 1 because when no numerical coefficient is written 1 is understood.

There are two kinds of coefficients:

(1) Numerical or a figure coefficient.
(2) Literal or a letter coefficient.

Thus in the equation $10W-8b+bx=60$

10 and 8 are numerical coefficients and b is a literal coefficient.

337. Problems. The following problems are given in order to help you to become familiar with the ideas of the preceding paragraphs.

1. What is an equation? What sign divides it into members? What signs divide it into terms? How many members are there? How many more terms than signs?

2. Write an equation having only as many terms as members. Write an equation having five terms.

3. Write the word coefficient and two other words meaning the same thing.

What are the two kinds of coefficients?

Write an equation having each kind of coefficient.

4. $3x^3+2x^2-tx=8$ has how many terms? 2 and 3 are what kind of coefficients? t is what kind of coefficient?

338. A Simple Equation. Equations are classified from the number of units in the exponent of the unknown quantity.

When the unknown quantity has no exponent, the exponent 1 is understood and the equation is said to be simple.

Therefore a simple equation is an equation whose unknown quantity has exponent 1 understood.

Thus $$3x+6x-2x=14$$

is a simple equation because the exponent of x, the unknown quantity, is unity.

$$5x^2-2x+4x=16$$

is not a simple equation because the unknown quantity x has an exponent 2 in one of the terms.

339. How a Simple Equation is Solved. To solve a simple equation,

(1) Collect the unknown terms into the first member.

(2) Collect the known terms into the second member.

(3) Divide the equation by the coefficient of the unknown quantity.

In order to collect it is often necessary to remove a term from one member and to write it in the other.

This term is then said to be transposed. Whenever this is done, *be sure to change the sign* of the terms transposed.

Thus $$20x-5x+6x-4=8+15x$$

when transposed becomes

$$20x-5x+6x-15x=8+4.$$

To collect terms is to add the minus terms together, and the plus terms together, and to set down the excess of plus or minus.

Thus, when the preceding terms are collected, the excess of plus over minus in the first member is $6x$.

We therefore write $6x=12$.

Dividing the equation by the coefficient of x, we have $x=2$.

340. Examples. Solve the following equations:

If the equation is fractional, multiply it by the least common denominator before collecting.

If the coefficient of the unknown after collecting, is minus, the terms obtained by division must be written with their signs changed, by the law of signs in division.

1. $8x-2x=24$

2. $13x-5x-7=13-2x$

3. $5x+12x+6=2x+36$

4. $29-18x+5x=79-3x$

5. $8-3x+9x=-5x+30$

6. $-9+31x-22x+6x=31-5x$

7. $2(3x-4+11x)=5x+38$

8. $25y-10y+7y-12=76-3y+2$

9. $\frac{3x}{2}+8=13$

10. $5x-\frac{7}{2}=\frac{13}{2}$

11. $\frac{4y}{3}-5=\frac{13}{3}$

12. $3x-\frac{3}{8}+2=5x-7$

13. $2x-\frac{3}{4}=2-\frac{x}{2}$

14. $3(6x-5x+8x-3)=36$

15. $4b-25=-9-12b$

16. $10a-21=32a+312-52a$

17. $18-x-5x-3=2x-7x$

18. $\frac{x}{2}+\frac{x}{5}-\frac{x}{3}+\frac{1}{2}=20$

19. $5x-1+6x-5=2x+12$

20. $2x-\frac{x}{6}+12\frac{1}{2}=\frac{x}{3}-\frac{2x}{2}$

21. $\frac{3}{2r}-7=\frac{5}{2r}-\frac{3}{r}$

22. $\frac{5x-2}{3}+\frac{3x+4}{4}=5\frac{1}{6}$

23. $\frac{11x+2}{14}+\frac{2-9x}{7}-1=3$

24. $\frac{17x+7}{9}+\frac{7x-\frac{2}{3}}{6}=7$

341. Solution of Equations Having Minus Fractions. Equations frequently contain fractions preceded by a minus sign.

Whenever such equations are cleared of fractions by multiplication by the least common denominator, all terms resulting from the multiplication of the minus fractions must be set down with changed signs because such terms are subtrahends.

Thus
$$\frac{2x-3}{5}-\frac{6x-4}{10}=-1$$
$$4x-6-6x+4=-10$$
$$-2x=-8$$
$$x=4$$

The numerator of the second fraction is $+6x-4$. But it is a subtrahend and therefore is set down with changed signs when multiplied by 10, the least common denominator.

Do not fail to notice that the minus between the fractions denotes the subtraction of the second fraction from the first, and that it is *not* the sign of the $6x$.

342. Examples. Solve the following equations and be sure to change the signs of all terms obtained from the multiplication of a minus fraction by the L.C.D.

1. $3x-\frac{x-2}{5}=20$
2. $4x-\frac{5x+8}{6}=5$
3. $12y-\frac{4y+26}{15}=10$
4. $\frac{6t+8}{4}-\frac{2t+1}{2}=1$
5. $6y-\frac{2y+13}{15}=5$
6. $5x-\frac{7x-2}{14}-\frac{2x-10}{7}=10$
7. $10+\frac{12x-9}{5}-\frac{4x-5}{3}=6x$
8. $8x-\frac{6-5x}{7}+\frac{2}{21}+\frac{6x-14}{3}-7=9$
9. $\frac{11x}{4}-\frac{5x}{6}-\frac{x+10}{3}=3$
10. $\frac{4b-18}{5}-\frac{5-8b}{3}-\frac{2b-189}{45}=40$

343. Trigonometric Equations. When an equation contains the cosine, sine, or tangent of an angle, it is called a trigonometric equation because these quantities are a part of the branch of mathematics known as trigonometry.

In the following equations the numerical values of the trigonometric quantities should be read from the 3-place table.

344. Examples. Solve the following equations. Results not integral should be expressed to two significant figures of the decimal.

1. $x \sin 45^\circ + \tan 45^\circ = 3.414$

2. $3x \sin 20^\circ - x \tan 5^\circ = 3.756$

3. $5x \cos 20^\circ + 6x \sin 32^\circ = 52$

4. $\frac{\sin 41^\circ + 16}{3} = 2x$

5. $\frac{3d-9}{5} = \tan 85^\circ$

6. $b \sin 34^\circ = \frac{8}{\tan 45}$

7. $\frac{.7y \tan 51^\circ}{2} = \frac{3}{4}$

8. $\frac{4 \cos 58^\circ}{2} = \frac{x}{4}$

9. $\frac{x}{8} = 10 \sin 24^\circ\, 30'$

10. $\frac{6 \tan 35^\circ\, 15'}{5} = 21h - 4$

11. $\frac{E}{120} = \frac{\sin 45^\circ}{\sin 90^\circ}$

12. $\tan 56^\circ = \frac{.89}{b}$

13. $\frac{P}{117} = 35.2 \cos 31^\circ\, 20'$

14. $\tan \theta = \frac{110 \sin 25^\circ}{55}$

15. $\frac{P}{\cos 35^\circ} = 1000 \sin 7^\circ$

16. $\frac{\cos 18^\circ\, 20'}{P} = \frac{1}{250 \times 75}$

17. $i = 120 \sin 51^\circ + 110 \sin 51^\circ$

18. $I = \sqrt{15 \cos 49^\circ}$

19. $I = \sqrt{20 \cos 45^\circ + 14 \cos 40^\circ}$

20. $50 \tan \theta = 8.74$

21. $K.\ V.\ A. = \frac{50}{\cos 17^\circ}$

22. $a = \frac{34.2 \cos 70^\circ}{5}$

23. $V \times 10^{-3} = \frac{\cos 29^\circ \tan 45^\circ}{\cos 15^\circ \tan 32^\circ\, 40'}$

24. $\cos 30^\circ = \frac{R}{\sqrt{90^2 + 58}}$

25. $T = 2.3 \sin (2 \times 40^\circ)$

26. $T = 3.2 \sin 40^\circ \cos 40^\circ$

27. $P \cos 48^\circ = \frac{31.4 \sin 20^\circ - .053}{.5}$

28. $144 \sin \theta = \frac{1728 \tan 34^\circ}{11}$

29. $12 \cos^2 25^\circ = \frac{R}{9} + 16$

30. $\frac{6 \cos^2 42^\circ + 5 \cos 34^\circ}{3} = 4x$

31. $\frac{3x}{4}-\frac{5x+8}{12}=3\sin 18^\circ 20'$ **32.** $c^2=5^2+8^2-2\times 5\times 8\cos 72^\circ$

33. $H-193.7=\sin 42^\circ \cos 42^\circ$ **34.** $11.2=\frac{\cos^2\theta+31.84}{3}$

35. $b=\frac{\tan 35^\circ-\tan 26^\circ}{1+\tan 35^\circ \tan 26^\circ}$

§ 3. TWO UNKNOWN QUANTITIES

345. Relation of Number of Equations to Number of Unknown Quantities. When an equation has two unknown quantities the values of the unknowns are unlimited and any particular values cannot therefore be determined.

Thus in $$a+b=48$$

b and a may have any number of values and their sum still equal 48; as for example

$$6+42=48$$
$$12+36=48.$$

If, however, a second equation containing a and b is given, in which a and b have exactly the same values as in the first equation, their values may be determined by solution as explained in the next paragraph.

Therefore in order to solve when there are *two unknowns, two equations are necessary,* and the unknowns in one equation must have exactly the same values as in the other.

When this is the case the equations are said to be simultaneous.

Simultaneous therefore means

(1) As many equations as unknown quantities.

(2) Exactly the same values of the unknowns in all the equations.

346. How to Solve. Where there are two unknowns and therefore two equations, they may be solved as follows:

(1) Multiply or divide so as to make the coefficients of one of the unknowns exactly the same in both equations.

(2) Add the two equations if the signs of the terms with equal coefficients are different. If the signs are alike, change all the signs of one of the equations, and add.

(3) Divide the equation obtained by adding or subtracting, by the coefficient of the unknown.

(4) In one of the given equations substitute the value thus determined and solve for the other unknown.

347. Model Solutions.

Signs different:

(1) $3a + 2b = 24$
(2) $5a - 3b = 2$
(3) $9a + 6b = 72$ ——————Mul. (1) by 3
(4) $10a - 6b = 4$ ——————Mul. (2) by 2
(5) $19a = 76$ ——————Add (3) and (4)
(6) $a = 4$ ——————Div. (5) by 19
(7) $12 + 2b = 24$ ——————In (1) subs. from (6)
(8) $2b = 12$ ——————Col. in (7)
(9) $b = 6$ ——————Div. (8) by 2

Signs alike:

(1) $7x + 4y = 34$
(2) $2x + 5y = 29$
(3) $14x + 8y = 68$ ——————Mul. (1) by 2
(4) ${}_{-}14x \underset{-}{+} 35y = {}_{-}203$ ——————Mul. (2) by 7
(5) $-27y = -135$ ——————Sub. (4) from (3)
(6) $y = 5$ ——————Div. (8) by -27
(7) $2x + 25 = 29$ ——————In (2) subs. from (6)
$2x = 4$ ——————Col. in (7)
$x = 2$ ——————Div. (8) by 2

348. Examples. Solve the following examples:

1. $x+2y=5$
$3x-y=1$

2. $2x+5y=19$
$5x+y=13$

3. $6x+y=22$
$4x-y=8$

4. $2x+3y=23$
$10x-5y=15$

5. $11x-3y=37$
$-2x+4y=14$

6. $-3x+7y=35$
$-5x-y=-43$

7. $4x-2y=10$
$-12x+15y=33$

8. $15x-9y=39$
$4x-3y=5$

9. $3\frac{1}{2}x-7y=21$
$8x-20y=40$

10. $15x+30y=15$
$8x-7y=54$

11. $-9x+8y=-5$
$16x-18y=24$

12. $14x+30y=40$
$24x+50y=60$

13. $-9b+5y=93$
$20b+8y=80$

14. $\frac{3x}{4}-\frac{5y}{12}=-24$

15. $\frac{1+2x}{3}-\frac{6+5y}{4}=8$
$\frac{3x}{8}+\frac{1}{2}-\frac{2y}{4}=1$

16. $\frac{1+2x}{11}+\frac{3+5y}{9}=5$
$x-y=13$

17. $5x+\frac{1}{2}+15y=33$
$\frac{24x}{5}-\frac{2}{5}+4y=10$

18. $3x+\frac{8-2y}{4}=14$
$\frac{10x}{3}-\frac{2}{3}+\frac{2+5y}{2}=27$

19. $\frac{5x}{3}-2y=3$
$3x+5y=14$

20. $\frac{7x}{2}-3y=26$
$\frac{3x}{4}+\frac{5y}{8}=\frac{1}{2}$

21. $\frac{13b}{4}-6y+2=40$
$\frac{b}{2}-\frac{y}{6}=4\frac{1}{3}$

22. $\frac{2t}{7}+\frac{8s}{3}=10$
$\frac{31-t}{8}-\frac{3s}{4}=\frac{3}{4}$

23. $\frac{8+d}{6}-4e=-1$
$\frac{3d-5}{7}+8e=7$

§ 4. FORMULAS

349. What a Formula is. When the relation of a quantity to one or more others is expressed by an equality sign between significant symbols denoting the quantities, the expression is called a formula.

A significant symbol is one which by its form suggests the quantity for which it is used, or which is suggestive because commonly used to denote a certain quantity.

Thus the relation of watts to current and e.m.f., is

$$W=IE$$

In this formula, W signifying watts and E signifying e.m.f. in volts are significant because each is the initial letter of the quantity denoted.

I denoting current or intensity in amperes, is significant for the same reason.

The conventional expression of the same relation might be

$$x=yz,$$

no symbol of which suggests the quantity represented. Although this last expression is an equation, it is not a formula because its symbols are *not* significant and neither it nor any other like expression will be considered in this section.

350. Transformation. The transformation of a formula is its solution for any of its quantities in terms of the others.

Thus Ohm's Law

$$(1) \qquad I=\frac{E}{R}$$

is transformed by solution into

$$(2) \qquad E=IR,$$

and (3) $$R = \frac{E}{I}.$$

In the three preceding equations, (1) is a formula for current in terms of voltage and resistance, (2) is a formula for voltage in terms of current and resistance, and (3) is a formula for resistance in terms of voltage and current.

In other words each is a formula for the quantity which is the first member, in terms of the quantities which are the second member.

351. Principles of Transformation. Following are the general principles by means of which almost any formula may be solved for any required quantity. In the statmment of these principles the quantity for which solution is required is called the unknown quantity.

(1) Observe where the unknown quantity is.

(2) If the unknown is under a radical, (1) transpose so that the radical is the only quantity in one of the members, (2) divide the equation by the coefficient of the radical, and (3) raise the equation to a power corresponding to the root index.

(3) If the unknown is inclosed in a parenthesis having a coefficient, (1) transpose so that the parenthesis forms one member, and (2) remove the parenthesis by dividing the equation by the coefficient of the parenthesis.

(4) When the unknown is in both terms of a fraction, or in the denominator only, multiply the equation by the denominator of the fraction.

(5) Collect all unknown terms into the first member and all other terms into the second, as in the solution of any equation.

(6) When the terms have been collected as specified in (5), if the unknown has a coefficient either integral or fractional, *divide* the equation by it with whatever sign it happens to have.

The preceding principles have been summarized for reference only and no attempt should be made to commit them to memory. Practice and repeated application will make them familiar. The sooner this familiarity is established, the sooner will you be able to use engineering handbooks and to read technical books and periodicals.

After completing the work in this book, not a week should pass in which a considerable time is not spent in reading and working through one or more topics involving applied mathematics, connected either with the work in which you are now engaged, or in which you are interested, or about which you wish to learn.

352. Examples in Transformation. Solve the following formulas for the specified quantities. When unable to determine what operation to perform first, refer to the principles of transformation in the preceding paragraph.

1. Law of the Lever.

$$\frac{P}{W}=\frac{d}{D}$$

Solve for P and W.

2. Law of Inclined Plane.

$$\frac{P}{W}=\frac{h}{L}$$

Solve for h and L.

3. Law of Pulleys.

$$P=\frac{W}{n}$$

Solve for W and n.

4. Law of Pulleys.

$$P(2^n-1)=W$$

Solve for P.

5. Law of Wheel and Axle.

$$\frac{P}{W}=\frac{R}{r}$$

Solve for P, R, and r.

6. Law of Pulleys.

$$2^nP=W$$

Solve for P.

7. Law of the Wedge.

$$\frac{R}{P}=\frac{L}{t}$$

Solve for R, P, and t.

8. Law of the Screw.

$$2\pi rP=pW$$

Solve for P, p, and r.

9. Law of Hydraulic Press.

$$\pi R^2P=\pi r^2W$$

Solve for R and W.

10. Law of Vertical Component.

$$y=F\sin\theta$$

Solve for $\sin\theta$ and F.

11. Law of Horizontal Component.

$$\frac{x}{\cos\theta}=F$$

Solve for $\cos\theta$ and x.

12. Velocity Ratio in a Screw-jack.

$$R=\frac{PV}{2\pi}$$

Solve for V.

13. Law of Specific Gravity.

$$62.3V=\frac{W}{sp.gr.}$$

Solve for W.

14. Law of Hypotenuse.

$$h=\sqrt{a^2+b^2}$$

Solve for a and b.

15. Law of Stretch of a Steel Spring.

$$\frac{Sr^4}{33\times10^8}=nW$$

Solve for S and r.

16. Law of Young's Modulus.

$$\frac{AeE}{L}=P$$

Solve for E.

17. Law of Inclined Plane.

$$\frac{P}{\sin\theta}=W$$

Solve for $\sin\theta$.

18. Law of Diameter of Shafting.

$$d=\sqrt[3]{\frac{50H}{N}}$$

Solve for H.

19. Law of Stress.

$$L=\frac{P_1-P}{\sin\theta}$$

Solve for $\sin\theta$.

20. Law of Reaction.

$$R=\frac{n}{4\cos^2\theta}$$

Solve for $\cos\theta$.

21. Law of Flow.

$$V=.97\sqrt{2gH}$$

Solve for H.

22. Law of Mechanical Efficiency.

$$\frac{1}{\text{I.H.P.}}=\frac{\text{M.E.}}{100\text{ B.H.P.}}$$

Solve for M.E.

23. Law of Differential Pulley.

$$W(R-r)=2PR$$

Solve for R, P, and W.

24. Law of Diameter of Ball-circle.

$$\frac{D}{2}=\frac{r+\frac{s}{2}}{\sin\frac{180^\circ}{n}}$$

Solve for D.

25. Power in Terms of Current and Voltage.

$$IE=W$$

Solve for I and E.

26. Power in Terms of Voltage and Resistance.

$$\frac{E^2}{R}=W$$

Solve for E.

27. Power in Terms of Current and Resistance.

$$I^2R=W$$

Solve for R and I.

28. Ohm's Law.

$$I=\frac{E}{R+r}$$

Solve for E, R, and r.

29. Ohm's Law.

$$I=\frac{E}{R}$$

Solve for E and R.

30. Circular Mil Area.

$$\text{C.M.}=D^2$$

Solve for D.

31. Square Mil Area.

$$\frac{\text{sq. M.}}{.7854}=\text{C.M.}$$

Solve for sq. M.

32. Weight Bare Copper Wire per Mile in Pounds.

$$W_c=\frac{\text{C.M.}}{62.5}$$

Write a formula for weight per foot.

33. Weight Bare Iron Wire per Mile in Pounds.

$$W_i=\frac{\text{C.M.}}{72.13}$$

Write a formula for weight per foot.

34. Temperature Fahrenheit in Terms of Centigrade.

$$\text{F}^\circ=\frac{9\text{C}^\circ}{5}+32^\circ$$

Solve for C°.

35. Size of Copper Wire for Direct Current.

$$\text{C.M.}=\frac{10.79L'I}{E}$$

Solve for E and I.

36. Size of Copper Wire for 3-wire System.

$$\text{C.M.}=\frac{10.79L'I}{4E}.$$

Solve for E and I.

37. Current in Terms of H.P. and per cent Efficiency of Motor.

$$I=\frac{746\text{ H.P.}}{\%\text{ ME.}}$$

Solve for H.P., per cent M, and E.

Solve the following for any of the quantities in the second member.

38. $Q=It$

39. $I=\frac{W}{Kt}$

40. $t=\frac{V}{IK}$

41. $R=\frac{KL}{d^2}$

42. $\text{Kw}=\frac{E^2}{1000R}$

43. $\text{Kw}=\frac{I^2R}{1000}$

44. $N=\frac{4Wr}{E^2}$

45. $Eff=\frac{R}{R+r}$

46. $H=.0009477EIt$

47. $W=\frac{\frac{\%GST\times746}{33000}}{\%M}$

48. $I=\frac{Hr\tan\theta}{N}$

49. $I=\frac{Ens}{rns+R}$

50. $I=\frac{E}{\frac{r}{nq}+R}$

51. $\tan\theta=\frac{I_L-I_c}{I_R}$

52. $.00381T=\frac{R_t-R_T}{R_T}$

53. $q=\frac{wq_1}{w+w_1}$

54. $M=\frac{W}{32.2}(v_2-v_1)$

55. $\frac{Wv^2E}{2g}=\frac{tR}{12}$

§ 5. THE QUADRATIC

353. Classification of Equations. The name by which an equation is called is determined by the number of units in the exponent of the unknown quantity after the equation has been cleared of fractions and radicals (indicated roots). An equation whose unknown quantity has no exexponent, and therefore exponent 1 is understood, is a simple equation, so called because its solution is simple.

An equation whose unknown quantity has an exponent 2 is a quadratic, so called because an exponent 2 denotes a square and the word " quadratic " means *square.*

Equations are therefore classified as follows:

Simple, $3x+45=x-1$
Quadratic, $4x^2-2x=5$
Cubic, $3x^3-8x=2$
Higher degree, $5x^5-8x^2=9$

354. Kinds of Quadratics. A quadratic is an equation which has the second power of the unknown quantity but no higher power when collected and having no fractions or radicals. There are two kinds of quadratics:

(1) Incomplete, $3x^2+9=24$.
(2) Complete, $2x^2-x=7$

An incomplete quadratic has no first power of the unknown quantity.

A complete quadratic has both the first and the second power of the unknown.

355. How an Incomplete Quadratic is Solved. The solution of a quadratic which has only the second power of the unknown quantity is a simple matter. For example:

(*a*) $6x^2+18=2x^2+46,$
(1) $4x^2=28,$
(2) $x^2=7,$
(3) $x=\pm 2.646.$

In (3) the sign following the equality sign is what is known as the double sign, meaning plus or minus. It must be placed before every square root obtained in the solution of a quadratic.

356. Problems. Determine the numerical value of x in the following equations:

1. $3x^2-2=18+x^2$

2. $4x^2=45-x^2$

3. $8+2x^2=28$

4. $5x^2-4=6x^2-14$

5. $7-6x^2=2x^2-63$

6. $3+12x^2=84+3x^2$

7. $10+2x^2=-7x^2+100$

8. $6x^2-18=4x^2+40$

357. How a Complete Quadratic is Solved. There are several ways of solving a complete quadratic, only one of which will be shown here. This method is called solution by *completing the square.*

What this method is and why it is so called is illustrated and explained below:

(*a*) $3x^2+2x=8$

(1) $x^2+\frac{2}{3}x=\frac{8}{3}$ Div. (a) by 3

(2) $x^2+\frac{2}{3}x+(\frac{1}{3})^2=\frac{8}{3}+\frac{1}{9}=\frac{25}{9}$ Add $\frac{1}{9}$ to (1)

(3) $x+\frac{1}{3}=\pm\frac{5}{3}$ Sq. Root (2)

(4) $x=\pm\frac{5}{3}-\frac{1}{3}$ Sub. $\frac{1}{3}$ in (3)

(5) $x=\frac{4}{3}$ or -2 Col. in (4)

Observe that the given equation was arranged in the order of the descending powers of x, with unknowns in the first member and knowns in the second member; the coefficient of x^2 was made unity by dividing equation (*a*) by 3; the first member of (2) was made a square by adding $\left(\frac{\text{coefficient of } x}{2}\right)^2$ to both members; the square root of the equation was extracted, giving (3), which was solved as a simple equation.

Whenever a complete quadratic is reduced to the form of equation (1) in which the coefficient of x^2 is unity, the first member may be made a perfect square by the addition of ($\frac{1}{2}$ coef. of x)^2 as in (2). It is then possible to transform the quadratic into a simple equation by extraction of the square root as in (3).

358. Rules for Solution of a Complete Quadratic.

(1) Collect the unknowns into the first member in the order of the descending powers; collect knowns into the second member.

(2) Divide the entire equation by the coefficient of the second power of the unknown with its sign.

(3) Add to both members ($\frac{1}{2}$ coef. of x)^2, indicating the addition in the first member and performing the addition in the second member.

(4) Extract the square root of both members, placing the double sign ± before the root of the second member; solve the resulting simple equation, using first the plus sign and then the minus sign.

359. Examples. Apply the rules of the preceding paragraph to the solution of the following equations. Do not try to commit the rules to memory, but refer to them in the book. Be sure to complete the operations required by each rule before applying the next rule.

1. $x^2-5x=-6$
2. $x^2+2x=8$
3. $2x^2+9x=5$
4. $2x^2-5x=-3$
5. $3x^2+5x=-2$
6. $4x^2+3x=10$
7. $3x^2=14x+24$
8. $6x^2+5x=50$
9. $30x^2-7x=2$
10. $8x^2=18x+18$
11. $15x^2-2x=1$
12. $6x^2-4x=16$
13. $15x^2-18x=\frac{8}{3}$
14. $24x^2=-7x+\frac{3}{4}$
15. $10x^2+4x-\frac{1}{2}=0$
16. $36x^2=24x-3$
17. $32x^2-2x=\frac{3}{2}$
18. $132x^2-9x=6$
19. $5x=25-42x^2$
20. $32x^2-\frac{25}{2}=-30x$

21. $x^2-12x+30=3$

22. $x^2+16x=17$

23. $6x^2-6x+9=5x^2+1$

24. $5x^2-16-6x=4x^2-2x-4$

25. $4x^2-25+19x=3x-4x+25$

26. $2x^2-2x-9=x^2-1$

27. $x^2+144=24x+169$

28. $x^2+\frac{x}{6}=\frac{1}{3}$

29. $x^2+\frac{4x}{3}+\frac{1}{3}=0$

30. $x^2-5=\frac{x}{2}$

31. $x^2+14\frac{3}{4}=16+\frac{2x}{3}$

32. $\frac{x^2}{2}-\frac{x}{3}-7\frac{5}{8}=-7$

33. $25x^2-77+20x=9x^2+4x$

34. $x^2-\frac{3x}{4}-5=16\frac{1}{4}$

35. $x^2-16-\frac{2x}{3}+14\frac{3}{4}=16$

36. $x+\frac{1}{x}=\frac{13}{6}$

37. $3x-\frac{3}{x}+8=10$

38. $\frac{4}{x}+5=6x$

39. $\frac{5}{x}+\frac{x-7}{x^2}=\frac{11}{9}$

40. $\frac{9}{x}+\frac{25x}{x-1}+9=0$

CHAPTER XIV

LOGARITHMS

SECTION 1. INTRODUCTION. SECTION 2. USE OF THE TABLE. SECTION 3. COMPUTATION BY LOGARITHMS.

§ 1. INTRODUCTION

360. Illustrations of a Logarithm. If a series of integral powers of 10 be written beginning with 10^4 and ending with 10^{-4} we have

$$\begin{aligned} 10^4 &= 10000 \\ 10^3 &= 1000 \\ 10^2 &= 100 \\ 10^1 &= 10 \\ 10^0 &= 1 \\ 10^{-1} &= .1 \\ 10^{-2} &= .01 \\ 10^{-3} &= .001 \\ 10^{-4} &= .0001 \end{aligned}$$

Observe that as we go down the column the exponents of 10 decrease by 1 and that each number obtained from the successive powers of 10 is therefore $\frac{1}{10}$ the preceding number. All these exponents of 10 are logarithms. 10000 may be obtained from 10 by raising 10 to the 4th power. 1000 may be obtained by raising 10 to the 3d power, and so on.

We may, therefore, multiply 10000 by 1000 by adding 4 and 3, the exponents of 10, which gives the 7th power of 10. In like manner if we could determine exactly what power of 10, 3250 is, and exactly what power of 10, 8628 is, we could determine the product

$$3250 \times 8628$$

by adding those powers of 10 and by writing down the number which the resulting power of 10 equals.

By reference to the powers of 10 at the beginning of this paragraph, it will be seen that 3250 and 8628 are greater than the 3d power of 10 and less than the 4th power. Therefore

$$3250 = 10^{3.+}$$

and $$8628 = 10^{3.+}$$

What is wanted, therefore, is a means of determining the decimal which must be added to the 3d power in each instance. These decimal parts of the powers of 10 have been determined and tabulated for all numbers used in computation as explained in the next paragraph. Such a tabulation is called a table or system of logarithms. The number from whose powers other numbers are obtained is called the base of the system.

361. Definition of a Logarithm. A logarithm is the exponent of the power to which the base of the system must be raised to give any other number.

Thus $$10^4 = 10000$$

Therefore $$\log 10000 = 4$$

$$10^{-3} = .001$$

Therefore $$\log .001 = -3$$

Hence $$\log 3250 = 3.+$$

And $$\log 8628 = 3.+$$

The system of logarithms in general use is the common or Briggs' system, the base of which is 10.

It will be evident that the logarithms of most numbers have two parts: one integral; the other, decimal. The integral part is the characteristic, the decimal part is the mantissa. The characteristic can be determined by inspection; the mantissa must be read from a table.

A table of logarithms is therefore a mantissa table.

362. How the Characteristic is Determined.

1. NUMBER GREATER THAN UNITY. The characteristic of the logarithm of a number greater than unity is 1 *less* than the number of integral figures in the number.

Thus $\log 8628 = 3.+$

$\log 86.28 = 1.+$

$\log 508.3 = 2.+$

The reason for the rule will be evident by reference to paragraph 360.

2. NUMBER LESS THAN UNITY. The characteristic of the logarithm of a number less than unity is *negative* in sign and in unit value is 1 *more* than the number of ciphers immediately following the decimal point in the number.

Thus $\log .069 = \bar{2}.+$

$\log .005 = \bar{3}.+$

$\log .784 = \bar{1}.+$

Observe that the minus is placed above the characteristic to show that it is negative while the mantissa is always plus (positive).

363. Examples. Enter the characteristics in the following table:

TABLE 152

CHARACTERISTICS

Number.	Characteristic.	Number.	Characteristic.
3967		18.23	
396.7		446.5	
3.967		.684	
39.67		.0096	
.0078		3.809	
5.402		21.63	
75.01		508.1	
.0009		.0724	
746.7		.8782	

§ 2. USE OF THE TABLE

364. How to Read a Mantissa. Tables of logarithms are called 3-place, 4-place, 5-place and so on, depending on the number of decimal places in the tabulated mantissas. For convenience decimal points are omitted in the table but must be prefixed to all mantissas when used.

Turn to the 4-place table in the back part of the book. The N-column has the first two figures of the number whose logarithm may be required. The third figure of the number is at the top of the columns from 1 to 9. The mantissa is the number in the column headed by the third figure and in the same line as the first two figures in the N-column.

The logarithm of 862 is therefore read as follows:

Find 86 in the N-column read the mantissa in the 2-column.

The logarithm of 325 is read as follows:

In line with 32 in the N-column read the mantissa in the 5-column.

A number whose logarithm is required is called a *natural* number.

In reading a mantissa be sure to disregard the decimal point in the natural number.

365. Examples. Fill in the omitted entries in the following table, entering both characteristic and mantissa.

TABLE 153
LOGARITHMS

No.	N	log	No.	N	log
1	928		11	.428	
2	137		12	.072	
3	25.4		13	.004	
4	341		14	.518	
5	.738		15	3.96	
6	6.92		16	.78	
7	48.7		17	.0029	
8	591		18	.381	
9	3.92		19	.0042	
10	.178		20	1.72	

366. Proportional Parts. When using a 4-place table to read the logarithm of a number having more than three figures, the table of proportional parts gives the reading for the additional figures.

In the table of proportional parts the figures of the number are at the left of the column; the additions to be made to the logarithm of the first three figures are at the right.

If reading for the fourth figure of a number, add the last figure of the reading directly to the last figure of the mantissa of the first three figures; for a fifth figure, add one-tenth the reading—that is, shift the reading *one* place to the right in setting it down for addition; for a sixth figure set down the reading shifted *two* places to the right, etc.

One or two illustrations will help to make this clear:

(1) Required the logarithm of 38.9765.

$$\begin{array}{lrl} & \log 38.9 = & 1.5900 \\ \text{P.P. reading for} & 7 = & 77 \\ \text{\quad `` \quad ``} & 6 = & 66 \\ \text{\quad `` \quad ``} & 5 = & \underline{55} \\ & \log 38.9765 = & 1.5908 \end{array}$$

Observe that only four decimal figures are retained in the logarithm because the reading is taken from a 4-place table.

(2) Required the logarithm of 176.098.

log 176 = 2.2455
P.P. reading for 9 = 22[*]
" " 8 = 20
log 176.098 = 2.2457

367. Examples. Set down the following numbers and their logarithms, showing all the work in full.

In the results retain only four places.

1. 37.71
2. 434.5
3. 6.51092
4. 9.38063
5. .000937
6. 5412.79
7. 315.685
8. .72349
9. 31.567
10. 803.656
11. 19.635
12. .5418
13. .03562
14. .00073
15. 9.7308
16. .58969
17. .34017
18. 388.402
19. 3.7609
20. 54.6008
21. 701.454
22. 8.0925
23. .0031972
24. 18.764
25. 2.58107
26. 2.6805
27. 34.008
28. 5.5064
29. 738.09
30. 384.17
31. 25.34
32. 1600.8
33. 3007.8
34. .2932
35. 900.64
36. .08307
37. 1.5005
38. .00095
39. 3845.09
40. .003208
41. .07608
42. 1.34007

---

[*] 22 is shifted one place to the right because 9 is the 5th figure of the number.

368. Antilogarithms. An antilogarithm in the Briggs' system is the number which any given power of 10 equals. If 1.7249 is a logarithm it is an exponent of 10. The antilogarithm of 1.7249 is therefore the number which equals $10^{1.7249}$.

Its reading from the table is just the reverse of the reading of the logarithm of a natural number. Since a table of Briggs' or common logarithms is a mantissa table only, the characteristic or integral part of the logarithm is disregarded in taking the reading.

Thus 1.7249

$\underline{3} = 530$

$6 = 7$

Therefore antilog $1.7249 = 53.07$

This may also be expressed as follows:

$$\log^{-1} 1.7249 = 53.07$$

Explanation. When, as is usually the case, the given mantissa is not found in the table, look for the next smaller mantissa. Save work by setting down only the figures that differ from the given figures.

In the preceding illustration, .7249 was not in the table. The next smaller mantissa was .7243, only the last figure of which was set down. The mantissa .7243 is in the zero-column in line with 53 in the N-column. The first three figures of the antilogarithm are therefore 530.

The difference between .7249 and .7243 is 6; by the proportional parts table this gives 7 as the fourth figure of the antilogarithm.

In 1.7249 the characteristic is +1. The antilogarithm therefore has *two* integral figures.

Therefore antilog $1.7249 = 53.07$.

§ 3. COMPUTATION BY LOGARITHMS

369. Operations with Logarithms. In this section logarithms will be used for the following operations:

Powers,
Roots,
Multiplication,
Division.

These operations are governed by the following laws:

Laws of Logarithms.

log power = exponent × log number;

$$\text{log root} = \frac{\text{log number}}{\text{index of root}};$$

log product = sum logs factors;

log quotient = log dividend − log divisor.

Formulas.

Power: $\log a^n = n \log a.$

Root: $\log \sqrt[n]{a} = \frac{\log a}{n}.$

Product: $\log ab = \log a + \log b.$

Quotient: $\log \frac{a}{b} = \log a - \log b.$

370. Powers and Roots. POWERS. By definition of an exponent $(a^3)^4 = a^3 \times a^3 \times a^3 \times a^3 = a^{12}$. The same result may be obtained by multiplying the two exponents 4 and 3 together.

Now $82.95^2 = (10^{1.9189})^2 = 10^{3.8378}.$

Therefore $\log 82.95^2 = 2 \log 82.95 = 2\times 1.9189$

$= 3.8378$

$\underline{6} = 688$

$2 = 3$

Therefore $82.95^2 = 6883.$

Roots. A root is determined by division of the logarithm by the root index.

Thus $\sqrt[3]{12.8} = \sqrt[3]{10^{1.1072}} = 10^{\frac{1,1072}{3}}$

$= 10^{.3691} = 2.34.$

Therefore $\log \sqrt[3]{12.8} = \frac{\log 12.8}{3} = \frac{1.1072}{3}$

$= .3691$

$\underline{2} = 2.34$

Therefore $\sqrt[3]{12.8} = 2.34.$

371. Product and Quotient. Multiplication. By definition of an exponent $a^2 \times a^3 = a \times a \times a \times a \times a = a^5$.

Therefore quantities having exponents are multiplied by adding their exponents,

Now $37.805 \times .6044 = 10^{1.5776} \times 10^{\bar{1}.7813}$

$= 10^{1.3589}$

Therefore $\log (37.805 \times .6044) = \log 37.805 + \log .6044$

$\log 37.805 = 1.5776$

$\log .6044 = \underline{\bar{1}.7813}$

$\log \text{product} = 1.3589$

$\underline{79} = 228$

$10 = 5$

$\text{product} = 22.85$

Division. Since division is the inverse of multiplication, the logarithm of a fraction or quotient is obtained by *subtracting* the logarithm of the divisor from the logarithm of the dividend.

372. Model Solution. Below are simple ways of performing the various computations by logarithms.

Powers: (1)

$$\begin{array}{rl} 3.562^3 = & \underline{\underline{45.2}} \\ .5515 & \\ \underline{24} & \\ .5517 & \\ \underline{3} & \\ 1.6551 & = 45.2 \end{array}$$

(2)

$$\begin{array}{rl} .7846^2 = & \underline{\underline{.6154}} \\ \bar{1}.8943 & \\ \underline{3} & \\ \bar{1}.8946 & \\ \underline{2} & \\ \bar{1}.7892 & \\ \underline{89} & = 615 \\ \bar{3} & = 4 \end{array}$$

Roots: (1)

$$\begin{array}{rl} \sqrt[3]{68.49} = & \underline{\underline{4.092}} \\ 1.8351 & \\ \underline{6} & \\ 3)\,\overline{1.8357} & \\ .6119 & \\ \underline{7} & = 409 \\ \bar{2} & = 2 \end{array}$$

(2)

$$\begin{array}{rl} \sqrt[3]{.7368} = & \underline{\underline{.9032}} \\ \bar{1}.8669 & \\ \underline{5} & \\ \bar{1}.8674 & \\ \underline{3 \qquad -3} & \\ 3)\,\overline{2.8674-3} & \\ \bar{1}.9558 & \\ \underline{7} & = 903 \\ \bar{1} & = 2 \end{array}$$

(3)

$$\begin{array}{rl} \sqrt[3]{.00892} = & \underline{\underline{.2074}} \\ 3)\,\underline{\bar{3}.9504} & \\ \bar{1}.3168 & \\ \underline{0} & = 207 \\ \bar{8} & = 4 \end{array}$$

In (2) 3 was not evenly contained in $\bar{1}$. Therefore 3 was added and its subtraction was indicated before dividing.

Whenever a *root index is not evenly contained* in a negative characteristic, add the index and denote its subtraction before dividing. If the addition of the index is not sufficient to eliminate the negative characteristic, add twice the index or three times the index and so on, as may be necessary.

Whenever a *root index is evenly contained,* divide directly.

Multiplication:

$$\begin{array}{rl}
17.86 \times .964 = & \underline{\underline{17.22}} \\
1.2504 & \\
15 & \\
\underline{\bar{1}.9841} & \\
1.2360 & \\
\underline{55} = & 172 \\
5 = & 2
\end{array}$$

Division:

$$\begin{array}{rl}
\dfrac{34.98}{6.54} = & \underline{\underline{5.356}} \\
1.5428 & \\
\underline{11} & \\
1.5439 & \\
\underline{.8150} & \\
.7289 & \\
\underline{4} = & 535 \\
5 = & 6
\end{array}$$

It is suggested that in computations by logarithms the work be displayed as in these model solutions.

373. Examples in Powers.

1. 17.5^2 **2.** 28.96^2 **3.** 1.784^3

4. 3.605^3 **5.** 2.769^4 **6.** 4.985^2

7. $.7604^2$ **8.** $.538^3$ **9.** $.8196^2$

10. 1.485^3 **11.** 2.9007^2 **12.** 12.75^2

13. 185^2 **14.** 3.008^3 **15.** 5.701^2

16. $.8429^3$ **17.** $.4206^3$ **18.** $.7291^2$

19. 1.9803^3 **20.** 9.82^2 **21.** 6.183^3

22. 3.786^3 **23.** 6.009^3 **24.** $.7584^2$

25. 21.3^3 **26.** $.2918^2$ **27.** 2.64^4

28. 3.18^3 **29.** $.761^2$ **30.** 2.08^5

31. 1.07^6 **32.** 9.09^2 **33.** $.849^3$

34. $.7846^3$ **35.** $.006^2$ **36.** $.9195^3$

374. Examples in Roots.

1. $\sqrt[3]{1728}$
2. $\sqrt[3]{985}$
3. $\sqrt{1728}$
4. $\sqrt{25.4}$
5. $\sqrt{.0765}$
6. $\sqrt{.0084}$
7. $\sqrt{.9831}$
8. $\sqrt[3]{.00764}$
9. $\sqrt[3]{41.68}$
10. $\sqrt[3]{1.83}$
11. $\sqrt[3]{248.7}$
12. $\sqrt[3]{.00482}$
13. $\sqrt[3]{.8154}$
14. $\sqrt[3]{.06529}$
15. $\sqrt{188.9}$
16. $\sqrt{78.62}$
17. $\sqrt[4]{4329}$
18. $\sqrt{25.4}$
19. $\sqrt{\pi}$
20. $\sqrt[3]{\pi}$
21. $\sqrt[1.4]{3.908}$
22. $\sqrt[2.2]{17.65}$
23. $\sqrt[2.7]{258.4}$
24. $\sqrt[.54]{2.65}$
25. $\sqrt[1.2]{8.32}$
26. $\sqrt[1.5]{15.96}$
27. $\sqrt[2.4]{184.2}$
28. $\sqrt[3.1]{2.048}$
29. $\sqrt[3.2]{7500}$
30. $\sqrt[4.2]{8260}$
31. $\sqrt[2.8]{1282}$
32. $\sqrt[1.7]{.849}$
33. $\sqrt[3.8]{.4281}$
34. $\sqrt[1.3]{.0623}$
35. $\sqrt[1.8]{.1792}$
36. $\sqrt[5.2]{8749}$
37. $\sqrt[3.4]{.6208}$
38. $\sqrt[2.1]{.0965}$
39. $\sqrt[.5]{.4592}$
40. $\sqrt[.3]{.000447}$

375. Examples in Multiplication and Division. Make the following computations on the slide-rule:

1. 34.5 × 1.72
2. 51.8 × 7.28
3. 1.829 × 29.12
4. 3.56 × 42.84
5. 17.91 × 8.17
6. 1.345 × 51.63
7. 5.008 × 9.72
8. 58.6 × 1.898

9. 5.64×4.297

10. 9.085×3.007

11. $\frac{63.4}{17.81}$

12. $\frac{5.629}{3.702}$

13. $\frac{641.9}{28.75}$

14. $\frac{8.923}{1.701}$

15. $\frac{28.47}{3.264}$

16. $\frac{1.908}{15.63}$

17. $\frac{12.72}{950.7}$

18. $\frac{34.08}{105.4}$

19. $\frac{7.84^2 \times 1.29^3}{3.672}$

20. $\frac{.374^3 \times 2.986^2}{\sqrt{431.5}}$

21. $.538 \times 1.82$

22. $.0763 \times 19.28$

23. $.00971 \times 212.9$

24. $.3754 \times 2.801$

25. 1.729×84.1

26. $.2963 \times 14.08$

27. $.0345 \times .2912$

28. $.00798 \times .7245$

29. $.0951 \times .00845$

30. $.6429 \times .01828$

31. $\frac{.7225 \times .784}{12.18 \times .927}$

32. $\frac{.3568 \times 48.1}{.0765 \times 20.18}$

33. $\frac{.6215 \times 216.4}{45.9 \times .00734}$

34. $\frac{1.789 \times .0634}{.734 \times 10.8}$

35. $\frac{12.738 \times .00745 \times 182}{43.74 \times .8201 \times 9.21}$

36. $\frac{75.5 \times 5.75 \times 25 \times 12.8}{33000}$

37. $\frac{5.96 \times 4.29 \times 3.09}{50.26 \times 8.43}$

38. $\frac{4.728 \times .0245 \times 1.364}{.392 \times 1.824 \times 21.7}$

39. $\frac{\sqrt{78.9 \times 185}}{.734 \times .549^2}$

40. $\frac{\sqrt[3]{1643 \times 1.284^2}}{\sqrt[3]{.0681} \times \sqrt[3]{.1889}}$

CHAPTER XV

THE SLIDE-RULE

SECTION 1. INTRODUCTION. SECTION 2. SINES. SECTION 3. TANGENTS. SECTION 4. LOGARITHMS. SECTION 5. SQUARES AND SQUARE ROOTS. SECTION 6. MULTIPLICATION. SECTION 7. DIVISION.

§ 1. INTRODUCTION

376. Relation of the Slide-rule to Logarithms. In the preceding chapter the use of logarithms in computation has been described and illustrated. It has been shown how the processes of multiplication and division are simplified into those of addition and subtraction respectively, and how any power or root of a number may be obtained by multiplication of the logarithm of the number by the exponent of the power or by division by the index of the root.

With practice all these operations may be performed on the slide-rule much more quickly and with greater certainty of result than with a table of logarithms.

By the use of a table of logarithms two or more numbers are multiplied together by adding their logarithms and reading the antilogarithm of the sum. To multiply with a slide-rule it is only necessary to place the logarithm of one factor on one piece of the rule, end to end with the logarithm of the other factor on the other piece of the rule and to read off as the product the graduation mark at the end of the combined lengths. Division is performed by a subtraction of the lengths.

In addition to its use for multiplication and division, the slide-rule gives squares and square roots, cubes and cube roots, sines, tangents, the logarithms of numbers, and antilogarithms.

377. Invention of the Slide-rule. So far as known the first rule for computation by the principles of logarithms was designed by John Gunter in 1620, six years after the invention of logarithms. Although scaled on the same principles as the modern rule, this was not a slide-rule for the reason that all scales were on one piece and the combined lengths were added by pricking them off on a straight line. As nearly as can be determined the slide-rule, as an instrument having one piece arranged to *slide* along another, was invented by William Oughtred somewhere between 1620 and 1630.*

The present arrangement of scales, dating from about 1850, was devised by Lieutenant Mannheim of the French army. A Mannheim rule is therefore one having the scales as shown in Fig. 185 and is not so named from any manufacturer. Some of the higher priced rules have modifications of the Mannheim arrangement, but the principles involved in their use are the same as in the regular Mannheim.

378. The Scales. All scales below show principal graduations only.

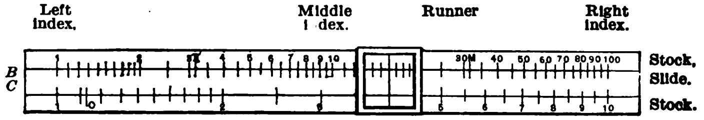

FIG. 185.

Face of Mannheim slide-rule with names of scales and parts.

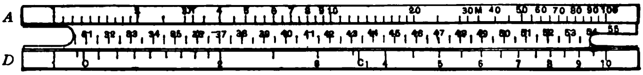

FIG. 186.

Same rule with slide and runner removed showing stock and channel; the channel on some rules has a continuation of the metric scale on the lower edge of the rule.

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* See Cajori, History of the Slide-rule.

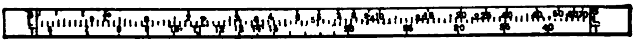


FIG. 187.

Reverse face of slide. Sine and tangent scales with equal parts scale or scale of logarithms between.

The scales shown in the cuts are on all slide rules in common use, but their arrangement varies with different makes. Most rules have one bevelled edge with a 10-inch scale and a plain edge with a metric scale. As shown in Fig. 185, which is a wax process reproduction of a plain Mannheim rule, a slide-rule has three parts: stock, slide, and runner. The stock has scales on one face only, except in the duplex rule; the slide has scales on both front and back. On the face of a rule having the Mannheim arrangement, there are four scales called *A*, *B*, *C*, and *D*. *A* and *D* on the stock, *B* and *C* on the slide. Observe that *A* and *B* are exactly alike and that the second half of each is a duplicate of the first half, the central graduation being called the middle index as shown in the cut.

C and *D* are also exactly alike, but extend the full length of the rule and therefore have no middle index. *C* and *D* being double the length of *A* and *B*, have a greater number of graduations and consequently can be read more closely than *A* and *B*.

Fig. 187 shows the three scales on the back of the slide. The sine and tangent scales are scales giving degrees and minutes of angle, the numerical values of whose functions are read on the *B* and *C* scales respectively, with the slide unreversed.

The *L* scale through the center of the back of the slide is a scale of logarithms of numbers on the *D* scale.

379. Graduation of the Face Scales. Examine the *D* scale. Observe that it has 10 primary graduations numbered consecutively from 1 to 10, each considerably shorter than the preceding. The graduations of the face scales are not uniform because they do not represent the numbers marked on them, but the logarithms of those numbers. Consequently as the scale advances the graduations continually decrease in length. The same principle is used in the graduation of the other face scales, all being graduated proportionally to the logarithms of the numbers represented on the scales.

The table following shows how to read the face scales:

TABLE 154

VALUES OF GRADUATIONS OF FACE SCALES

Scales.	Range.	Graduations.		
		Principal.	Intermediate.	Smallest.
C and *D*	1 to 2	.1	.05	.01
	2 to 4	.1	none	.02
	4 to 10	.1	.05	.05
A and *B*	1 to 2	.1	none	.02
	2 to 5	.1	.05	.05
	5 to 10	.5	none	.1

It should be evident that if the left index be taken as 10 and *not* 1, all graduations will have 10 times the tabulated value; if the left index be taken as 100, all graduations will have 100 times the tabulated value, etc.

Do not be satisfied to glance at this table and then take up the work in the next paragraph. Study the rule and the table until any number can be located on the different scales.

§ 2. SINES

380. Graduation of the Sine Scale. On a plain Mannheim rule the sine or S scale is the upper scale on the back of the slide when the rule is turned over. If the scale is not marked S on your rule it may be distinguished by its numbered graduations 40′, 50′, and 1° to 70°. Observe that the last two graduations, which are 80° and 90°, are not numbered for the reason that they are so close together. The sine or S scale therefore gives a range of angles from about 35′ to 90°. In other words it is a scale of *angles* and not a scale of sines.

Observe that the numbered graduations are subdivided as follows:

From 1° to 10° into 5′ divisions;
10° to 20° into 10′ divisions;
20° to 40° into 30′ divisions;
40° to 70° into 1° divisions;
70° to 80° into 2° divisions.

381. How a Sine is Read. On a Mannheim rule a sine is read on the B scale under the right A index, as follows:

(1) With the rule turned over move the slide so as to place the given angle on the S scale, under the index mark on the upper edge of the notch at the right end of the rule, or under the index mark on the transparent plate if your rule is so made.

(2) Reverse the rule and move the runner to the right A index.

(3) Read the sine under the runner, on the B scale.

Rule for Pointing Off a Sine. Place point cipher before all sine readings on *left B*; place point only, before all sine readings on *right B* except sin 90°, which equals 1.

382. Examples in Reading Sines. Read and tabulate the sines of the angles in Table 155.

TABLE 155
SINES

Number.	Angle.	Sine.	Number.	Angle.	Sine.
1	18°		11	20° 45′	
2	25°		12	15° 10′	
3	30°		13	28° 35′	
4	45°		14	51° 40′	
5	45′		15	70°	
6	12° 20′		16	90°	
7	34° 30′		17	80°	
8	60°		18	65° 40′	
9	40° 45′		19	5° 15′	
10	8° 40′		20	49° 50′	

§ 3. TANGENTS

383. Graduation of the Tangent Scale. On a plain Mannheim rule the tangent or *T* scale is the lower scale on the back of the slide when the rule is turned over. The scale may be identified if not marked *T*, by the numbered graduations from 6 to 40, meaning 6° to 40°.

Observe that the graduations begin at slightly less than 5° 45′ and end with 45°. The tangent scale is therefore a scale of angles and not tangents.

Remove the slide and note the graduations:

From 5° 43′ to 20° into 5′ divisions;
20° to 45° into 10′ divisions.

384. How a Tangent is Read. 1. ANGLES LESS THAN 5° 43′. As may be verified by a table of tangents, the tangents of angles less than 5° 43′ have the same value to three places as the sines. They are therefore read with the *S* and *B* scales.

2. ANGLES BETWEEN 5° AND 45°. Tangents of angles within the range of the tangent scale are read on the *C* scale over the left *D* index as follows:

(1) With the rule turned over move the slide so as to place the given angle under the index mark on the trans-

parent plate or on the lower edge of the notch in the *left* end of the rule.

(2) Turn the rule over and move the runner to the *left* *D* index.

(3) Under the runner read the tangent on the *C* scale.

3. ANGLES FROM 45° TO 80°. By definition the tangent equals the reciprocal of the cotangent;

or $$\tan\theta = \frac{1}{\cot\theta}.$$

But the word "cotangent" means the complement's tangent or tangent of 90° minus the angle.

Therefore $$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\tan(90^\circ - \theta)}.$$

Tangents of angles between 45° and 80° are therefore read as follows:

(1) Subtract the given angle from 90°.

(2) With the rule turned over set the complementary angle on the tangent scale, to the index mark on the celluloid plate or on the lower edge of the notch in the left end of the rule.

(3) Turn the rule over and set the runner to the *right* *C* index.

(4) Under the runner read the required reciprocal (and therefore the tangent of the given angle) *on the D scale.*

Summary of Tangent Readings.

Angle Less than 5° 43′: Read tangent as a sine, on *B* under the *right* *A* index.

Angle from 5° 43′ to 45°: Read tangent on *C*, over the *left* *D* index.

Angle from 45° to 80°: Set on *T*, 90° minus the angle; read tangent on *D*, under the *right* *C* index.

Angle from 80° to 90°: Read in a table of tangents.

Rule for Pointing off a Tangent. Place point directly before all tangent readings of angles from 5° 43′ to 45° except tan 45° which equals 1; place point immediately following the first figure of all tangent readings of angles from 45° to 80°.

385. Examples in Reading Tangents. Read the tangents of the following angles on the slide-rule and tabulate in the same form as in paragraph 381.

1. 16° 30′
2. 24° 10′
3. 30°
4. 45°
5. 4° 30′
6. 21° 40′
7. 3° 25′
8. 15° 45′
9. 27° 50′
10. 12° 20′
11. 2° 30′
12. 70°
13. 1° 45′
14. 69° 20′
15. 60°
16. 75°
17. 50° 30′
18. 32° 25′
19. 8° 40′
20. 3° 40′
21. 63° 40′
22. 18° 20′
23. 29° 50′
24. 78° 15′
25. 54° 30′
26. 2° 50′
27. 48° 20′
28. 71° 30′
29. 36° 25′
30. 1° 30′

§ 4. LOGARITHMS

386. Law for Characteristic. In determining the logarithm of a number, whether from a table of logarithms or by the slide-rule, the characteristic should be set down before taking the reading, otherwise it may be overlooked.

The rule for characteristics is the same as in the preceding chapter: (1) The characteristic of the logarithm of a number *greater* than unity is *one less* than the number of integral figures in the number.

(2) The characteristic of the logarithm of a decimal is *negative* (denoted by a minus sign over it) and has *one more* unit than the number of ciphers immediately following the decimal point in the number whose logarithm is required.

387. How a Logarithm is Read. MANNHEIM RULE. On a Mannheim rule the mantissa of the logarithm of a number is read with the *L* and *D* scales as follows:

(1) Place the hair line over the number on the *D* scale.

(2) Bring the left index of the slide, to the hair line.

(3) Turn the rule over and read the logarithm on the *L* scale under the *L* index on the lower edge of the notch. If your rule has a wide notch at the back with a transparent celluloid plate covering the scales, read the logarithm on the *L* scale, under the plate index.

DUPLEX RULE. On a duplex rule the *L* scale is on the edge of the rule. The mantissa of the logarithm of a number is read as follows:

(1) Move the runner to the number on *D*. This means place the hair line over the number.

(2) Read the mantissa on the *L* scale, under the index mark on the lower edge of the runner.

LL DUPLEX. On the log log or *LL* duplex rule the *L* scale parallels the *D* scale on the stock and the mantissa of the logarithm of a number is therefore read on *L* under the hair line when the runner is moved to the number on *D*.

388. Graduation of the L Scale. Examine the *L* scale. Observe that it has 10 numbered graduations reading from right to left when the scale is on the back of the slide as in the plain Mannheim rule, and from left to right when on the stock.

Each numbered graduation is divided in half, each half being read .05.

Each half has four equal divisions which are read .01, .02, .03, .04.

Finally each of the four equal divisions of each half of each numbered graduation is subdivided into four equal parts which are therefore read .002, .004, .006, .008.

The *numbered* graduations are read as 10ths.

389. Examples in Reading Logarithms. On the slide-rule read to three places the logarithms of the following numbers. Rule the table in the work-book and fill in the omitted entries.

TABLE 156

LOGARITHMS

No.	Number.	log	No.	Number.	log
1	34		11	.78	
2	52		12	1.12	
3	6.4		13	.14	
4	7.5		14	.095	
5	1.2		15	.0008	
6	12.5		16	.0079	
7	1.25		17	.94	
8	18.1		18	.532	
9	5.4		19	.0645	
10	86.7		20	.455	

§ 5. SQUARES AND SQUARE ROOTS

390. Integral Figures of Number. In computations by slide-rule it is seldom possible to point off the results by the arithmetical rule regarding the number of places, for the reason that the rule at best gives not more than four figures. When therefore the position of the decimal point in a reading cannot be determined by inspection, it may be located by a consideration of the integral figures in the numbers employed.

Following is the law of integral figures:

(1) The number of integral figures in a number greater than 1 is the number of figures at the left of the decimal point.

(2) The number of integral figures in a *decimal* is rated as follows: If the first figure of the decimal immediately follows the decimal point, the number has zero integral figures; if one or more ciphers immediately follow the decimal point, the number of integral figures in the decimal equals

minus the number of ciphers between the decimal point and the first figure.

Thus .9 has zero integral figures;
.086 has -1 integral figures;
.00738 has -2 integral figures.

391. Examples in Integral Figures. Fill in the omitted entries in the following table in the work-book:

TABLE 157

INTEGRAL FIGURES

No.	Number.	Integ. Fig.	No.	Number.	Integ. Fig.
1	13.75		11	.90	
2	1896.1		12	.78504	
3	2.89		13	.00008	
4	.734		14	.0713	
5	.0853		15	.000009	
6	.982		16	.346	
7	.00094		17	.0097	
8	.00781		18	.756	
9	.0964		19	.0832	
10	.483		20	.0081	

392. How to Read a Square. A square was obtained in the preceding chapter by doubling the logarithm and reading the antilogarithm of the product. On the slide-rule the square of a number is read *with no movement of the slide,* as follows:

(1) Move the runner to the number on the *D* scale.

(2) Read the square on the *A* scale under the runner.

The explanation of the reading of a square is simple. Since the 1 to 10 graduation of the *A* scale is exactly half the length of the 1 to 10 graduation of the *D* scale, any length under the runner on *A* from the left index represents twice the logarithm of the aligned number on *D*. The graduation reading at the end of the length on A is therefore the square of the aligned graduation number on D.

Rule for Pointing Off. (1) The number of integral figures in a square on *left A* is *one less* than twice the number of integral figures in the given number.

(2) The number of integral figures in a square on *right A* is twice the number of integral figures in the given number.

393. Examples in Reading Squares. With the slide-rule read the squares of the following numbers:

1. 9
2. 12
3. 15
4. 13
5. 17
6. 19
7. 16
8. 18
9. 21
10. 25
11. 36
12. 2.8
13. 41
14. 63
15. 54
16. 72
17. 39
18. 6.2
19. 8.3
20. 9.1
21. 17
22. .35
23. .085
24. .64
25. .0092
26. .755
27. .0069
28. .814
29. .00075
30. 5.48

394. How to Read a Square Root. The square root of a number may be read on the slide-rule as follows:

(1) Separate the given number or consider it to be separated into periods of two figures each in both directions from the decimal point.

(2) If the first period at the left has *one* figure, set the runner to the number on *left A*; if the first period has *two* figures, set the runner to the number on *right A*.

(3) Read the square root under the runner on D.

Each period in the number gives one figure in the root.

A period of two ciphers gives one cipher in the root.

395. Examples in Reading Square Roots. Read the square roots of the following numbers on the slide-rule:

1. 39
2. 169
3. 384
4. 216
5. 789
6. 504
7. 3.42
8. 131
9. 156
10. 845
11. 1024
12. 1219
13. 5410
14. 4850
15. 34.9
16. 5.42
17. 54.2
18. 17.5
19. 1.75
20. 3.009
21. 1.63
22. 9.78
23. 14.4
24. 26.7
25. .0083
26. .954
27. .425
28. .078
29. .0059
30. 824

§ 6. MULTIPLICATION

396. How to Multiply. The product of two or more numbers may be read with the *A* and *B* scales, or the *C* and *D* scales. Since the latter have double the number of graduations of the *A* and *B* scales, better readings can be obtained from them and the instructions which follow are therefore for the *C* and *D* scales. The values of the graduations of these scales were tabulated in paragraph 379.

Following are rules for multiplying two numbers together on the slide-rule:

(1) Set the runner to the first factor on *D*.

(2) Set the *left* *C* index to the runner. Look along the rule to see whether the second factor on *C* is off (projects beyond) the rule. If it is, move the *right* *C* index to the runner.

(3) Set the runner to the second factor on *C*.

(4) Read the product on *D* under the runner.

Rule for Pointing Off a Product. The number of integral figures in a product read with the *C* and *D* scales when the *slide* projects to the *left*, equals the *sum* of the integral figures in the factors; when the *slide* projects to the *right*, the number of integral figures equals *one less* than the sum of the integral figures in the factors. This may be tabulated as follows:

TABLE 158

INTEGRAL FIGURES IN A *C* AND *D* PRODUCT

Projection of Slide.	Integ. Figs. in Prod.
Left	Sum Integ. Figs. Factors
Right	Integ. Figs. Factors−1

397. How to Diagram a Multiplication. The diagram below shows the setting for reading the product 32.5×2.8. The symbols in the diagram have the following meaning:

L_1 = left index;
R_1 = right index;
$C = C$ scale;
$D = D$ scale;
R = runner;
F_1 = first factor;
F_2 = second factor.

Multiplication.

C	*1	F_2
D	F_1	Prod.

* See (2) paragraph 396.

C	L_1	2.8
D	32.5	91.

$2+1-1=2.$

⟶

Observe the equation for pointing off and the arrow showing the projection of the slide.

398. Examples in Multiplication. Determine the following products on the slide-rule and check by logarithmic computation.

Diagram the setting for not less than six examples, in three of which the slide projects to the right, and in the other three to the left. In all examples show the direction of the slide and the equation for pointing off.

1. 12×38

2. 24×29

3. 18×27

4. 11.5×21.8

5. 34×19.2

6. 2.6×17.5

7. 4.8×13.4

8. 5.4×19.6

9. 7.2×6.54

10. $.48 \times 11.7$

11. $.815 \times .208$

12. $.73 \times 16.2$

13. $135 \times .0073$

14. $652 \times .059$

15. 7.18×21.3

16. $19.1 \times .0054$

17. $705 \times .0046$

18. $.064 \times .0185$

19. $52.3 \times .24$

20. $.138 \times 73$

21. $.0485 \times 38.2$

22. $.538 \times .91$

23. $.092 \times 1.76$

24. $.0067 \times .346$

25. $.283 \times .0045$

26. $7.18 \times .965$

27. 14.2×22.8

28. $4.64 \times .0055$

29. $.539 \times 1.9$

30. $.125 \times \sin 60^\circ$

31. $18.2 \sin 45^\circ$

32. $2.18 \sin 30^\circ$

33. $3.85 \sin 28^\circ\ 30'$

34. 24* $\cos 31^\circ\ 20'$

35. $7.25 \cos 41^\circ\ 50'$

36. $61.4 \cos 54^\circ\ 40'$

---

* cos denotes the cosine, which is the complement's sine.

§ 7. DIVISION

399. How to Divide. Division is performed on the slide-rule by setting runner and slide so that the logarithm of the divisor may be subtracted from the logarithm of the dividend, as follows:

(1) Set the runner to the dividend on *D*.

(2) Set the divisor on *C* under the runner.

(3) Set the runner to the *C* index.

(4) Read the quotient on *D* under the runner.

Briefly: Set the divisor on *C* over the dividend on *D* and read the quotient on *D* under the *C* index.

Diagramatically:

C	Divisor	1
D	Dividend	Quotient

1 in the diagram denotes the *C* index which may be the right or the left depending on which one is " on the rule."

400. Rule for Pointing Off a Quotient. The number of integral figures in a quotient read with the *C* and *D* scales, equals the following:

(1) The number of integral figures in the dividend minus the number of integral figures in the divisor when the slide projects to the *left.*

(2) One *more* than the difference between the number of integral figures in dividend and divisor when the slide projects to the *right.*

TABLE 159

INTEGRAL FIGURES IN A *C* AND *D* QUOTIENT

Projection of Slide.	Integ. Figs. in Quotient.
Left	$\left.\begin{matrix}\text{Integ. Figs.}\\\text{dividend}\end{matrix}\right\} - \left\{\begin{matrix}\text{Integ. Figs.}\\\text{divisor}\end{matrix}\right.$
Right	Difference plus 1

The following diagram exhibits the reading of

$$875 \div 34.2 = \frac{875}{34.2}:$$

C	34.2	1
D	875	25.6

$$3-2+1=2$$
$$\longrightarrow$$

401. Examples in Division. With the slide-rule determine the following quotients. Diagram not less than six settings, three with the slide to the right and three with the slide to the left, with equations for pointing off. Check the results by logarithms.

1. $55.2 \div 35$
2. $84.6 \div 29.2$
3. $1728 \div 12$
4. $256 \div 8.1$
5. $6.83 \div .75$
6. $398 \div 11.4$
7. $.781 \div .0087$
8. $\frac{36.7}{4.92}$
9. $\frac{5.58}{.127}$
10. $\frac{4.38}{.078}$
11. $\frac{.684}{.873}$
12. $\frac{.0428}{.0212}$
13. $\frac{782}{48.3}$
14. $\frac{.645}{.232}$
15. $\frac{17.9}{.0065}$
16. $\frac{21.85}{3.97}$
17. $\frac{.348}{.055}$
18. $\frac{.0038}{.095}$
19. $4.28^2 \div 15.3$
20. $1735 \div 292.7$
21. $34.9 \div 5.84^2$
22. $6.37^2 \div 18.9$
23. $9.29 \div .763$
24. $18.34 \div 2.17^2$

25. $.378 \div .045$

26. $\frac{97}{.841}$

27. $\frac{.752}{.0483}$

28. $\frac{3.14}{12.5}$

29. $\frac{.582}{.129^2}$

30. $\frac{17.6^2}{2.48}$

31. $\frac{3.47^2}{.963}$

32. $\frac{.0812}{.00325}$

33. $\frac{.976}{.274^2}$

34. $\frac{1.79}{.242^2}$

35. $\frac{55.8}{3.08^2}$

36. $\frac{425.5}{16.3^2}$

CHAPTER XVI

THE SOLUTION OF A TRIANGLE

SECTION 1. FUNCTIONS OF ANGLES. SECTION 2. SOLUTION OF A RIGHT TRIANGLE. SECTION 3. SOLUTION OF AN OBLIQUE TRIANGLE.

§ 1. FUNCTIONS OF ANGLES

402. An Angle. The amount of opening or **divergence** of two straight lines which have one point in common is called an **angle.** The divergence is expressed in degrees, a **degree** being one-ninetieth of a right angle.

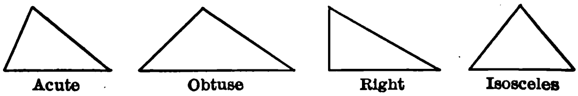
Angles are classified as follows:

Acute,	less than 90°;
Obtuse,	greater than 90° and less than 180°;
Right,	equal to 90°;
Straight,	equal to 180°.

403. A Triangle. When three straight lines inclose space, a **triangle** is formed. A triangle has six parts:

3 angles, and 3 sides.

The different kinds of triangles are shown below:



Acute Obtuse Right Isosceles

FIG. 188.

An **acute** triangle has all angles acute.

An **obtuse** triangle has one obtuse angle.

A **right** triangle has one right angle, the side opposite the right angle being called the hypotenuse.

An **isosceles** triangle has two equal sides and may be acute, obtuse, or right.

To **solve** a triangle is to find the value of its unknown parts or those whose numerical value is not given. Solution therefore necessitates formulation and computation.

Solution is possible only when at least *one side* and two other parts are known.

404. Functions of Angles. In a right triangle there are six **functions** of the acute angles. These functions are the ratios of the sides to each other, and are called functions because the numerical value of the ratios depends on the size of the acute angles.

A **function** is a quantity whose value depends on one or more other quantities.

For the same number of degrees, that is, for angles of the same size, the ratios have the same respective values, whether the triangles are large or small. Three of these ratios are as follows:

1. $\frac{\text{opposite}}{\text{hypotenuse}}$ or $\frac{\text{opp.}}{\text{hypot.}}$
2. $\frac{\text{adjacent}}{\text{hypotenuse}}$ or $\frac{\text{adj.}}{\text{hypot.}}$
3. $\frac{\text{opposite}}{\text{adjacent}}$ or $\frac{\text{opp.}}{\text{adj.}}$

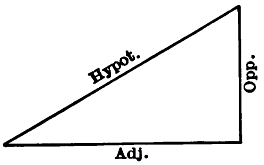


Fig. 189.

In solving a right triangle, one or more of these ratios is always used.

Following are the names of the ratios above, as numbered:

1. Sine;
2. Cosine;
3. Tangent.

We therefore have the following definitions of the functions of acute angles of a right triangle:

The **sine** of either acute angle equals the **opposite** side divided by the **hypotenuse.**

The **cosine** of either acute angle equals the **adjacent** side divided by the **hypotenuse.**

The **tangent** of either acute angle equals the **opposite** side divided by the **adjacent.**

In more convenient form,

$$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$$

405. Notation of a Right Triangle. The symbols by which the sides and angles of a triangle are named are called its notation.

The drawing shows a right triangle with the parts lettered as is customary in books treating of the solution of triangles.

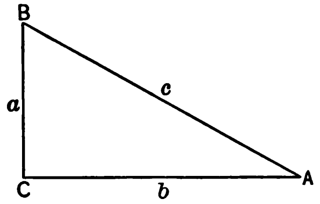


FIG. 190.

As shown, the lengths of the sides are denoted by small letters corresponding to the capitals by which the opposite angles are denoted.

The right angle is denoted by C, and therefore the hypotenuse which is opposite the right angle, is denoted by c.

The acute angles are denoted by A and B, which may be placed as shown, or interchanged.

These letters must always be placed *outside* the triangle as shown in the figure, the capitals being at the vertices and the small letters half way between.

406. The Function Equations or Formulas. The functions of angles are abbreviated and defined as follows:

TABLE 160

FUNCTIONS

Name.	Abbreviation.	Definition.
sine	sin	$\frac{\text{opp}}{\text{hypot}}$
cosine	cos	$\frac{\text{adj}}{\text{hypot}}$
tangent	tan	$\frac{\text{opp}}{\text{adj}}$

If the definitions of the function are applied to the triangle here shown, the following formulas may be written:

$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

$$\tan A = \frac{a}{b}$$

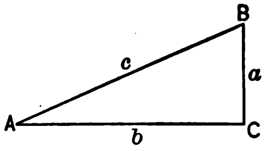


FIG. 191.

Draw the triangle and copy the table and formulas in the work-book.

407. Examples. Solve the following examples in the work-book:

1. Draw the triangle in the preceding paragraph. Write formulas for sin B, cos B, tan B.

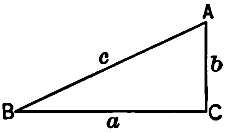


FIG. 192.

2. Draw Fig. 192 about three times as large in the work-book, and write the formulas for sine, cosine, and tangent of each acute angle.

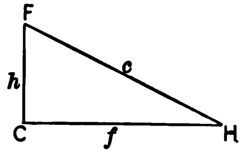


Fig. 193.

3. Draw this triangle in the work-book, the same size as here shown, and write the formulas for the sine, cosine, and tangent of each acute angle.

4. Draw a right triangle whose right angle is *C* and whose acute angles are *T* and *S*, and write the formulas for the sine, cosine, and tangent of both *T* and *S*.

5. Draw a right triangle with right angle *C* and acute angles *V* and *R*, and write the function formulas for each acute angle.

6. (1) Rule the table below in the work-book.

(2) Draw the right triangles specified.

(3) Write three function formulas for each acute angle.

(4) Enter the ratios in the table.

Table 161

RIGHT TRIANGLES

No.	Rt ∠	Acute ∠s.		sin		cos		tan	
		1	2	1	2	1	2	1	2
1	*C*	*D*	*E*						
2	*L*	*W*	*V*						
3	*C*	*O*	*F*						
4	*C*	*M*	*N*						
5	*P*	*Q*	*A*						
6	*H*	*R*	*B*						
7	*C*	*E*	*G*						
8	*I*	*K*	*B*						
9	*V*	*F*	*A*						
10	*C*	*B*	*A*						

408. Numerical Values of Function Ratios. (1) Draw the triangle here shown and write the function formulas for each acute angle.

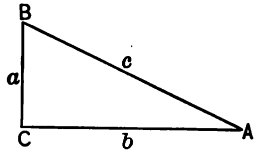


FIG. 194.

(2) In these formulas substitute the following values for the sides of the triangles:

$a=24$
$c=30$
$b=18$

(3) Reduce each fraction obtained in (2) to a decimal of three places. In order to determine the third figure of the decimal it will be necessary to carry the division to four places. If the fourth figure is 5 or greater increase the third figure by 1. If the fourth figure is less than 5 do not increase the third figure of the decimal.

409. Examples in 3-place Functions. In the examples below, the letters denote the sides of right triangles, c being the hypotenuse. Draw each triangle, taking care to make the sides in about the proportion denoted by the data. Write the function formulas for the acute angles, substitute the numerical values, and reduce each ratio to a three-place decimal.

1. $c=42$
$b=35$
$a=$

2. $a=89$
$b=$
$c=112$

3. $b=126$
$a=84$
$c=$

4. $c=185$
$a=125$
$b=$

5. $c=324$
$b=180$
$a=$

6. $b=$
$c=500$
$a=346$

7. $a=425$
$b=650$
$c=$

8. $c=$
$s=96$
$t=114$

9. $v=162$
$r=274$
$c=$

10. $f=348$
$d=526$
$c=$

11. $a=$
$b=671$
$c=1242$

12. $r=354$
$n=608$
$c=$

13. $g =$
$k = 14.56$
$c = 21.65$

14. $a =$
$r = 35.1$
$c = 43.4$

15. $a = 89.6$
$b = 176$
$c =$

16. (1) Accurately measure the sides of this triangle with the U. S. scale, and draw it in the work-book with each dimension doubled.

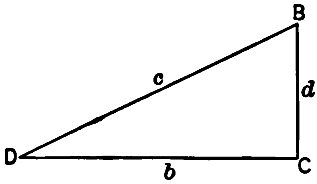


Fig. 195.

(2) Write formulas for sin D, cos D, and tan D.

(3) In each formula, substitute the lengths of the sides as drawn in the work-book.

(4) Reduce the fraction or ratio in each formula to a decimal of 3 places. In order to do this, the fourth decimal figure must be determined. If this is 5 or more, increase the third decimal figure by 1.

17. (1) Measure the sides of the triangle in Problem 16, accurately with the metric scale, and draw the triangle in the work-book with each dimension one and one-half times as long.

(2) Write the function formulas for each acute angle and substitute in them the metric lengths of the sides as drawn, reducing each fraction to a 3-place decimal.

(3) In the function formulas substitute the metric lengths as measured on the triangle in the text, and reduce each fraction to a 3-place decimal.

18. Rule this table in the work-book, and in it tabulate the measurements and the decimals in Problems 16 and 17:

Table 162

TABULATION OF PROBLEMS 16 AND 17

Prob.	Scale.	b	c	d	sin D	cos D	tan D
16	U. S.						
	Metric						
17	U. S.						
	Metric						

410. Function Tables. In the tabulation of Problem 18, decimal values of the same function are practically the same, although one triangle was drawn with its sides double the lengths of the triangle in the text, and the other triangle was drawn with its sides $\frac{3}{2}$ the length of the triangle in the text.

These decimal values would have been exactly the same for the same function, were it possible to measure a figure with absolute accuracy.

This means that the sine, cosine, and tangent of all angles of the same size have the same respective numerical values, regardless of the size of the triangle.

A table of trigonometric functions is a tabulation of the decimal values of the sine, cosine, tangent, or other functions of angles.

Some tables contain eight functions, as follows:

TABLE 163

THE EIGHT FUNCTIONS OF AN ANGLE

No.	Function.	Abbreviation.	Definition.
1	sine	sin	$\frac{\text{opp}}{\text{hyp}}$
2	cosine	cos	$\frac{\text{adj}}{\text{hyp}}$
3	tangent	tan	$\frac{\text{opp}}{\text{adj}}$
4	cosecant	csc	$\frac{1}{\sin}$
5	secant	sec	$\frac{1}{\cos}$
6	cotangent	cot	$\frac{1}{\tan}$
7	versine	vers	$1-\cos$
8	coversine	cvs	$1-\sin$

The functions in the preceding list are called trigonometric functions because they are treated of in that branch of mathematics known as trigonometry, or the measurement or solution of triangles. This chapter is therefore a chapter from trigonometry.

411. How to Read an Angle from its Function. A trigonometric function table makes it possible to determine the magnitude of any angle from any of its functions when expressed in decimal form.

How this is done is shown in the following illustrations:

(1) Angle Read from Sine.

$$\begin{aligned} \sin A &= .714 \\ &\underline{.707} = 45^\circ \\ &\frac{7}{12} \times 60 = 35' \\ A &= 45^\circ\ 35' \end{aligned}$$

Explanation. As .714 is not found in the sine column in the table, the next smaller sine .707 is set down under .714, giving an angle of 45° and 7 difference in sine. A glance at the table shows that sin 46° is 12 more than sin 45°.

This means that at this point in the table, 12 difference in sine makes 1° or 60′ difference in angle. Therefore 7 difference in sine makes a difference in angle, of $\frac{7}{12} \times 60' = 35'$.

Therefore $A = 45^\circ\ 35'$.

(2) Angle Read from Tangent.

$$\begin{aligned} \tan V &= 1.91 \\ &\underline{1.88} = 62^\circ \\ &\frac{3}{8} \times 60' = 22.5' \\ V &= 62^\circ\ 22.5' \end{aligned}$$

Explanation. As 1.91 is not found in the tangent column in the table, the next smaller tangent, 1.88, is set down under 1.91, giving an angle of 62° and 3 difference in tangent. In the table, tan 63° is 8 more than tan 62°. Therefore 8 difference in tangent makes 1° or 60′ difference in angle.

Therefore 3 difference in tangent makes a difference in angle, of $\frac{3}{8}\times 60 = 22.5'$.

Therefore $V = 62^\circ\ 22.5'$.

(3) Angle Read from Cosine.

$$\begin{aligned} \cos R &= .199 \\ &\ \underline{.191 = 79^\circ} \\ &\ \ \frac{8}{17}\times 60' = 28' \\ R &= 78^\circ\ 32' \end{aligned}$$

Explanation. The table shows that the *angle decreases as the cosine increases.*

The cosine given, .199, is 8 larger than .191. Therefore the angle is $\frac{8}{17}\times 60'$ or 28′ *smaller* than 79°, and therefore is 78° 32′.

412. Interpolated Tables. The determination of the minutes in a table reading to degrees, and the seconds in a table reading minutes—is called interpolation. In most tables of five and six places, the addition or subtraction to be made for any reading is shown in a table of proportional parts at the side or at the bottom of the page.

Tables which include the proportional parts are called interpolated tables.

413. Rule for Proportional Parts. In a table reading to the degree the proportional parts are minutes, as shown in paragraph 411. In tables reading to the minute the proportional parts are seconds.

Any trigonometric table will show that

(1) Co-functions *decrease* as angles *increase;*

(2) Other functions increase as angles increase.

Therefore the following rule:

When determining angles from *co-functions, subtract* proportional parts.

When determining angles from other functions, *add* proportional parts.

414. Determination of Angles. In paragraphs 411, 412, and 413 the process of reading an angle from its functions has been illustrated and explained.

Under the same example numbers as in paragraph 409, set down each function which was computed in that paragraph, and determine the angles by the 3-place table. Arrange the work in the same form as in paragraph 411.

415. How to Read Functions of Angles. Below are illustrations of the reading of a sine, cosine, and tangent.

1. sin 29° 42′

$$\begin{aligned} \sin 29^\circ &= .485 \\ \frac{42}{60}\times 15 &= \underline{11} \\ \sin 29^\circ\, 42' &= .496 \end{aligned}$$

Explanation. The table shows sin 30° is 15 more than sin 29°. Therefore 1° or 60′, make 15 difference in sine. Therefore 42′ will make $\frac{42}{60}\times 15 = 11$ difference in sine.

2. tan 40° 17′

$$\begin{aligned} \tan 40^\circ &= .839 \\ \frac{17}{60}\times 30 &= \underline{9} \\ \tan 40^\circ\, 17' &= .848 \end{aligned}$$

3. cos 69° 51′

$$\begin{aligned} \cos 69^\circ &= .358 \\ \frac{51}{60}\times 16 &= \underline{14} \\ & .344 \end{aligned}$$

In reading a co-function be sure to subtract the proportional part as here shown.

416. Examples in Reading Functions. With the 3-place table, read and tabulate the sine, cosine, and tangent of each angle in the table below, arranging the work in the same form as in paragraph 415.

TABLE 164

FUNCTIONS OF ANGLES

No.	A	sin	cos	tan	No.	A	sin	cos	tan
1	18° 15′				16	15° 28′			
2	23° 34′				17	45° 5′			
3	27° 50′				18	72° 9′			
4	42° 22′				19	14° 23′			
5	10° 19′				20	28° 7′			
6	41° 34′				21	56° 12′			
7	56° 18′				22	80° 29′			
8	8° 27′				23	55° 31′			
9	33° 19′				24	78° 41′			
10	50° 30′				25	85° 16′			
11	12° 45′				26	51° 35′			
12	16° 27′				27	18° 51′			
13	43° 19′				28	39° 4′			
14	31° 39′				29	15° 15′			
15	60° 15′				30	67° 54′			

417. Solution of the Trigonometric Formulas. The following formulas are obtained by the definitions of sine, cosine, and tangent:

(1) $\sin A = \frac{a}{c}$

(2) $\cos A = \frac{b}{c}$

(3) $\tan A = \frac{a}{b}$

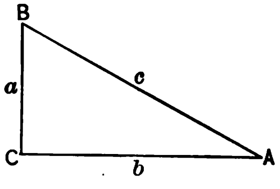


FIG. 196.

If each formula be multiplied by the denominator of the fraction, we have

$$c \sin A = \frac{a}{c} \times c. \quad c \cos A = \frac{b}{c} \times c. \quad b \tan A = \frac{a}{b} \times b.$$

Observe that a multiplier of a trigonometric function must always be placed before the function.

Canceling in each formula and reversing the members, we have

$$(4) \quad a = c \sin A. \quad (5) \quad b = c \cos A. \quad (6) \quad a = b \tan A.$$

If each formula be divided by the function we have

$$\frac{a}{\sin A} = \frac{c \sin A}{\sin A}. \quad \frac{b}{\cos A} = \frac{c \cos A}{\cos A}. \quad \frac{a}{\tan A} = \frac{b \tan A}{\tan A}.$$

Canceling in each formula and reversing the members we have

$$(7) \quad c = \frac{a}{\sin A}. \quad (8) \quad c = \frac{b}{\cos A}. \quad (9) \quad b = \frac{a}{\tan A}.$$

418. Examples in Solution of Formulas. Solve the following examples in the same way as shown in paragraph 417, but do not write explanations.

1. Write the formula for sin B and solve it for b and c.

2. Formulate cos B and solve for each term of the fraction.

3. Formulate tan B and solve for each term of the ratio.

4. Draw a triangle having right angle C and acute angles R and S.

Formulate sin R and solve for each term of the fraction.

5. From the figure of Problem 4, formulate tan R and solve for each term of the fraction.

6. From the figure of Problem 4, formulate cos R and solve for both terms in the second member.

7. From the figure of Problem 4, write the formulas for sin S, cos S, and tan S and solve for both terms of the ratios.

8. Draw a triangle with right angle V and acute angles B and D.

Formulate sin D, cos D, and tan D, and solve each for each term of the fractions.

9. Draw a triangle whose angles are L, H, and R, with $H = 90°$. Formulate three functions of L and solve for the sides of the triangle.

10. From the triangle of Problem 9, formulate three functions of R and solve for the sides of the triangle.

§ 2. SOLUTION OF A RIGHT TRIANGLE

419. Principles of the Solution of a Right Triangle. If the work of the preceding paragraphs has been studied so as to be clearly understood, the solution of triangles will present few difficulties.

Following are the principles governing the determination of the **solution formulas** by the use of which any right triangle may be solved.

Known parts are those whose numerical values are known.

1. No triangle can be solved unless three parts are known, one of which must be a side.
2. No formula can be solved if it has more than one unknown quantity.
3. When two sides are known, the acute angles may be determined from a function formula whose second member is the ratio of the known sides.
4. When one acute angle is known, subtract it from 90° to obtain the other angle.
5. Formulate an *unknown* side in a fraction or ratio with a known side, as a function of a known angle, and solve the formula for the unknown side before substituting numerical values.
6. If the unknown part is in the numerator, multiply the formula by the denominator.
7. If the unknown part is in the denominator, multiply the formula by the denominator.

Divide the resulting formula by the multiplier or coefficient of the unknown.

8. The multiplier of a quantity is called a coefficient.

9. The given or known numerical values are called the data.

10. A *solution formula* is one whose first member is an unknown part or function, and whose second member has only known parts or functions of known parts, or both.

420. Arrangement of Work. The work of the next paragraph will be more easily read and inspected if arranged as shown below.

The entries above the single line are the data; those between the single line and the double underlining are the results obtained by solution.

Model Solution.

$$\begin{array}{l} a=12.5\\ b=23 \\ \hline A=28^\circ\ 30' \\ B=61^\circ\ 30' \\ \underline{\underline{c=26.2}} \end{array}$$

$$\tan A=\frac{a}{b}$$

$$\tan A=\frac{12.5}{23}=\begin{array}[t]{l} .543\\ \underline{.532}=28^\circ \\ \frac{11}{22}\times 60'=30' \end{array}$$

$$A=\underline{\underline{28^\circ\ 30'}}$$

$$\sin A=\frac{a}{c}$$

$$c=\frac{a}{\sin A}$$

$$c=\frac{12.5}{.478}$$

$$c=\underline{\underline{26.2}}$$

$$\frac{30}{60}\times 16=\begin{array}[t]{r} .470\\ \underline{8}\\ .478 \end{array}$$

$$\begin{array}{r} .478\ )\ 12.500\ (\ 26.2\\ \underline{956}\\ 2940\\ \underline{2868}\\ 720\end{array}$$

Observe that in this arrangement, data, results, formulas, and substitutions are at the left, and computations at the right.

421. Examples in the Right Triangle. In solving the following triangles it is suggested that not less than ten be carefully drawn with a straight-edge, pains being taken that the parts are in the proportion indicated by the data.

When ten have been drawn and solved, the others should be solved directly from the data without drawing the triangles.

If there is any difficulty, the operations shown and explained in the preceding pages, and the principles of solution in paragraph 419 will probably remove it.

When known sides are expressed in U. S. or metric measure, denote results both in U. S. and metric units.

1. $A = 25^\circ\ 18'$
$c = 250$ m.

2. $b = 17.55$ mm.
$A = 34^\circ\ 30'$

3. $a = 35.5$ cm.
$B = 70^\circ\ 28'$

4. $A = 20^\circ\ 15'$
$c = 112$

5. $a = 75$ cm.
$b = 180$ cm.

6. $a = 11.5$ in.
$c = 20$ in.

7. $a = 78.2$
$b = 210$

8. $c = 11.8$ dm.
$a = 8.2$ dm.

9. $b = 1180$ mm.
$c = 3.5$ m.

10. $B = 42^\circ\ 28'$
$a = 17.6$ ft.

11. $A = 30^\circ\ 18'$
$a = 12.5$ Hm.

12. $B = 72^\circ\ 54'$
$b = 38.1$ Km.

13. $c = 102.3$ yd.
$a = 271$ ft.

14. $b = 92.2$ in.
$B = 63^\circ\ 18'$

15. $A = 27^\circ\ 21'$
$c = 1055$ Dm.

16. $b = 32.03$ rd.
$a = 500$ ft.

17. $B = 70^\circ\ 40'$
$c = 3\frac{1}{8}$ in.

18. $a = 2\frac{1}{16}''$
$c = 3\frac{5}{8}''$

19. $B = 36^\circ\ 52'$
$c = 10.09$ mi.

20. $c = 18.02$ in.
$a = 1\frac{1}{8}$ ft.

21. $A = 21^\circ\ 45'$
$b = 112.5$

22. $b = 72.1$ ft.
$a = 20.5$ m.

23. $a = 41.2$
$B = 48^\circ\ 31'$

24. $b = 50.6$ cm.
$A = 54^\circ\ 7'$

25. $a = 10.1$ in.
$A = 74^\circ\ 21'$

26. $c = 48.4$
$B = 80^\circ\ 5'$

27. $c = 12.4$ dm.
$B = 60^\circ$

28. $c = 40.7$ in.
$b = 36.1$ in.

29. $B = 18^\circ\ 9'$
$a = 72.1$ ft.

30. $b = 44.6$ cm.
$a = 34.1$ cm.

422. Problems in the Right Triangle. Solve the following problems in the work-book and illustrate each by a carefully drawn figure. Take all readings in the 4-place table unless you have a table of more places.

Carefully check all readings and results with the slide-rule.

1. Pitch of a Screw. The pitch of a screw is the distance between the threads or the distance from the center of the point of one thread to the center of the point of the next, measured on a line parallel to the axis of the screw.

A screw has 8 threads to the inch. Formulate and compute the pitch in thousandths of an inch, using the following notation:

P = pitch of screw in inches;
n = number of threads to the inch.

2. Depth of a Sharp V-Thread. The sketch shows a section of a screw having a sharp V-thread.

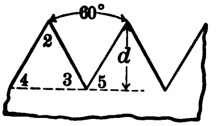


Fig. 197.

(1) Determine the number of degrees in angles 4, 2, and 3, and enter on the figure in the work-book.

(2) How does the length of each of the three sides of the triangle compare with the pitch P?

(3) Formulate the depth of the thread, d, in terms of P, and the sine, cosine, and tangent of 5.

(4) Compute d by each formula, for a screw having 4 threads to the inch.

3. By the sine formula of Problem 2, compute the omitted entries in the following table, checking the results by the tangent formula.

Table 165

PITCH AND DEPTH OF SHARP V-THREADS

No.	n	P	d	No.	n	P	d
1	18			6	16		
2	12			7	11		
3	10			8	20		
4	4½			9	3½		
5	4			10	14		

4. Depth of a U. S. Standard Thread. A U. S. thread as shown in the drawing has its sides at an angle of 60°. The top is taken off a distance equal to ⅛ of the pitch, measured along the side of the thread. The bottom is left uncut, the same distance, forming a flat.

Formulate and compute the width of the flat both at the point of the thread and at the bottom of the thread when the screw has 9 threads to the inch.

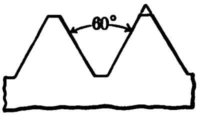


Fig. 198.

Formulate and compute the depth.

5. Formulate and compute the thickness of a U. S. standard thread at the root when the screw has 13 threads to the inch.

6. The Metric Standard Screw Thread. The metric thread has the same proportions as the U. S. standard thread except that the dimensions are in millimeters.

Formulate and compute the width of the flat in millimeters when the pitch is .75 mm.

7. Formulate and compute the depth of the thread in millimeters, under the same conditions as in Problem 6.

8. Formulate and compute the thickness of a metric standard thread at the bottom under the same conditions as in Problem 6.

9. By the formulas of Problems 6, 7, and 8, compute and fill in the omitted entries in the following table in which,

P = pitch of screw in millimeters;
w = width of flat;
d = depth of thread;
t = thickness of thread at bottom.

Table 166

DIMENSIONS OF METRIC STANDARD THREAD

No.	P	w	d	t	No.	P	w	d	t
1	.5				6	2.5			
2	.75				7	3			
3	1.25				8	3.5			
4	1.5				9	4.5			
5	1.75				10	6.5			

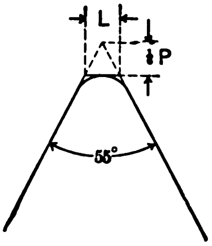


FIG. 199.

10. Whitworth Standard Thread. The drawing shows the enlarged point of a Whitworth standard thread.

Formulate and compute the distance *L* when the screw has 7 threads to the inch.

11. Angle of Acme Screw Thread. The drawing shows a section of an Acme thread.

In a thread of 6-pitch, as made by Pratt & Whitney Co.,

$$F=0.0617'';$$
$$T=0.1101'';$$
$$d=0.0933''.$$

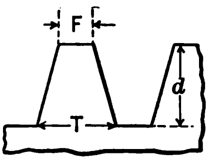


FIG. 200.

Formulate and compute *A*, the angle of the thread.

An Acme thread is therefore a how-many degree thread?

12. Depth of Acme Screw Thread. Formulate and compute the depth of an Acme thread having $2\frac{1}{2}$ threads to the inch, when

$$F=.1482;$$
$$T=.2570.$$

13. The depth as computed in Problem 12 is how many hundredths of an inch more than $\frac{1}{2}$ the pitch?

14. If .01″ is subtracted from the depth as given in Problem 11, the remainder equals what part of the pitch?

15. Relation of Depth to Pitch. The relation of depth to pitch in Acme threads of all pitches is always the same, as indicated by the results obtained in Problems 13 and 14.

Therefore formulate depth in terms of pitch, using the notation

d = depth of thread in inches;
P = pitch of thread in inches.

16. In an Acme thread having $1\frac{3}{8}$ threads to the inch,

$$T=.3986;$$
$$F=.3707P.$$

Compute the depth by the formula of Problem 12 and check the result by the formula of Problem 15.

17. Brown & Sharpe 29° Worm Thread. The drawing shows a sectional view of this thread.

When there are 6 threads to the inch

$F = .056;$
$T = .115.$

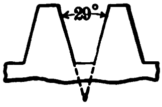


Fig. 201.

Formulate and compute the depth.

18. Comparison of Acme and of B. & S. 29° Worm Thread. Fill in the omitted entries in the table below, using the data and results of Problems 11 and 17.

Table 167

29° Acme and B. & S. 29° Worm, 6 Pitch

	B. & S.	Acme.	Difference.
F			
T			
d			

19. Depth of an Acme Tap Thread. The Acme tap thread as illustrated has the same form as the Acme screw thread, but the proportions are slightly different.

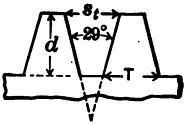


Fig. 202.

On a 6-pitch thread the dimensions are as follows:

$F = .0566;$
$T = .1101.$

Formulate and compute the depth of the thread and the width of the space at the root and at the point of the thread for a tap having 6 threads to the inch.

20. Formulate, compute, and fill in the omitted entries in Table 168, in which

F = flat in inches at top of thread;
T = thickness in inches at root of thread;
S_b = space in inches between threads at bottom;
S_t = space in inches between threads at top.

TABLE 168

ACME 29° TAP THREADS

No.	No. Threads to Inch.	F	T	S_b	S_t	d
1	2	.1801	.3199			
2	4	.0875	.1625			
3	7	.0478	.0951			
4	8	.0411	.0839			
5	9	.0361	.0751			

21. Wire Measurement of Acme 29° Threads. The drawing shows a sectional view of a wire lying in the groove of an Acme 29° tap thread, the top of the wire being flush with the top of the thread.

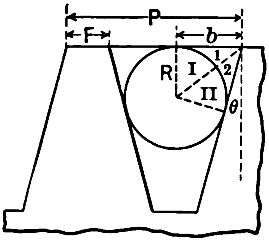


FIG. 203.

(1) Right triangles are equal when a side and the hypotenuse of one are equal respectively to a side and the hypotenuse of the other.

Does $\triangle I = \triangle II$? Why?

Therefore what is the relation of angles 1 and 2?

But $1+2=90°-$what?

Therefore $1=$how many degrees?

(2) Formulate and compute D, the diameter of the wire, when the pitch is $\frac{1}{8}''$.

22. With a scale of 10 to 1, draw an Acme 29° screw thread having 6 threads to the inch.

Formulate and compute R, the radius of the measuring wire, as shown in Fig. 203.

23. British Association Screw Threads. This thread, shown in section in Fig. 204, has all its dimensions in millimeters.

For all pitches, $d=.6P.$

Formulate and compute D, and determine exactly what distance has been rounded off the point of the thread on a screw whose pitch $=.53$ mm.

24. In the British Association thread the radius of the top and bottom for all pitches is determined by the formula

$$r=\frac{2}{11}P.$$

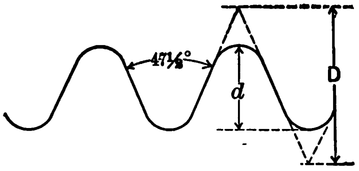


Fig. 204.

Compute r when the screw has a pitch of .66 mm.

25. Layout of a 29° Angle. The following method is sometimes employed for laying out a 29° angle.

Draw a circle of any known diameter, say 4 inches.

Draw the diameter AB.

Take $\frac{1}{4}$ the diameter in the compasses and with one leg on A, strike off arcs of the circumference in both directions, cutting the circumference at O and V.

Draw OB and VB.

Draw AV and by trigonometry determine the number of degrees in the angle OBV.

26. Miter of a Pentagon. This drawing from the American Carpenter and Builder for December, 1911, shows a carpenter's square placed on a pentagon to illustrate how the miter at the corner may be determined.

Give the computation for determining the number of degrees in the miter angle, and by formulation and computation determine whether the square as placed will give the correct miter.

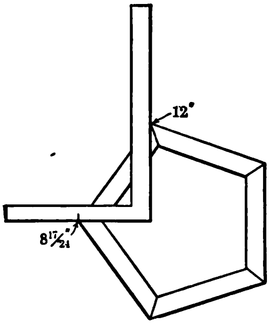


Fig. 205.

27. Setting of Cutter. The drawing shows an angular cutter in position for cutting the spiral on a milling cutter. The face angles of the cutter are as follows:

$$OVA = 48°;$$
$$OVL = 12°.$$

As shown, the cutter is placed off center with the 12° side radial, so that

$$d = .1D,$$

when

d = distance in inches cutter is off center.

D = diameter of work in inches.

Radial depth of cut = $\frac{1}{32}''$.

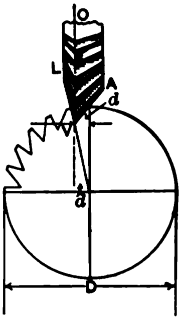


FIG. 206.

Formulate d in terms of D and some function of the central angle and determine by computation whether the formula above is correct, when $D = 1\frac{3}{4}$ inches.

28. A Thread-gauge. The sketch shows a thread-gauge for measurement of either a 60° thread or a 55° thread, depending on whether angle B is 60° or 55°.

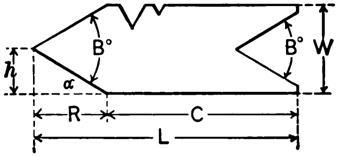


FIG. 207.

On the sketch,

$$\alpha \doteq \text{what part of } B?$$

Draw the figure in the work-book and denote the angle α by B over the proper denominator, and h as a part of W.

Formulate R in terms of W and two different functions of B, when $B = 60°$.

29. Length of a 60° Thread-gauge. By reference to Problem 28, formulate and compute L, the total length of the gauge, when

$$B = 60°;$$
$$W = \tfrac{3}{4}'';$$
$$C = 1\tfrac{9}{16}''.$$

30. Length of a 55° Thread-gauge. Draw a dimensioned sketch and formulate and compute the length of a thread gauge when $B=55°$ and the other dimensions are the same as in the data of Problem 29.

31. Apothem of a Regular Polygon. The drawing shows a regular polygon with the apothem h.

Observe that h is a side of a right triangle, and is opposite an angle, which, as half the angle of the polygon, may be denoted by $\frac{\theta}{2}$.

Formulate h from the figure as dimensioned, denoting the half side by $\frac{s}{2}$.

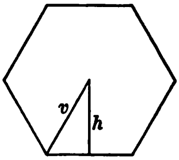


Fig. 208.

32. Draw a regular heptagon (see paragraph 128) whose side is 25.6 inches, and compute its apothem by the formula of Problem 31, both by the slide-rule and by logarithms.

33. In the figure of Problem 32, if a line is drawn from the center of the polygon to each vertex, how many triangles are formed?

Therefore the area of the heptagon equals how many times the area of one of the triangles?

Formulate the area of the heptagon as indicated by the preceding question, using the notation,

$A_7=$ area heptagon;
$A_\Delta=$ area triangle.

Formulate A_Δ in terms of base and altitude, substitute in the formula for A_7 and compute the area of the heptagon both by the slide-rule and by logarithms.

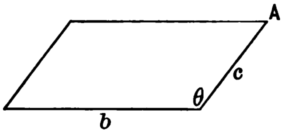


Fig. 209.

34. In the parallelogram here shown,

$b=14.5''$;
$c=9\frac{3}{4}''$;
$\theta=122°\ 15'$.

(1) Draw the altitude of the parallelogram from the point A.
(2) Formulate its value in terms of c and θ.
(3) Formulate and determine the area of the parallelogram
    (a) By arithmetical computation;
    (b) By the slide-rule;
    (c) By logarithms.

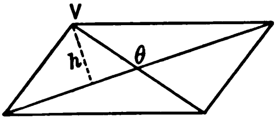


Fig. 210.

35. In the parallelogram shown in the cut, the long diagonal equals 24.5 inches, the short diagonal equals 18 inches, and θ equals 125°.

Formulate and compute the area of the parallelogram.

36. An equilateral triangle is inscribed in a circle whose radius is $5\frac{3}{4}$ inches.

Formulate and compute the distance from the center of the circle to the side of the triangle.

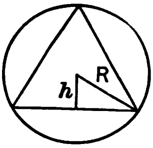


Fig. 211.

37. Using the same data as in Problem 36, formulate and compute (1) the length of the side of the triangle; (2) the area of the triangle.

38. An equilateral triangle whose side is $8\frac{1}{4}''$ is inscribed in a circle.

Formulate and compute (1) the diameter of the circle; (2) the area of the triangle.

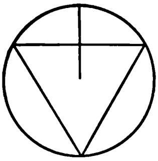


Fig. 212.

39. The rise of an arc which is subtended by a chord is the perpendicular distance to the arc from the center of the chord.

The figure represents an inscribed equilateral triangle.

Formulate and compute the rise of the arc of the circle when the radius is 3.98 inches.

40. In the figure *h* is parallel to *c*, and *a* is parallel to *v*.

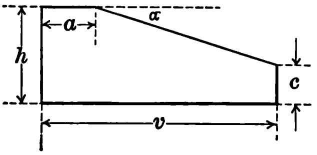


Fig. 213.

Formulate and compute the angle α (alpha) when

$a = 4\frac{3}{4}''$, $v = 12\frac{1}{8}''$,
$h = 5\frac{1}{4}''$, $c = 2\frac{1}{16}''$.

41. In the figure

$$b = 42\tfrac{1}{2}'',$$
$$c = 21.19'',$$
and
$$\theta = 67^\circ\ 8'.$$

Formulate and compute the depth of the cut.

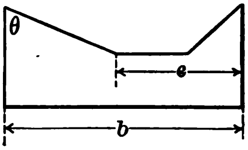


FIG. 214.

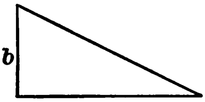


FIG. 215.

42. In this right triangle the hypotenuse is twice the length of the side b. Determine the acute angles.

43. The arch in the figure has a span $L = 5'\ 3''$, the radius being $10'\ 4''$. What is the rise?

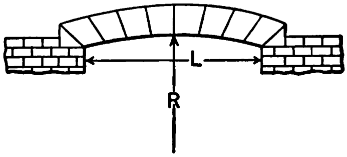


FIG. 216.

44. In the figure R and r, the radii of the two pulleys, are perpendicular at the points of tangency of the belt and b is perpendicular to the radius R.

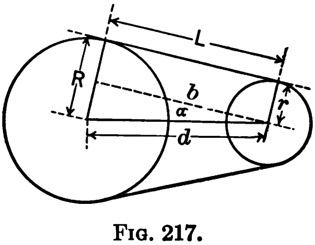


FIG. 217.

Formulate and compute α when

$$L = 11\tfrac{1}{2}',$$
$$R = 15\tfrac{3}{4}'',$$
and
$$r = 11\tfrac{1}{2}''.$$

45. Offset Bend. The drawing shows a reversed or offset pipe bend.

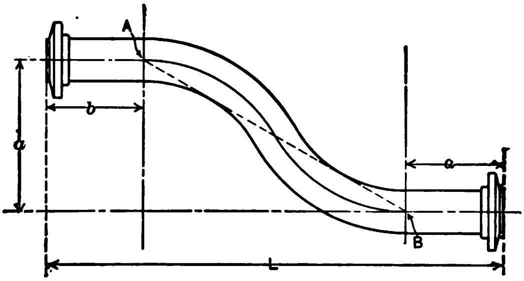


Fig. 218.

The straight portions of the pipe, called ends, are parallel to each other and tangent to the curve at each end.

The bent part of the pipe consists of two arcs of equal length and equal radius.

Draw the figure in the work-book as follows:

(1) Draw the ends at a sufficient off-set to each other to make a large figure, and denote the center line in each.

(2) Draw the straight line *AB* and locate *C* at its center.

(3) From the middle points of *AC* and *BC* draw perpendiculars terminating at *O* and *V* in perpendiculars to the "ends" from *A* and *B*.

(4) From the centers *O* and *V*, with the compasses describe arcs to form the reversed pipe bend and its center line, making the center line light and the pipe lines heavy.

(5) Dimension the drawing in the same notation as shown in the cut.

When the drawing is finished read through the directions again to make sure everything specified has been done.

Use the completed drawing for the work required in Problems 46 to 50 inclusive.

46. Formulate and compute the length of the straight line *AB*, with the following data:

$$a = b = 3'';$$
$$d = 15'';$$
$$L = 40''.$$

47. On the figure draw a line from A, perpendicular to the perpendicular from B.

Compute the angle θ between this line and the line AB, when the data are the same as in Problem 46.

48. Using the same figure and data as in Problem 47, formulate and compute the radius of the curved part of the pipe.

49. Using the same figure and data as in Problem 47, formulate and compute the total length of the pipe from end to end.

50. Using the same figure as in Problem 47 and the data in the table below, formulate, compute, and tabulate the following:

(1) Angle θ;
(2) Straight distance from A to B;
(3) R, Radius of bend;
(4) L_t, Total length of bent piece.

Table 169

OFFSET PIPE-BENDS

No.	a	b	d	L	θ	AB	R	L_t
1	8″	8″	3′ 8″	6′ 4″				
2	12″	12″	2′ 2″	5′ 3″				
3	9″	18″	28″	80″				
4	15″	7″	3′ 6″	8½′				
5	20″	16″	4′ 4″	12′ 5″				

51. Standard Radius. In pipe bends the standard radius for all sizes of pipe from 4″ to 20″ in diameter inclusive is six times the diameter of the pipe, or

$$R = 6D,$$

when R = radius of the bend

and d = diameter of the pipe.

With diameters from 4″ to 12″, a radius equal to three times the diameter may be used, or $R = 3D$.

Assuming the bends in Problem 50 to be of standard radius, compute the diameters of the pipes.

52. One-eighth Pipe Bend. A one-eighth bend is a bend through $\frac{1}{8}$ of a circle or 45°.

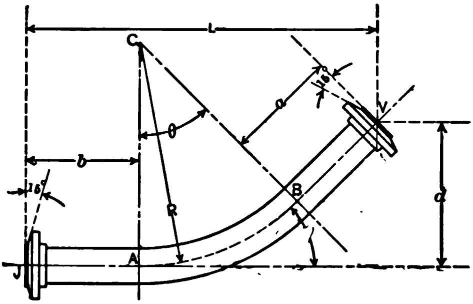


Fig. 219.

The length of the ends or straight parts of the pipe may be the same or different, and for this reason are shown in a different notation.

Draw the figure carefully in the work-book with dimensions as shown, omitting *VE*, *BF*, and *BH*.

Observe that the center of curvature is the point of intersection of the perpendiculars from the tangent points of the ends.

53. On the figure of the preceding problem, make the following construction:

(1) Draw *VE* perpendicular to *JO*, *E* being the point of intersection with *JO*.

(2) Draw *BF* perpendicular to *CA*, *F* being the point of intersection.

(3) Draw *BH* perpendicular to *VE*.

Use the figure as now completed for Problems 54 to 60 inclusive, with the following data:

$$a = b = 5'', \qquad L = 44\tfrac{1}{2}'', \qquad \theta = 45°.$$

54. Formulate *BF* in terms of *L*, *b*, and *BH*.

55. Formulate *BH* as a side of the triangle *VBH*.

56. By the formula of Problem 54, compute the length of *BH*.

57. On the diagram denote *CB* by *R*. Formulate *R* in triangle *CBF*.

58. Compute the length of the radius of the bend.

59. Formulate the length of the bend *AB* in terms of *R*, and simplify.

60. Formulate and compute the total length of the bent piece.

61. Using the same figure as in immediately preceding problems and the data in the table below, formulate, compute, and tabulate the following, in which L_t denotes the total length of the bent piece:

TABLE 170

45° PIPE-BENDS

No.	*a*	*b*	*L*	*BF*	*R*	L_t	*d*
1	7″	7″	2′ 10″				
2	6″	8″	3′				
3	8″	10″	3′ 8″				
4	10″	12″	4′				
5	15″	18″	5′ 3″				

62. Offset Bend with Straight between the 45° Arcs. When an offset bend is made as shown, the straight length between the arcs is usually made 12 inches long, this being a convenient length for gripping in making the bend.

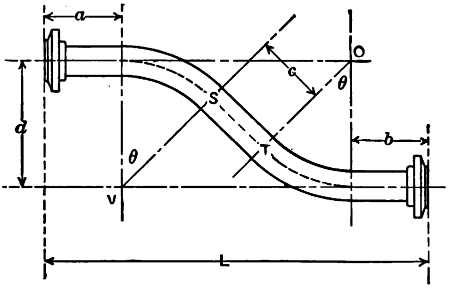


FIG. 220.

Draw the figure as follows:

(1) At an angle of 45° with the horizontal draw *ST*, the straight central part of the bend; also the lines forming the angles marked *θ*, making *SV* and *TO* perpendicular to *ST*.

(2) With radii VS and OT, describe the curved portion of the pipe, and draw the straight ends.

(3) Enter A and B on the figure as shown in Fig. 218.

(4) Complete the figure, making the pipe outlines heavy and the center line light.

Use the completed figure for the work required in Problems 63 to 66 inclusive.

63. (1) From S draw a line perpendicular to, and terminating in, a horizontal line TC at C.

(2) Formulate and compute the length of TC when $ST=12$ inches.

(3) Draw SJ perpendicular to AV, terminating in AV at J.

(4) Formulate SJ in terms of L and other horizontal distances.

(5) Compute SJ.

64. Formulate and compute R when

$$L=60''; \qquad a=8'';$$
$$b=11''; \qquad ST=12''.$$
$$\theta=45^\circ.$$

65. Using the same data as in Problem 64, formulate and compute the length of the two 45° arcs.

66. Formulate and compute the total length of the bent piece when the data are the same as in the two preceding problems.

67. The distance across the flats of a hexagonal nut is $1\frac{1}{8}''$.

Formulate and compute the distance across the corners.

68. The drawing represents a lathe center with the following dimensions:

$$L=3\tfrac{3}{4}'';$$
$$D=.655''.$$

Taper of shank = No. 7 B. & S.

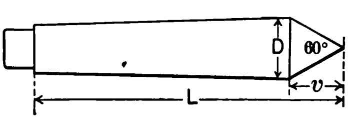


FIG. 221.

Formulate and compute v; also compute A, the diameter at the small end of the taper.

69. The angle of a taper is the inclination of the taper to the axis of the piece.

Draw the figure of Problem 52 on page 156, and formulate and compute the angle of each taper.

70. The drawing shows an arc of length C; also the rectification of the arc in the straight line C which is tangent to the arc at O.

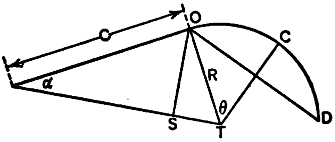


FIG. 222.

Formulate and compute the angle θ when

$$R = 19\tfrac{3}{4}''$$

and

$$OD = 25''.$$

71. Using the data and result of Problem 70, compute the length of C.

72. With data unchanged, formulate and compute the length OS in the figure of Problem 70.

73. A circular brass plate 18 inches in diameter has 24 holes equally spaced around the periphery, the center of the holes being 1½ inches from the edge of the plate.

Formulate and compute the shortest distance from the center of one hole to the center of the next.

74. Round stock is to be used for making bolts with a hexagonal head.

The required distance across the flats of the head is 1¼ inches.

Formulate and compute the diameter of stock required.

75. Formulate and compute the diameter of stock required for square head bolts, measuring 1¼ inches across the flats.

76. The cut illustrates a straight-gauged arch.

Formulate and compute c and h when

$$\theta = 60°;$$
$$d = 12'';$$
$$s = 3'\,2''.$$

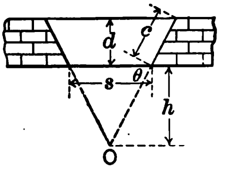


FIG. 223.

77. The cut shows an ordinary two-pitch or couple roof in which both pitches are the same and both sides of the roof have the same dimensions.

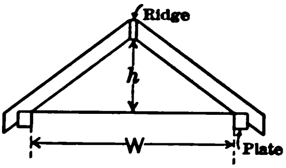


FIG. 224.

By pitch is meant the ratio of the height of the ridge to the width of the building. In the cut

$$\text{Pitch} = \frac{h}{W}.$$

When $W = 24$ feet 4 inches, formulate and compute the angles at which the rafters must be cut at ridge and plate for the following pitches: $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

78. Make a drawing to scale showing the position of the carpenter's square for the cuts in the rafters for roofs of the various pitches in Problem 77.

423. The Isosceles Triangle. A triangle is called isosceles when two sides are equal, "isos" meaning equal. The vertex of an isosceles triangle is the vertex of the angle formed by the two equal sides; the base is the side opposite the vertex.

When three parts are known, of which at least one is a side, an isosceles triangle is easily solved if a perpendicular is drawn from the vertex to the base. This bisects the base, the vertex angle, and the triangle. The unknown parts may therefore be determined by solving one of the equal right triangles.

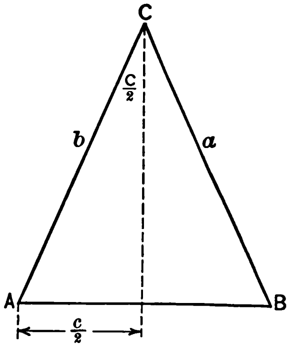


FIG. 225.

Fig. 225 shows the usual way of lettering, in other words the *notation* of, an isosceles triangle.

C = the angle at the vertex;

$A = B$ = the angles opposite the equal sides, a and b.

This notation is used in the examples which follow.

424. Examples. Solve the following isosceles triangles by the simplest method. Check all work with the slide-rule.

1. $a=34.2$
$C=28^\circ\ 20'$

2. $b=5.18$
$A=53^\circ\ 30'$

3. $a=9.73$
$A=72^\circ\ 10'$

4. $a=17.8$
$B=60^\circ\ 20'$

5. $C=58^\circ\ 40'$
$b=75.6$

6. $C=67^\circ\ 30'$
$a=3.86$

7. $C=82^\circ\ 10'$
$c=7.929$

8. $a=54.52$
$c=39.37$

9. $c=2814$
$b=4925$

10. $A=51^\circ\ 50'$
$c=3.463$

11. $C=78^\circ\ 10'$
$a=4.295$

12. $B=24^\circ\ 30'$
$a=5537$

13. $A=47^\circ$
$c=18.84$

14. $C=101^\circ$
$c=97.56$

15. $A=71^\circ\ 30'$
$b=1.183$

16. $a=7216$
$B=84^\circ\ 25'$

17. $C=99^\circ\ 12'$
$c=4562$

18. $c=193$
$a=621$

19. $C=78^\circ\ 54'$
$b=80.8$

20. $A=75^\circ\ 13'$
$c=695$

21. $b=.784$
$C=103^\circ$

22. $c=.381$
$B=35^\circ\ 40'$

23. $a=4056$
$c=2893$

24. $C=85^\circ$
$c=.8175$

25. $a=\sqrt{38.42}$
$c=5.789$

26. $b=6.49$
$c=\sqrt{22.78}$

27. $B=37^\circ\ 18'$
$c=\sqrt[3]{2095}$

28. $C=79^\circ\ 20'$
$c=\sqrt{541.6}$

29. $a=\sqrt{18.72}$
$c=\sqrt{39.58}$

30. $c=\sqrt[3]{8295}$
$B=56^\circ\ 15'$

31. $A=63^\circ$
$a=.09784$

32. $b=.06087$
$c=.08116$

33. $C=34^\circ\ 21'$
$c=9.73\times 18.4$

34. $a=\sqrt[3]{1728}$
$C=60^\circ$

§ 3. THE OBLIQUE TRIANGLE

425. Definition. An oblique triangle is one which is not an isosceles, equilateral, or right triangle. By drawing one or more perpendiculars so as to form a right triangle, an oblique triangle may be solved by the methods previously used. The solution is simplified, however, by the use of the laws formulated in the next paragraph.

426. How to Solve an Oblique Triangle. Four laws are used for the solution of an oblique triangle, but only the two which follow are required:

1. Law of Sines.
2. Law of Cosines.

These laws apply in the four cases tabulated below:

TABLE 171

SOLUTION OF OBLIQUE TRIANGLES

Case.	Known Parts		Law to be Applied
	Sides.	Angles.	
I	2	1 Opposite ∠	Sines
II	1	2 Adjacent ∠s	Sines
III	2	1 Included ∠	Cosines
IV	3	None	Cosines

427. Law of Sines. In any plane triangle the sines of the angles are proportional to the opposite sides.

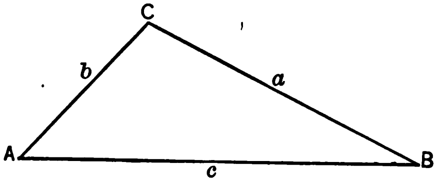


FIG. 226.

Thus in the figure,

$$(1)\quad \frac{\sin A}{\sin B}=\frac{a}{b}.$$

$$(2)\quad \frac{\sin B}{\sin C}=\frac{b}{c}.$$

$$(3)\quad \frac{\sin A}{\sin C}=\frac{a}{c}.$$

If these three formulas are compared with the table in paragraph 426, it will be evident why they apply in Cases I and II.

For example if two sides and an opposite angle are known, the known parts may be substituted in one of the formulas, giving an equation with only one unknown quantity whose value may therefore be determined by computation. If one side and the two adjacent angles are known, the third angle is determined by subtraction from 180°, and the law of sines is used as in Case I.

428. Model Solution.

$a=94.6$
$c=75.2$
$A=41^\circ\ 50'$
$C=32^\circ$
$B=$
$b=$

$$\frac{\sin C}{\sin A}=\frac{c}{a}$$

$$\sin C=\frac{c\sin A}{a}$$

$$\begin{array}{r} 1.8762 \\ 9.8241 \\ \hline 11.7003 \\ 1.9759 \\ \hline 9.7244 \\ \underline{\quad 2}=32^\circ \end{array}$$

The solution for B and b does not require illustration.

429. Law of Cosines. This law is as follows:

In any plane triangle the square of any side equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle.

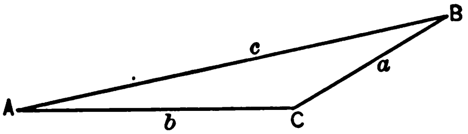


FIG. 227.

Thus in the figure,

(4) $c^2=a^2+b^2-2ab\cos C.$

(5) $a^2=c^2+b^2-2bc\cos A.$

(6) $b^2=a^2+c^2-2ac\cos B.$

Comparison of these formulas with the table in paragraph 426, will show why they apply in Cases III and IV. For example if two sides and the included angle are known the third side may be computed after substitution in one of the formulas. The solution may then be finished by the law of sines.

If three sides are known, any of the three angles may be determined by substitution in the formula having the cosine of the angle. The solution may then be finished by the law of sines.

430. Functions of an Obtuse Angle. As previously defined, an obtuse angle is an angle greater than 90° and less than 180°. The functions of an obtuse angle are determined as follows:

(1) Subtract the angle from 180°.

(2) In the table of functions read the required function of the difference obtained in (1).

(3) Prefix to the reading the sign tabulated.

TABLE 172

SIGNS OF FUNCTIONS OF AN OBTUSE ANGLE

Function.	Sign.	Function.	Sign.
sin	+	csc	+
cos	−	sec	−
tan	−	cot	−

The functions of 127° 28′ are therefore determined as follows:

$$\begin{array}{rl} 179^\circ\ 60' & \sin 52^\circ\ 32' = .7937 \\ \underline{127\ \ \ \ 28\ } & -\cos 52^\circ\ 32' = -\ .6083 \\ 52^\circ\ 32' & -\tan 52^\circ\ 32' = -1.30 \end{array}$$

431. Suggestions Regarding Solution. In the solution of oblique triangles be sure to observe the following:

(1) Study the example to determine what parts are known.

(2) If two angles are known, subtract their sum from 180° to determine the third angle.

(3) Refer to Table 171 to determine what case the example represents and what law to apply.

(4) Substitute the known values in that formula which will have only one unknown quantity after the substitution.

If the formula is solved for the unknown in terms of the other quantities, before substitution, it will save unnecessary writing of figures.

(5) Use only the simplest method, generally logarithmic, and check results by the slide-rule.

(6) Remember that the *sine* of an *obtuse* angle is *plus;* the *cosine* and the *tangent* are *minus.*

432. Examples in Oblique Triangles.

1. $a = 17.8$
$c = 24.5$
$A = 42^\circ\ 50'$

2. $b = 4.08$
$c = 6.91$
$B = 71^\circ\ 30'$

3. $a = 64.3$
$B = 43^\circ\ 40'$
$c = 49.6$

4. $a = 354$
$b = 327$
$c = 198$

5. $a = 632$
$b = 751$
$C = 73^\circ\ 40'$

6. $a = 44.6$
$c = 75.9$
$b = 68.1$

7. $a = 13.48$
$B = 63^\circ\ 40'$
$C = 37^\circ\ 50'$

8. $a = 238$
$b = 309$
$B = 68^\circ\ 20'$

9. $b = 514.7$
$B = 42^\circ\ 25'$
$a = 435.9$

10. $A = 45^\circ\ 30'$
$b = 5.63$
$c = 8.91$

11. $a = .7582$
$c = .469$
$b = .8706$

12. $a = .349$
$c = .5095$
$B = 68^\circ\ 27'$

13. $a = 420.5$
$B = 78^\circ\ 20'$
$C = 40^\circ\ 45'$

14. $b = 75.37$
$c = 60.4$
$A = 55^\circ\ 10'$

15. $a = 738$
$b = 946$
$c = 861.5$

16. $a = 17.49$
$c = 25.07$
$B = 71^\circ\ 20'$

17. $a = .1098$
$b = .3441$
$B = 34^\circ\ 49'$

18. $a = 11.06$
$C = 36^\circ\ 10'$
$B = 49^\circ\ 26'$

19. $a = 1.405$
$b = 2.096$
$C = 127^\circ\ 41'$

20. $A = 24^\circ\ 37'$
$B = 76^\circ\ 4'$
$c = 2147$

21. $c = 22.07$
$b = 34.92$
$B = 71^\circ\ 18'$

22. $c = 741$
$a = 785.4$
$b = 429.3$

23. $c = 5.25$
$A = 40^\circ\ 33'$
$B = 35^\circ\ 27'$

24. $a = 15.97$
$b = 6.34$
$C = 70^\circ\ 32'$

25. $c = 4.432$
$b = 5.216$
$B = 62^\circ\ 38'$

26. $a = 67.8$
$c = 52.9$
$C = 34^\circ\ 17'$

27. $a = 7.08$
$c = 5.29$
$b = 4.53$

28. $a = 1.95$
$c = 2.84$
$B = 34^\circ\ 20'$

29. $a = .425$
$b = .346$
$c = .537$

30. $b = 2.517$
$c = 3.012$
$B = 33^\circ\ 10'$

31. $C = 75^\circ\ 50'$
$b = 4.163$
$c = 6.257$

32. $B = 63^\circ\ 30'$
$A = 49^\circ\ 40'$
$c = 13.25$

33. $b = .2516$
$c = .4309$
$A = 51^\circ\ 18'$

34. $a = .529$
$c = .347$
$B = 60^\circ\ 34'$

35. $b = .0731$
$A = 40^\circ\ 36'$
$c = .1095$

36. $a = 175.6$
$B = 51^\circ\ 30'$
$C = 42^\circ\ 53'$

37. $a = .9514$
$b = .6752$
$c = .8076$

38. $A = 81^\circ\ 35'$
$c = .0435$
$b = .0608$

39. $a = .1278$
$c = .2093$
$B = 49^\circ\ 42'$

40. $A = 128^\circ\ 51'$
$c = 78.21$
$b = 54.07$

41. $a = 11.506$
$b = 4.75$
$C = 115^\circ\ 25'$

42. $c = .0731$
$a = 1.056$
$B = 51^\circ\ 26'$

43. $a = 3.25$
$c = 4.625$
$b = 2.907$

44. $c = 2.432$
$b = 4.216$
$B = 78^\circ\ 30'$

45. $c = .3165$
$a = .5634$
$B = 118^\circ\ 48'$

46. $a = 45.41$
$c = 65.83$
$A = 54^\circ\ 32'$

47. $A = 72^\circ\ 20'$
$b = .01967$
$c = .03048$

48. $a = 5.58$
$c = 7.042$
$B = 41^\circ\ 55'$

49. $a = 64.31$
$B = 58^\circ\ 27'$
$C = 21^\circ\ 8'$

50. $a = .04268$
$c = .07143$
$b = .09586$

TABLE I.—FOUR-PLACE LOGARITHMS OF NUMBERS

N.	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5515	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5900
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	22	**21**
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20	**19**
1	2.0	1.9
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	18	**17**
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	16	**15**
1	1.6	1.5
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	14	**13**
1	1.4	1.3
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

Note.—Proportional parts begin with N. 19. Determine P.P. from N. 10 to N. 19 by multiplying the actual difference between the mantissa read and the mantissa immediately following, by all the figures of the natural number (except the first three) with a decimal point before them.

TABLE I.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	12	**11**
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

	9	**8**
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

TABLE I.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9	P. P.
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	
N.	0	1	2	3	4	5	6	7	8	9	

	7	**6**
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5	**4**
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

TABLE II.—FOUR-PLACE TRIGONOMETRIC FUNCTIONS

ANGLE	SINE		COSINE		TANGENT		COTANGENT		ANGLE
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
0 00	.0000	——	1.0000	0.0000	.0000	——	——	——	**90** 00
10	.0029	7.4637	1.0000	0000	.0029	7.4637	343.77	2.5363	50
20	.0058	7648	1.0000	0000	.0058	7648	171.89	2352	40
30	.0087	9408	1.0000	0000	.0087	9409	114.59	0591	30
40	.0116	8.0658	.9999	0000	.0116	8.0658	85.940	1.9342	20
50	.0145	1627	.9999	0000	.0145	1627	68.750	8373	10
1 00	.0175	8.2419	.9998	9.9999	.0175	8.2419	57.290	1.7581	**89** 00
10	.0204	3088	.9998	9999	.0204	3089	49.104	6911	50
20	.0233	3668	.9997	9999	.0233	3669	42.964	6331	40
30	.0262	4179	.9997	9999	.0262	4181	38.188	5819	30
40	.0291	4637	.9996	9998	.0291	4638	34.368	5362	20
50	.0320	5050	.9995	9998	.0320	5053	31.242	4947	10
2 00	.0349	8.5428	.9994	9.9997	.0349	8.5431	28.636	1.4569	**88** 00
10	.0378	5776	.9993	9997	.0378	5779	26.432	4221	50
20	.0407	6097	.9992	9996	.0407	6101	24.542	3899	40
30	.0436	6397	.9990	9996	.0437	6401	22.904	3599	30
40	.0465	6677	.9989	9995	.0466	6682	21.470	3318	20
50	.0494	6940	.9988	9995	.0495	6945	20.206	3055	10
3 00	.0523	8.7188	.9986	9.9994	.0524	8.7194	19.081	1.2806	**87** 00
10	.0552	7423	.9985	9993	.0553	7429	18.075	2571	50
20	.0581	7645	.9983	9993	.0582	7652	17.169	2348	40
30	.0610	7857	.9981	9992	.0612	7865	16.350	2135	30
40	.0640	8059	.9980	9991	.0641	8067	15.605	1933	20
50	.0669	8251	.9978	9990	.0670	8261	14.924	1739	10
4 00	.0698	8.8436	.9976	9.9989	.0699	8.8446	14.301	1.1554	**86** 00
10	.0727	8613	.9974	9989	.0729	8624	13.727	1376	50
20	.0756	8783	.9971	9988	.0758	8795	13.197	1205	40
30	.0785	8946	.9969	9987	.0787	8960	12.706	1040	30
40	.0814	9104	.9967	9986	.0816	9118	12.251	0882	20
50	.0843	9256	.9964	9985	.0846	9272	11.826	0728	10
5 00	.0872	8.9403	.9962	9.9983	.0875	8.9420	11.430	1.0580	**85** 00
10	.0901	9545	.9959	9982	.0904	9563	11.059	0437	50
20	.0929	9682	.9957	9981	.0934	9701	10.712	0299	40
30	.0958	9816	.9954	9980	.0963	9836	10.385	0164	30
40	.0987	9945	.9951	9979	.0992	9966	10.078	0034	20
50	.1016	9.0070	.9948	9977	.1022	9.0093	9.7882	0.9907	10
6 00	.1045	9.0192	.9945	9.9976	.1051	9.0216	9.5144	0.9784	**84** 00
10	.1074	0311	.9942	9975	.1080	0336	9.2553	9664	50
20	.1103	0426	.9939	9973	.1110	0453	9.0098	9547	40
30	.1132	0539	.9936	9972	.1139	0567	8.7769	9433	30
40	.1161	0648	.9932	9971	.1169	0678	8.5555	9322	20
50	.1190	0755	.9929	9969	.1198	0786	8.3450	9214	10
7 00	.1219	9.0859	.9925	9.9968	.1228	9.0891	8.1443	0.9109	**83** 00
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
ANGLE	COSINE		SINE		COTANGENT		TANGENT		ANGLE

TABLE II.—*Continued*

ANGLE		SINE		COSINE		TANGENT		COTANGENT		ANGLE	
°	′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	°	′
7	00	.1219	9.0859	.9925	9.9968	.1228	9.0891	8.1443	0.9109	**83**	00
	10	.1248	0961	.9922	9966	.1257	0995	7.9530	9005		50
	20	.1276	1060	.9918	9964	.1287	1096	7.7704	8904		40
	30	.1305	1157	.9914	9963	.1317	1194	7.5958	8806		30
	40	.1334	1252	.9911	9961	.1346	1291	7.4287	8709		20
	50	.1363	1345	.9907	9959	.1376	1385	7.2687	8615		10
8	00	.1392	9.1436	.9903	9.9958	.1405	9.1478	7.1154	0.8522	**82**	00
	10	.1421	1525	.9899	9956	.1435	1569	6.9682	8431		50
	20	.1449	1612	.9894	9954	.1465	1658	6.8269	8342		40
	30	.1478	1697	.9890	9952	.1495	1745	6.6912	8255		30
	40	.1507	1781	.9886	9950	.1524	1831	6.5606	8169		20
	50	.1536	1863	.9881	9948	.1554	1915	6.4348	8085		10
9	00	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	**81**	00
	10	.1593	2022	.9872	9944	.1614	2078	6.1970	7922		50
	20	.1622	2100	.9868	9942	.1644	2158	6.0844	7842		40
	30	.1650	2176	.9863	9940	.1673	2236	5.9758	7764		30
	40	.1679	2251	.9858	9938	.1703	2313	5.8708	7687		20
	50	.1708	2324	.9853	9936	.1733	2389	5.7694	7611		10
10	00	.1736	9.2397	.9848	9.9934	.1763	9.2463	5.6713	0.7537	**80**	00
	10	.1765	2468	.9843	9931	.1793	2536	5.5764	7464		50
	20	.1794	2538	.9838	9929	.1823	2609	5.4845	7391		40
	30	.1822	2606	.9833	9927	.1853	2680	5.3955	7320		30
	40	.1851	2674	.9827	9924	.1883	2750	5.3093	7250		20
	50	.1880	2740	.9822	9922	.1914	2819	5.2257	7181		10
11	00	.1908	9.2806	.9816	9.9919	.1944	9.2887	5.1446	0.7113	**79**	00
	10	.1937	2870	.9811	9917	.1974	2953	5.0658	7047		50
	20	.1965	2934	.9805	9914	.2004	3020	4.9894	6980		40
	30	.1994	2997	.9799	9912	.2035	3085	4.9152	6915		30
	40	.2022	3058	.9793	9909	.2065	3149	4.8430	6851		20
	50	.2051	3119	.9787	9907	.2095	3212	4.7729	6788		10
12	00	.2079	9.3179	.9781	9.9904	.2126	9.3275	4.7046	0.6725	**78**	00
	10	.2108	3238	.9775	9901	.2156	3336	4.6382	6664		50
	20	.2136	3296	.9769	9899	.2186	3397	4.5736	6603		40
	30	.2164	3353	.9763	9896	.2217	3458	4.5107	6542		30
	40	.2193	3410	.9757	9893	.2247	3517	4.4494	6483		20
	50	.2221	3466	.9750	9890	.2278	3576	4.3897	6424		10
13	00	.2250	9.3521	.9744	9.9887	.2309	9.3634	4.3315	0.6366	**77**	00
	10	.2278	3575	.9737	9884	.2339	3691	4.2747	6309		50
	20	.2306	3629	.9730	9881	.2370	3748	4.2193	6252		40
	30	.2334	3682	.9724	9878	.2401	3804	4.1653	6196		30
	40	.2363	3734	.9717	9875	.2432	3859	4.1126	6141		20
	50	.2391	3786	.9710	9872	.2462	3914	4.0611	6086		10
14	00	.2419	9.3837	.9703	9.9869	.2493	9.3968	4.0108	0.6032	**76**	00
°	′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	°	′
ANGLE		COSINE		SINE		COTANGENT		TANGENT		ANGLE	

TABLE II.—*Continued*

ANGLE	SINE		COSINE		TANGENT		COTANGENT		ANGLE
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
14 00	.2419	9.3837	.9703	9.9869	.2493	9.3968	4.0108	0.6032	**76** 00
10	.2447	3887	.9696	9866	.2524	4021	3.9617	5979	50
20	.2476	3937	.9689	9863	.2555	4074	3.9136	5926	40
30	.2504	3986	.9681	9859	.2586	4127	3.8667	5873	30
40	.2532	4035	.9674	9856	.2617	4178	3.8208	5822	20
50	.2560	4083	.9667	9853	.2648	4230	3.7760	5770	10
15 00	.2588	9.4130	.9659	9.9849	.2679	9.4281	3.7321	0.5719	**75** 00
10	.2616	4177	.9652	9846	.2711	4331	3.6891	5669	50
20	.2644	4223	.9644	9843	.2742	4381	3.6470	5619	40
30	.2672	4269	.9636	9839	.2773	4430	3.6059	5570	30
40	.2700	4314	.9628	9836	.2805	4479	3.5656	5521	20
50	.2728	4359	.9621	9832	.2836	4527	3.5261	5473	10
16 00	.2756	9.4403	.9613	9.9828	.2867	9.4575	3.4874	0.5325	**74** 00
10	.2784	4447	.9605	9825	.2899	4622	3.4495	5378	50
20	.2812	4491	.9596	9821	.2931	4669	3.4124	5331	40
30	.2840	4533	.9588	9817	.2962	4716	3.3759	5284	30
40	.2868	4576	.9580	9814	.2994	4762	3.3402	5238	20
50	.2896	4618	.9572	9810	.3026	4808	3.3052	5192	10
17 00	.2924	9.4659	.9563	9.9806	.3057	9.4853	3.2709	0.5147	**73** 00
10	.2952	4700	.9555	9802	.3089	4898	3.2371	5102	50
20	.2979	4741	.9546	9798	.3121	4943	3.2041	5057	40
30	.3007	4781	.9537	9794	.3153	4987	3.1716	5013	30
40	.3035	4821	.9528	9790	.3185	5031	3.1397	4969	20
50	.3062	4861	.9520	9786	.3217	5075	3.1084	4925	10
18 00	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	**72** 00
10	.3118	4939	.9502	9778	.3281	5161	3.0475	4839	50
20	.3145	4977	.9492	9774	.3314	5203	3.0178	4797	40
30	.3173	5015	.9483	9770	.3346	5245	2.9887	4755	30
40	.3201	5052	.9474	9765	.3378	5287	2.9600	4713	20
50	.3228	5090	.9465	9761	.3411	5329	2.9319	4671	10
19 00	.3256	9.5126	.9455	9.9757	.3443	9.5370	2.9042	0.4630	**71** 00
10	.3283	5163	.9446	9752	.3476	5411	2.8770	4589	50
20	.3311	5199	.9436	9748	.3508	5451	2.8502	4549	40
30	.3338	5235	.9426	9743	.3541	5491	2.8239	4509	30
40	.3365	5270	.9417	9739	.3574	5531	2.7980	4469	20
50	.3393	5306	.9407	9734	.3607	5571	2.7725	4429	10
20 00	.3420	9.5341	.9397	9.9730	.3640	9.5611	2.7475	0.4389	**70** 00
10	.3448	5375	.9387	9725	.3673	5650	2.7228	4350	50
20	.3475	5409	.9377	9721	.3706	5689	2.6985	4311	40
30	.3502	5443	.9367	9716	.3739	5727	2.6746	4273	30
40	.3529	5477	.9356	9711	.3772	5766	2.6511	4234	20
50	.3557	5510	.9346	9706	.3805	5804	2.6279	4196	10
21 00	.3584	9.5543	.9336	9.9702	.3839	9.5842	2.6051	0.4158	**69** 00
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
ANGLE	COSINE		SINE		COTANGENT		TANGENT		ANGLE

TABLE II.—*Continued*

ANGLE		SINE		COSINE		TANGENT		COTANGENT		ANGLE	
°	′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	°	′
21	00	.3584	9.5543	.9336	9.9702	.3839	9.5842	2.6051	0.4158	**69**	00
	10	.3611	5576	.9325	9697	.3872	5879	2.5826	4121		50
	20	.3638	5609	.9315	9692	.3906	5917	2.5605	4083		40
	30	.3665	5641	.9304	9687	.3939	5954	2.5386	4046		30
	40	.3692	5673	.9293	9682	.3973	5991	2.5172	4009		20
	50	.3719	5704	.9283	9677	.4006	6028	2.4960	3972		10
22	00	.3746	9.5736	.9272	9.9672	.4040	9.6064	2.4751	0.3936	**68**	00
	10	.3773	5767	.9261	9667	.4074	6100	2.4545	3900		50
	20	.3800	5798	.9250	9661	.4108	6136	2.4342	3864		40
	30	.3827	5828	.9239	9656	.4142	6172	2.4142	3828		30
	40	.3854	5859	.9228	9651	.4176	6208	2.3945	3792		20
	50	.3881	5889	.9216	9646	.4210	6243	2.3750	3757		10
23	00	.3907	9.5919	.9205	9.9640	.4245	9.6279	2.3559	0.3721	**67**	00
	10	.3934	5948	.9194	9635	.4279	6314	2.3369	3686		50
	20	.3961	5978	.9182	9629	.4314	6348	2.3183	3652		40
	30	.3987	6007	.9171	9624	.4348	6383	2.2998	3617		30
	40	.4014	6036	.9159	9618	.4383	6417	2.2817	3583		20
	50	.4041	6065	.9147	9613	.4417	6452	2.2637	3548		10
24	00	.4067	9.6093	.9135	9.9607	.4452	9.6486	2.2460	0.3514	**66**	00
	10	.4094	6121	.9124	9602	.4487	6520	2.2286	3480		50
	20	.4120	6149	.9112	9596	.4522	6553	2.2113	3447		40
	30	.4147	6177	.9100	9590	.4557	6587	2.1943	3413		30
	40	.4173	6205	.9088	9584	.4592	6620	2.1775	3380		20
	50	.4200	6232	.9075	9579	.4628	6654	2.1609	3346		10
25	00	.4226	9.6259	.9063	9.9573	.4663	9.6687	2.1445	0.3313	**65**	00
	10	.4253	6286	.9051	9567	.4699	6720	2.1283	3280		50
	20	.4279	6313	.9038	9561	.4734	6752	2.1123	3248		40
	30	.4305	6340	.9026	9555	.4770	6785	2.0965	3215		30
	40	.4331	6366	.9013	9549	.4806	6817	2.0809	3183		20
	50	.4358	6392	.9001	9543	.4841	6850	2.0655	3150		10
26	00	.4384	9.6418	.8988	9.9537	.4877	9.6882	2.0503	0.3118	**64**	00
	10	.4410	6444	.8975	9530	.4913	6914	2.0353	3086		50
	20	.4436	6470	.8962	9524	.4950	6946	2.0204	3054		40
	30	.4462	6495	.8949	9518	.4986	6977	2.0057	3023		30
	40	.4488	6521	.8936	9512	.5022	7009	1.9912	2991		20
	50	.4514	6546	.8923	9505	.5059	7040	1.9768	2960		10
27	00	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	**63**	00
	10	.4566	6595	.8897	9492	.5132	7103	1.9486	2897		50
	20	.4592	6620	.8884	9486	.5169	7134	1.9347	2866		40
	30	.4617	6644	.8870	9479	.5206	7165	1.9210	2835		30
	40	.4643	6668	.8857	9473	.5243	7196	1.9074	2804		20
	50	.4669	6692	.8843	9466	.5280	7226	1.8940	2774		10
28	00	.4695	9.6716	.8829	9.9459	.5317	9.7257	1.8807	0.2743	**62**	00
°	′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	°	′
ANGLE		COSINE		SINE		* COTANGENT		TANGENT		ANGLE	

TABLE II.—*Continued*

ANGLE		SINE		COSINE		TANGENT		COTANGENT		ANGLE	
°	′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	°	′
28	00	.4695	9.6716	.8829	9.9459	.5317	9.7257	1.8807	0.2743	**62**	00
	10	.4720	6740	.8816	9453	.5354	7287	1.8676	2713		50
	20	.4746	6763	.8802	9446	.5392	7317	1.8546	2683		40
	30	.4772	6787	.8788	9439	.5430	7348	1.8418	2652		30
	40	.4797	6810	.8774	9432	.5467	7378	1.8291	2622		20
	50	.4823	6833	.8760	9425	.5505	7408	1.8165	2592		10
29	00	.4848	9.6856	.8746	9.9418	.5543	9.7438	1.8040	0.2562	**61**	00
	10	.4874	6878	.8732	9411	.5581	7467	1.7917	2533		50
	20	.4899	6901	.8718	9404	.5619	7497	1.7796	2503		40
	30	.4924	6923	.8704	9397	.5658	7526	1.7675	2474		30
	40	.4950	6946	.8689	9390	.5696	7556	1.7556	2444		20
	50	.4975	6968	.8675	9383	.5735	7585	1.7437	2415		10
30	00	.5000	9.6990	.8660	9.9375	.5774	9.7614	1.7321	0.2386	**60**	00
	10	.5025	7012	.8646	9368	.5812	7644	1.7205	2356		50
	20	.5050	7033	.8631	9361	.5851	7673	1.7090	2327		40
	30	.5075	7055	.8616	9353	.5890	7701	1.6977	2299		30
	40	.5100	7076	.8601	9346	.5930	7730	1.6864	2270		20
	50	.5125	7097	.8587	9338	.5969	7759	1.6753	2241		10
31	00	.5150	9.7118	.8572	9.9331	.6009	9.7788	1.6643	0.2212	**59**	00
	10	.5175	7139	.8557	9323	.6048	7816	1.6534	2184		50
	20	.5200	7160	.8542	9315	.6088	7845	1.6426	2155		40
	30	.5225	7181	.8526	9308	.6128	7873	1.6319	2127		30
	40	.5250	7201	.8511	9300	.6168	7902	1.6212	2098		20
	50	.5275	7222	.8496	9292	.6208	7930	1.6107	2070		10
32	00	.5299	9.7242	.8480	9.9284	.6249	9.7958	1.6003	0.2042	**58**	00
	10	.5324	7262	.8465	9276	.6289	7986	1.5900	2014		50
	20	.5348	7282	.8450	9268	.6330	8014	1.5798	1986		40
	30	.5373	7302	.8434	9260	.6371	8042	1.5697	1958		30
	40	.5398	7322	.8418	9252	.6412	8070	1.5597	1930		20
	50	.5422	7342	.8403	9244	.6453	8097	1.5497	1903		10
33	00	.5446	9.7361	.8387	9.9236	.6494	9.8125	1.5399	0.1875	**57**	00
	10	.5471	7380	.8371	9228	.6536	8153	1.5301	1847		50
	20	.5495	7400	.8355	9219	.6577	8180	1.5204	1820		40
	30	.5519	7419	.8339	9211	.6619	8208	1.5108	1792		30
	40	.5544	7438	.8323	9203	.6661	8235	1.5013	1765		20
	50	.5568	7457	.8307	9194	.6703	8263	1.4919	1737		10
34	00	.5592	9.7476	.8290	9.9186	.6745	9.8290	1.4826	0.1710	**56**	00
	10	.5616	7494	.8274	9177	.6787	8317	1.4733	1683		50
	20	.5640	7513	.8258	9169	.6830	8344	1.4641	1656		40
	30	.5664	7531	.8241	9160	.6873	8371	1.4550	1629		30
	40	.5688	7550	.8225	9151	.6916	8398	1.4460	1602		20
	50	.5712	7568	.8208	9142	.6959	8425	1.4370	1575		10
35	00	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	**55**	00
°	′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	°	′
ANGLE		COSINE		SINE		COTANGENT		TANGENT		ANGLE	

TABLE II.—*Continued*

ANGLE	SINE		COSINE		TANGENT		COTANGENT		ANGLE
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
35 00	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	**55** 00
10	.5760	7604	.8175	9125	.7046	8479	1.4193	1521	50
20	.5783	7622	.8158	9116	.7089	8506	1.4106	1494	40
30	.5807	7640	.8141	9107	.7133	8533	1.4019	1467	30
40	.5831	7657	.8124	9098	.7177	8559	1.3934	1441	20
50	.5854	7675	.8107	9089	.7221	8586	1.3848	1414	10
36 00	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	**54** 00
10	.5901	7710	.8073	9070	.7310	8639	1.3680	1361	50
20	.5925	7727	.8056	9061	.7355	8666	1.3597	1334	40
30	.5948	7744	.8039	9052	.7400	8692	1.3514	1308	30
40	.5972	7761	.8021	9042	.7445	8718	1.3432	1282	20
50	.5995	7778	.8004	9033	.7490	8745	1.3351	1255	10
37 00	.6018	9.7795	.7986	9.9023	.7536	9.8771	1.3270	0.1229	**53** 00
10	.6041	7811	.7969	9014	.7581	8797	1.3190	1203	50
20	.6065	7828	.7951	9004	.7627	8824	1.3111	1176	40
30	.6088	7844	.7934	8995	.7673	8850	1.3032	1150	30
40	.6111	7861	.7916	8985	.7720	8876	1.2954	1124	20
50	.6134	7877	.7898	8975	.7766	8902	1.2876	1098	10
38 00	.6157	9.7893	.7880	9.8965	.7813	9.8928	1.2799	0.1072	**52** 00
10	.6180	7910	.7862	8955	.7860	8954	1.2723	1046	50
20	.6202	7926	.7844	8945	.7907	8980	1.2647	1020	40
30	.6225	7941	.7826	8935	.7954	9006	1.2572	0994	30
40	.6248	7957	.7808	8925	.8002	9032	1.2497	0968	20
50	.6271	7973	.7790	8915	.8050	9058	1.2423	0942	10
39 00	.6293	9.7989	.7771	9.8905	.8098	9.9084	1.2349	0.0916	**51** 00
10	.6316	8004	.7753	8895	.8146	9110	1.2276	0890	50
20	.6338	8020	.7735	8884	.8195	9135	1.2203	0865	40
30	.6361	8035	.7716	8874	.8243	9161	1.2131	0839	30
40	.6383	8050	.7698	8864	.8292	9187	1.2059	0813	20
50	.6406	8066	.7679	8853	.8342	9212	1.1988	0788	10
40 00	.6428	9.8081	.7660	9.8843	.8391	9.9238	1.1918	0.0762	**50** 00
10	.6450	8096	.7642	8832	.8441	9264	1.1847	0736	50
20	.6472	8111	.7623	8821	.8491	9289	1.1778	0711	40
30	.6494	8125	.7604	8810	.8541	9315	1.1708	0685	30
40	.6517	8140	.7585	8800	.8591	9341	1.1640	0659	20
50	.6539	8155	.7566	8789	.8642	9366	1.1571	0634	10
41 00	.6561	9.8169	.7547	9.8778	.8693	9.9392	1.1504	0.0608	**49** 00
10	.6583	8184	.7528	8767	.8744	9417	1.1436	0583	50
20	.6604	8198	.7509	8756	.8796	9443	1.1369	0557	40
30	.6626	8213	.7490	8745	.8847	9468	1.1303	0532	30
40	.6648	8227	.7470	8733	.8899	9494	1.1237	0506	20
50	.6670	8241	.7451	8722	.8952	9519	1.1171	0481	10
42 00	.6691	9.8255	.7431	9.8711	.9004	9.9544	1.1106	0.0456	**48** 00
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
ANGLE	COSINE		SINE		COTANGENT		TANGENT		ANGLE

TABLE II.—*Continued*

Angle	Sine		Cosine		Tangent		Cotangent		Angle
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
42 00	.6691	9.8255	.7431	9.8711	.9004	9.9544	1.1106	0.0456	48 00
10	.6713	8269	.7412	8699	.9057	9570	1.1041	0430	50
20	.6734	8283	.7392	8688	.9110	9595	1.0977	0405	40
30	.6756	8297	.7373	8676	.9163	9621	1.0913	0379	30
40	.6777	8311	.7353	8665	.9217	9646	1.0850	0354	20
50	.6799	8324	.7333	8653	.9271	9671	1.0786	0329	10
43 00	.6820	9.8338	.7314	9.8641	.9325	9.9697	1.0724	0.0303	47 00
10	.6841	8351	.7294	8629	.9380	9722	1.0661	0278	50
20	.6862	8365	.7274	8618	.9435	9747	1.0599	0253	40
30	.6884	8378	.7254	8606	.9490	9772	1.0538	0228	30
40	.6905	8391	.7234	8594	.9545	9798	1.0477	0202	20
50	.6926	8405	.7214	8582	.9601	9823	1.0416	0177	10
44 00	.6947	9.8418	.7193	9.8569	.9657	9.9848	1.0355	0.0152	46 00
10	.6967	8431	.7173	8557	.9713	9874	1.0295	0126	50
20	.6988	8444	.7153	8545	.9770	9899	1.0235	0101	40
30	.7009	8457	.7133	8532	.9827	9924	1.0176	0076	30
40	.7030	8469	.7112	8520	.9884	9949	1.0117	0051	20
50	.7050	8482	.7092	8507	.9942	9975	1.0058	0025	10
45 00	.7071	9.8495	.7071	9.8495	1.0000	0.0000	1.0000	0.0000	45 00
° ′	Nat.	Log.	Nat.	Log.	Nat.	Log.	Nat.	Log.	° ′
Angle	Cosine		Sine		Cotangent		Tangent		Angle

TABLE III.—THREE-PLACE NATURAL TRIGONOMETRIC FUNCTIONS

Deg.	sin	cos	tan	Deg.	sin	cos	tan
0	.000	1.000	.000	46	.719	.695	1.04
1	.017	.999	.017	47	.731	.682	1.07
2	.035	.999	.035	48	.743	.669	1.11
3	.052	.999	.052	49	.755	.656	1.15
4	.070	.998	.070	50	.766	.643	1.19
5	.087	.996	.087	51	.777	.629	1.23
6	.105	.995	.105	52	.788	.616	1.28
7	.122	.993	.123	53	.799	.602	1.33
8	.139	.990	.141	54	.809	.588	1.38
9	.156	.988	.158	55	.819	.574	1.43
10	.174	.985	.176	56	.829	.559	1.48
11	.191	.982	.194	57	.839	.545	1.54
12	.208	.978	.213	58	.848	.530	1.60
13	.225	.974	.231	59	.857	.515	1.66
14	.242	.970	.249	60	.866	.500	1.73
15	.259	.966	.268	61	.875	.485	1.80
16	.276	.961	.287	62	.883	.469	1.88
17	.292	.956	.306	63	.891	.454	1.96
18	.309	.951	.325	64	.899	.438	2.05
19	.326	.946	.344	65	.906	.423	2.14
20	.342	.940	.364	66	.914	.407	2.25
21	.358	.934	.384	67	.921	.391	2.36
22	.375	.927	.404	68	.927	.375	2.48
23	.391	.921	.424	69	.934	.358	2.61
24	.407	.914	.445	70	.940	.342	2.75
25	.423	.906	.466	71	.946	.326	2.90
26	.438	.899	.488	72	.951	.309	3.08
27	.454	.891	.510	73	.956	.292	3.27
28	.469	.883	.532	74	.961	.276	3.49
29	.485	.875	.554	75	.966	.259	3.73
30	.500	.866	.577	76	.970	.242	4.01
31	.515	.857	.601	77	.974	.225	4.33
32	.530	.848	.625	78	.978	.208	4.70
33	.545	.839	.649	79	.982	.191	5.14
34	.559	.829	.675	80	.985	.174	5.67
35	.574	.819	.700	81	.988	.156	6.31
36	.588	.809	.727	82	.990	.139	7.12
37	.602	.799	.754	83	.993	.122	8.14
38	.616	.788	.781	84	.995	.105	9.51
39	.629	.777	.810	85	.996	.087	11.4
40	.643	.766	.839	86	.998	.070	14.3
41	.656	.755	.869	87	.999	.052	19.1
42	.669	.743	.900	88	.999	.035	28.6
43	.682	.731	.933	89	.999	.017	57.3
44	.695	.719	.966	90	1.000	.000	Inf.
45	.707	.707	1.000				

TABLE IV.—DECIMAL EQUIVALENTS OF PARTS OF AN INCH

Fraction.	Decimal.	Fraction.	Decimal.	Fraction.	Decimal.
$\frac{1}{64}$	.01563	$\frac{21}{64}$	.32813	$\frac{45}{64}$	.70313
$\frac{1}{32}$	.03125	$\frac{11}{32}$	.34375	$\frac{23}{32}$	.71875
$\frac{3}{64}$	.04688	$\frac{23}{64}$	.35938	$\frac{47}{64}$	.73438
1-16	.0625	**3-8**	.375	**3-4**	.75
$\frac{5}{64}$	.07813	$\frac{25}{64}$	.39063	$\frac{49}{64}$	.76563
$\frac{3}{32}$	.09375	$\frac{13}{32}$	.40625	$\frac{25}{32}$	.78125
$\frac{7}{64}$	.10938	$\frac{27}{64}$	.42188	$\frac{51}{64}$	.79688
1-8	.125	**7-16**	.4375	**13-16**	.8125
$\frac{9}{64}$	.14063	$\frac{29}{64}$	.45313	$\frac{53}{64}$	.82813
$\frac{5}{32}$	.15625	$\frac{15}{32}$	.46875	$\frac{27}{32}$	.84375
$\frac{11}{64}$	.17188	$\frac{31}{64}$	.48438	$\frac{55}{64}$	.85938
3-16	.1875	**1-2**	.5	**7-8**	.875
$\frac{13}{64}$	.20313	$\frac{33}{64}$	.51563	$\frac{57}{64}$	.89063
$\frac{7}{32}$	.21875	$\frac{17}{32}$	.53125	$\frac{29}{32}$	.90625
$\frac{15}{64}$	.23438	$\frac{35}{64}$	.54688	$\frac{59}{64}$	.92188
1-4	.25	**9-16**	.5625	**15-16**	.9375
$\frac{17}{64}$	.26563	$\frac{37}{64}$	.57813	$\frac{61}{64}$	.95313
$\frac{9}{32}$	.28125	$\frac{19}{32}$	.59375	$\frac{31}{32}$	.96875
$\frac{19}{64}$	.29688	$\frac{39}{64}$	.60938	$\frac{63}{64}$	.98438
5-16	.3125	**5-8**	.625	**1**	1.00000
		$\frac{41}{64}$	.64063		
		$\frac{21}{32}$	.65625		
		$\frac{43}{64}$	.67188		
		11-16	.6875		

TABLE V.—WIRE GAUGE SIZES

IN DECIMALS OF AN INCH

Number of Gauge.	Birmingham or Stubs Iron Wire Gauge.	American or Brown & Sharpe Wire Gauge.	Washburn & Moen Manufacturing Co. and John A. Roebling's Sons Co. Wire Gauge.	Trenton Iron Co. Wire Gauge.	American Screw Co. Screw Wire Gauge.	British Imperial or English Legal Standard Wire. Gauge.
0000000	....	....	....	....	....	.500
000000	....	....	.4600	....	....	.464
00000	....	....	.4300	.450	....	.432
0000	.454	.460000	.3938	.400	....	.400
000	.425	.409642	.3625	.360	.0315	.372
00	.380	.364796	.3310	.330	.0447	.348
0	.340	.324861	.3065	.305	.0578	.324
1	.300	.289297	.2830	.285	.0710	.300
2	.284	.257627	.2625	.265	.0842	.276
3	.259	.229423	.2437	.245	.0973	.252
4	.238	.204307	.2253	.225	.1105	.232
5	.220	.181940	.2070	.205	.1236	.212
6	.203	.162023	.1920	.190	.1368	.192
7	.180	.144285	.1770	.175	.1500	.176
8	.165	.128490	.1620	.160	.1631	.160
9	.148	.114423	.1483	.145	.1763	.144
10	.134	.101897	.1350	.130	.1894	.128
11	.120	.090742	.1205	.1175	.2026	.116
12	.109	.080808	.1055	.105	.2158	.104
13	.095	.071962	.0915	.0925	.2289	.092
14	.083	.064084	.0800	.0806	.2421	.080
15	.072	.057068	.0720	.070	.2552	.072
16	.065	.050821	.0625	.061	.2684	.064
17	.058	.045257	.0540	.0525	.2816	.056
18	.049	.040303	.0475	.045	.2947	.048
19	.042	.035890	.0410	.040	.3079	.040
20	.035	.031961	.0348	.035	.3210	.036
21	.032	.028462	.03175	.031	.3342	.032
22	.028	.025346	.0286	.028	.3474	.028
23	.025	.022572	.0258	.025	.3605	.024
24	.022	.020101	.0230	.0225	.3737	.022
25	.020	.017900	.0204	.020	.3868	.020
26	.018	.015941	.0181	.018	.4000	.018
27	.016	.014195	.0173	.017	.4132	.0164
28	.014	.012641	.0162	.016	.4263	.0148
29	.013	.011257	.0150	.015	.4395	.0136
30	.012	.010025	.0140	.014	.4526	.0124
31	.010	.008928	.0132	.013	.4658	.0116
32	.009	.007950	.0128	.012	.4790	.0108
33	.008	.007080	.0118	.011	.4921	.0100
34	.007	.006305	.0104	.010	.5053	.0092
35	.005	.005615	.0095	.0095	.5184	.0084
36	.004	.005000	.0090	.009	.5316	.0076
37	....	.004453	.0085	.0085	.5448	.0068
38	....	.003965	.0080	.008	.5579	.0060
39	....	.003531	.0075	.0075	.5711	.0052
40	....	.003144	.0070	.007	.5842	.0048

From the Cambria Handbook.

TABLE VI.—MATERIALS OF CONSTRUCTION

No.	Material.	Specific Gravity.	Average Weight in Lbs.	
			Cu.in.	Cu.ft.
1	Aluminum, cast	2	.075	125
2	" hammered	2.7	.099	170.6
3	* Ash, American white	....	....	38
4	" red	....	....	40
5	Asphalt	1 to 1.8	....	87.3
6	* Beech	....	....	43
7	* Birch	....	....	45.5
8	Brass, cast	7.8 to 8.4	.29	504
9	" rolled	8.4	.303	524
10	Brick, best pressed	....	....	150
11	" common, hard	....	....	125
12	" soft	....	....	100
13	Brickwork, pressed brick	....	....	140
14	" medium brick	....	....	125
15	" ordinary brick	....	....	112
16	Bronze, copper 8, tin 1	8.5	.306	529
17	* Cedar, American	....	....	34.5
18	Cement, hydraulic, ground loose, Rosendale	....	....	56
19	Portland, loose	....	....	88
20	* Cherry	....	....	42
21	Copper, cast	8.6 to 8.8	.314	542
22	" rolled	8.8 to 9	.321	555
23	* Elm	....	....	35
24	Glass, common window	2.52	....	157
25	Granite	2.56 to 2.9	....	170
26	Gravel	....	....	117
27	* Hemlock	....	....	25
28	* Hickory	.85	....	53
29	Iron, cast	6.9 to 7.5	.26	450
30	" wrought	7.79	.278	480
31	Lead, commercial	11.38	....	709.5
32	Limestone and marble	2.46 to 2.84	....	164.4
33	Mahogany, Spanish	.85	....	53
34	" Honduras	.56	....	35

* Seasoned.

TABLE VI.—MATERIALS OF CONSTRUCTION—*Continued*

No.	Material.	Specific Gravity.	Average Weight in Lbs.	
			Cu.in.	Cu.ft.
35	*Maple		....	49
36	Mortar, hardened	1.4 to 1.9	....	103
37	*Oak, live	....	....	59
38	" red American	....	....	40
39	" white American	....	....	50
40	*Pine, white	....	....	25
41	" yellow, Northern	....	....	34
42	" " Southern	....	....	45
43	*Poplar	....	....	39
44	Sand, quartz	2.75	....	90 to 106
45	" wet	....	....	120 to 140
46	" well shaken	....	....	99 to 117
47	Sandstone, building	2.41	....	151
48	Slate, American	2.7 to 2.9	....	175
49	*Spruce	....	....	25
50	Steel, structural	7.85	.28	490
51	*Walnut, black	....	....	38
52	Water	1	....	†62.355
53	Zinc, cast	6.9	....	376
54	" rolled	7.2	....	449

* Seasoned. † At 62° Fahrenheit, barometer 30 inches.

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